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I_2-statistical convergence of double sequences in 2-normed spaces

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Abstract

In this paper we introduce $\mathcal{I}_2$-statistical convergence and $\mathcal{I}_2$-statistical Cauchy of double sequences in 2-normed spaces and investigate their relationship.

1 Introduction

The concept of statistical convergence for sequences of real numbers was introduced by Fast [7] in 1951 and then several generalizations and applications of this notion have been investigated by various authors (see [15,6,1]). $\mathcal{I}$-convergence was introduced by Kostyrko et al [3] as a generalization of statistical convergence which is based on the structure of ideal I of subset of the set of natural numbers [17,18,4]. Recently Sahiner et al [19] and Gürdal and Acik [13] studied ideal convergence and I-Cauchy sequences respectively in 2-normed spaces and Dündar [5] studied the concepts of I_2-Cauchy for double sequence in 2-normed spaces. Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers. If E is a subset of positive integers $\mathbb{N}$ and $j \in \mathbb{N}$,

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then the quotient $d_j(E) = \frac{card(E \bigcap \{1,...,j\})}{j}$ is called the j'th *partial density* of E. Note that d_j is a probability measure on $\mathcal{P}(\mathbb{N})$, with support $\{1,...,j\}$. $d(E) = \lim_{j\to\infty} d_j(E)$ (if it exists) is called the *natural density* of $E \subseteq \mathbb{N}$ [8]. Recall that a sequence $x = (x_k)$ of elements of X is said to be *statistically convergent* to $l \in X$ if the set $A(\epsilon) = \{k \in \mathbb{N} : \|x_k - l\| \geq \epsilon\}$ for each $\epsilon > 0$ has natural density zero. In other words, for each $\varepsilon > 0$,

$$\lim_n \frac{1}{n} card(\{k \leq n : \|x_k - l\| \geq \varepsilon\}) = 0$$

We write

$$st - \lim x = l$$

Now, we recall some basic definitions about ideals.
A family $\mathcal{I} \subseteq \mathcal{P}(Y)$ of subsets of a nonempty set Y is said to be an ideal in Y if:
i) $\emptyset \in \mathcal{I}$
ii) $A, B \in \mathcal{I} \Rightarrow A \bigcup B \in \mathcal{I}$
iii) $A \in \mathcal{I}, B \subseteq A \Rightarrow B \in \mathcal{I}$
$\mathcal{I}$ is called a *nontrivial ideal* if $X \notin \mathcal{I}$. A nontrivial ideal $\mathcal{I}$ in Y is called *admissible* if $\{x\} \in \mathcal{I}$ for each $x \in Y$. A nontrivial ideal $\mathcal{I}$ in $\mathbb{N} \times \mathbb{N}$ is called *strongly admissible* if $\{i\} \times \mathbb{N}$ and $\mathbb{N} \times \{i\}$ belong to $\mathcal{I}$ for each $i \in \mathbb{N}$. It is evident that a strongly admissible ideal is admissible also. Let $\mathcal{I}_0$=$\{A \subset \mathbb{N} \times \mathbb{N} : (\exists m(A) \in \mathbb{N})(i, j \geq m(A) \Rightarrow (i, j) \in \mathbb{N} \times \mathbb{N} - A)\}$. Then $\mathcal{I}_0$ is a nontrivial strongly admissible ideal and clearly an ideal $\mathcal{I}$ is strongly admissible if and only if $\mathcal{I}_0 \subseteq \mathcal{I}$.

Definition 1.1. *A double sequence $x = (x_{ij})$ in X is said to be $\mathcal{I}_2$-convergent to $l \in X$ if for all $\varepsilon > 0$ we have*

$$A(\epsilon) = \{(i, j) \in \mathbb{N} \times \mathbb{N} : \|x_{ij} - l\| \geq \epsilon\} \in \mathcal{I}_2$$

In this case, we say that x is $\mathcal{I}_2$-convergent and write it as $\mathcal{I}_2 - \lim x_{ij} = l$.

The notion of linear 2-normed spaces has been investigated by Gâhler in 1960's [9] and has been developed extensively in different subjects by others [10,12,16].

Definition 1.2. *Let X be a real linear space of dimension greater than 1 and let $\|.,.\|$ be a non-negative real-valued function on $X \times X$ satisfying the following conditions:*
G1) $\|x, y\| = 0$ if and only if x and y are linearly dependent vectors.
G2) $\|x, y\| = \|y, x\|$ for all x, y in X.
G3) $\|\alpha x, y\| = |\alpha|\|x, y\|$ where α is real.
G4) $\|x + y, z\| \leq \|x, z\| + \|y, z\|$ for all x, y, z in X

$\|.,.\|$ is called a 2-norm on X and the pair $(X, \|.,.\|)$ is called a linear 2-normed space.
Every linear 2-normed space $(X, \|.,.\|)$ of dimension different from one is a locally convex topological vector space. In fact, for a fixed $b \in X$, $p_b(x) = \|x, b\|, x \in X$, is a seminorm and the family $P = \{p_b : b \in X\}$ of seminorms generates a locally convex topology on X.

Some of the basic properties of 2-norm were introduced in [16,2].

Lemma 1.3. *[12] Let $v = \{v_1, ..., v_k\}$ be a basis of X. A double sequence $(x_{mn})_{m,n\in\mathbb{N}}$ in X is convergent to x in X if and only if $\lim_{m,n\to\infty} \|x_{mn} - x, v_i\| = 0$ for every $i = 1, ..., k$.*

We can define the norm $\|.\|_\infty$ on X by

$$\|x\|_\infty := \max\{\|x, v_i\| : i = 1, ..., k\}.$$

Lemma 1.4. *[12] A double sequence $(x_{mn})_{m,n\in\mathbb{N}}$ in X is convergent to x in X if and only if $\lim_{m,n\to\infty} \|x_{mn} - x\|_\infty = 0$*

Example 1.5. Let $X = \mathbb{R}^2$ be equipped with the 2-norm $\|x, y\|:=$ the area of the parallelogram determined by the vectors x and y, which may be given explicitly by the formula

$$\|x, y\| = |x_1y_2 - x_2y_1| \quad , \quad x = (x_1, x_2) \quad y = (y_1, y_2).$$

Take the standard basis $\{i, j\}$ for $\mathbb{R}^2$.
Then, $\|x, i\| = |x_2|$ and $\|x, j\| = |x_1|$, and so the derived norm $\|.\|_\infty$ with respect to $\{i, j\}$ is

$$\|x\|_\infty = \max\{|x_1|, |x_2|\}, \qquad x = (x_1, x_2).$$

Thus, here the derived norm $\|.\|_\infty$ is exactly the uniform norm on $\mathbb{R}^2$. Thus the derived norm is is equivalent to Euclidean norm on $\mathbb{R}^2$.

In [15], Mursaleen introduced the concept of statistically convergent and Das [3] defined $\mathcal{I}$-convergent of a double sequence in a 2-normed space.

Definition 1.6. *Let $(X, \|.,.\|)$ be 2-normed space. The sequence $(x_{ij})_{i,j}$ is statistically convergent to a number l if for each $\varepsilon > 0$ and for every $z \in X$,*

$$\lim_{n,m} \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l, z\| \geq \varepsilon\}) = 0$$

Definition 1.7. *Let $(X, \|.,.\|)$ be 2-normed space. The sequence $(x_{ij})_{i,j}$ is $\mathcal{I}_2$-convergence to l if for all non zero $z \in X$ and for each $\varepsilon > 0$*

$$A(\varepsilon, z) = \{(i, j) \in \mathbb{N} \times \mathbb{N} : \|x_{ij} - l, z\| \geq \varepsilon\} \in \mathcal{I}_2.$$

2 $\mathcal{I}_2$-statistical convergence of double sequences in 2-normed spaces

In [22], Gürdal introduced the concepts of $\mathcal{I}$-statistical convergence and $\mathcal{I}$-statistically Cauchy sequence for a single sequence in a 2-normed space. In this section we introduce the notion of $\mathcal{I}_2$-statistical convergence double sequences in 2-normed spaces. Throughout the paper we assume X to be a 2-normed space.

Definition 2.1. *Let $\mathcal{I}_2$ be a nontrivial ideal in $\mathbb{N} \times \mathbb{N}$. The sequence (x_{ij}) in X is said to be $\mathcal{I}_2$-statistically convergent to l in X if for each $\varepsilon > 0$, $\delta > 0$ and nonzero $z \in X$ we have*

$$\{(m, n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l, z\| \geq \varepsilon\}) \geq \delta\} \in \mathcal{I}_2$$

or, in other words, for each $\varepsilon > 0$,

$$A_{mn}(\varepsilon, z) = \{i \leq m, j \leq n : \|x_{ij} - l, z\| \geq \varepsilon\}, \delta_2(A_{mn}(\varepsilon, z)) = \frac{card(A_{mn}(\varepsilon, z))}{mn}$$

for nonzero $z \in X$,

$$\mathcal{I}_2 - \lim_{n,m} \delta_2(A_{mn}(\varepsilon, z)) = 0$$

We write $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij} - l, z\| = 0$ *or* $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij}, z\| = \|l, z\|$. *The number* l *is called* $\mathcal{I}_2 - \lim$ *of the sequence* (x_{ij}).

Remark 2.2. Note that if $z = 0$, then the above set is empty.

Theorem 3.1. Let $x = (x_{ij})_{i,j\in\mathbb{N}}$ be an $\mathcal{I}_2$-statistically convergence double sequence in 2-normed space X and $l_1, l_2 \in X$. If $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij}, z\| = \|l_1, z\|$ and $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij}, z\| = \|l_2, z\|$, then $l_1 = l_2$.

Proof. Suppose, to get a contradiction, that $l_1 \neq l_2$. Then there exists a nonzero $z \in X$ such that $l_1 - l_2$ and z are linearly independent. Let $\varepsilon > 0$ and $\delta > 0$ be given.

$$\frac{1}{mn} card(\{i \leq m, j \leq n : \|l_1 - l_2, z\| \geq \varepsilon\}) = 2\delta$$

we have:

$$\|l_1 - l_2, z\| = \|l_1 - x_{ij} + x_{ij} - l_2, z\| \leq \|x_{ij} - l_1, z\| + \|x_{ij} - l_2, z\|$$

$2\delta = \frac{1}{mn} card(\{i \leq m, j \leq n : \|l_1 - x_{ij} + x_{ij} - l_2, z\|) \geq \varepsilon\} \leq$
$\frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l_1, z\| \geq \varepsilon\}) +$
$\frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l_2, z\|) \geq \varepsilon\})$
Thus for each nonzero $z \in X$
$\{(m, n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l_1, z\| \geq \varepsilon\}) \prec \delta\} \subseteq \{(m, n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l_2, z\| \geq \varepsilon\}) \geq \delta\}$. Since $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij}, z\| = \|l_1, z\|$, we have

$$\delta_{\mathcal{I}_2}(\{(m, n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l_2, z\| \geq \varepsilon\}) \geq \delta\}) = 0$$

which contradicts $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij}, z\| = \|l_2, z\|$ and so $l_1 = l_2$.

Corollary 2.3. *If* $(x_{ij})_{i,j\in\mathbb{N}}$,$(y_{ij})_{i,j\in\mathbb{N}}$ *are double sequences in a 2-normed space* $(X, \|.,.\|)$ *and* $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij}, z\| = \|a, z\|, \mathcal{I}_2 - st_2 - \lim_{i,j} \|y_{ij}, z\| = \|b, z\|$ *then*

(i)$\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij} + y_{ij}, z\| = \|a + b, z\|$
(ii)$\mathcal{I}_2 - st_2 - \lim_{i,j} \|\alpha x_{ij}, z\| = \|\alpha a, z\|$, *where* $\alpha \in \mathbb{R}$
proof.(i) *For each nonzero* $z \in X$, *by assumption, we have* $\delta_{\mathcal{I}_2}(A_1) = 0$ *and* $\delta_{\mathcal{I}_2}(A_2) = 0$

$$A_1 = A_1(\varepsilon, z) := \{(m,n) \in \mathbb{N}\times\mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij}-a, z\| \geq \varepsilon\}) \geq \frac{\delta}{2}\}$$

$$A_2 = A_2(\varepsilon, z) := \{(m,n) \in \mathbb{N}\times\mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|y_{ij}-b, z\| \geq \varepsilon\}) \geq \frac{\delta}{2}\}$$

it is clear that

$$A := \{(m,n) \in \mathbb{N}\times\mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|(x_{ij}+y_{ij})-(a+b), z\| \geq \varepsilon\}) \geq \delta\} \subseteq A_1 \bigcup A_2$$

Since $\delta_{\mathcal{I}_2}(A_1 \bigcup A_2) = 0$, $\delta_{\mathcal{I}_2}(A) = 0$ *and this completes the proof.*
(ii)*Assume that* $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij}, z\| = \|a, z\|$. *Thus*

$$\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - a, z\| \geq \frac{\varepsilon}{\alpha}\}) \geq \delta\} \in \mathcal{I}_2$$

To prove $\mathcal{I}_2 - st_2 - \lim_{i,j} \|\alpha x_{ij}, z\| = \|\alpha a, z\|$, *it suffices to show that*

$$\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|\alpha x_{ij} - \alpha a, z\| \geq \varepsilon\}) \geq \delta\} \in \mathcal{I}_2$$

Hence, by the above equality we get for every nonzero $z \in X$,

$$\mathcal{I}_2 - st_2 - \lim_{i,j} \|\alpha x_{ij}, z\| = \|\alpha a, z\|.$$

Lemma 2.4. *Let* $V = \{v_1, ..., v_k\}$ *be a basis of* X. *A double sequence* $(x_{ij})_{i,j\in\mathbb{N}}$ *is* $\mathcal{I}_2$*-statistically convergent to* l *in* X *if and only if* $\mathcal{I}_2 - st_2 - \lim_{i,j\to\infty} \|x_{ij} - l, v_n\| = 0$ *for every* $n = 1, ..., k$.
Proof. *If* $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij}, z\| = \|l, z\|$, *then this is clear by definition. Now it suffices to prove that for every nonzero* $z \in X$, *we have* $\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij} - l, z\| = 0$. *For every nonzero* $z \in X$, *there exists* $\alpha_1, \ldots, \alpha_k \in \mathbb{R}$ *such that*

$z = \alpha_1 v_1 + \ldots + \alpha_k v_k$ and, by the definition of the 2-norm, we have

$$\|x_{ij} - l, z\| \leq |\alpha_1| \, \|x_{ij} - l, v_1\| + \ldots + |\alpha_k| \, \|x_{ij} - l, v_k\|.$$

Then
$\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l, z\| \geq \varepsilon\}) \geq \delta\} \subseteq$
$\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l, v_1\| \geq \frac{\varepsilon}{|\alpha_1|}\}) \geq \delta\}$
$\cup \ldots \cup \{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn} card(\{i \leq m, j \leq n : \|x_{ij} - l, v_k\| \geq \frac{\varepsilon}{|\alpha_k|}\}) \geq \delta\}$
Since the right hand side of the above inclusion belongs to $\mathcal{I}_2$, the left hand side belongs to $\mathcal{I}_2$ and this completes the proof.

Lemma 2.5. *Let $V = \{v_1, ..., v_k\}$ be a basis of X. A double sequence $(x_{ij})_{i,j \in \mathbb{N}}$ is $\mathcal{I}_2$-statistically convergent to l in X if and only if $\mathcal{I}_2 - st_2 - \lim_{i,j \to \infty} \max\{\|x_{ij} - l, v_n\| = 0, n = 1, ..., k\} = 0$.*

Using the definition of the norm $\|.\|_\infty$ on $X : \|x\|_\infty := \max\{\|x, v_i\| : i = 1, ..., k\}$ we have a similar lemma:

Lemma 2.6. *Let $V = \{v_1, ..., v_k\}$ be a basis of X. A double sequence $(x_{ij})_{i,j \in \mathbb{N}}$ is $\mathcal{I}_2$-statistically convergent to l in X if and only if $\mathcal{I}_2 - st_2 - \lim_{i,j \to \infty} \|x_{ij} - l, v_n\|_\infty = 0$ for every $n = 1, ..., k$.*

Associated to the derived norm $\|.\|_\infty$, define the open balls

$$B_V(x, \varepsilon) := \{y : \|x - y\|_\infty := \max\{\|x - y, v_i\| : i = 1, ..., k\} \prec \varepsilon\}$$

Using these balls, we get the following Lemma:

Lemma 2.7. *Let V be a basis of X. A double sequence $(x_{ij})_{i,j \in \mathbb{N}}$ is $\mathcal{I}_2$-statistically convergent to l in X if and only if $\delta_{\mathcal{I}_2}(A_2(\varepsilon, z)) = 0$, where $A_2(\varepsilon, z) = \{i \leq m, j \leq n : x_{ij} \notin B_V(x, \varepsilon)\}$.*

Now, we define the notion of $\mathcal{I}_2$-statistically Cauchy double sequence in 2-normed space.

Definition 2.8. *Let $\mathcal{I}_2$ be a nontrivial ideal in $\mathbb{N}\times\mathbb{N}$. The sequence (x_{ij}) in X is said to be $\mathcal{I}_2$-statistically Cauchy in X if for each $\varepsilon > 0$, $\delta > 0$ and nonzero $z \in X$ there exist p , q such that*

$$\{(m,n) \in \mathbb{N}\times\mathbb{N} : \frac{1}{mn} card(\{i \le m, j \le n : \|x_{ij} - x_{pq}, z\| \ge \varepsilon\}) \ge \delta\} \in \mathcal{I}_2.$$

Theorem 3.2. Any $\mathcal{I}_2$-statistically Cauchy double sequence in a 2-normed space X is $\mathcal{I}_2$-statistically convergent if and only if every $\mathcal{I}_2$-statistically Cauchy double sequence with norm $\|.\|_\infty$ is $\mathcal{I}_2$-statistically convergent.

Proof. By the above lemma we have

$$\mathcal{I}_2 - st_2 - \lim_{i,j\to\infty} \|x_{ij} - l, z\| = 0 \quad \Longleftrightarrow \quad \mathcal{I}_2 - st_2 - \lim_{i,j\to\infty} \|x_{ij} - l, z\|_\infty = 0.$$

It suffices to show that the double sequence (x_{ij}) is $\mathcal{I}_2$-statistically Cauchy in 2- normed space if and only if it is $\mathcal{I}_2$-statistically Cauchy with norm $\|.\|_\infty$.
But (x_{ij}) is $\mathcal{I}_2$-statistically Cauchy in 2-norm if and only if
$\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij} - x_{pq}, z\| = 0$ for every non zero $z \in X$ if and only if
$\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij} - x_{pq}, v_n\| = 0$ for every $n = 1, ..., k$ if and only if
$\mathcal{I}_2 - st_2 - \lim_{i,j} \|x_{ij} - x_{pq}\|_\infty = 0$ if and only if
(x_{ij}) is $\mathcal{I}_2$-statistically Cauchy double sequence with norm $\|.\|_\infty$.

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