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Intuitionistic Fuzzy k-Γ-Hyperideals of Γ-Semihyperrings

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Abstract

We introduce the concept of intuitionistic fuzzy k-Γ-hyperideals of Γ-semihyperrings. Then, we investigate some fundamental properties of intuitionistic fuzzy k-Γ-hyperideals of Γ-semihyperrings.

1 Introduction

The fuzzy set was introduced by Zadeh [26] in 1965, as a function from a nonempty set X to the unit interval $[0, 1]$. Later, many researchers have researched on the generalization of the notion of fuzzy sets with applications in computer, logic and many ramifications of pure and applied mathematics. Atanassov [4] introduced and studied the concept of an intuitionistic fuzzy set which is a generalization of the notion of the fuzzy set. Namely, the fuzzy sets give the degree of membership of an element in a given set, while the intuitionistic fuzzy sets give both a degree of membership and a degree of nonmembership.

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The concept of a hyperstructure was first introduced by Marty [16] in 1934, as a generalization of ordinary algebraic structures. This theory was studied in the following decades and nowadays by many mathematicians, e.g., [5, 6, 7, 25]. Vougiouklis [24] introduced the concept of a semihyperring, as a generalization of a semiring, where both the addition and the multiplication are hyperoperations.

The notion of a Γ-ring was introduced by Nobusawa [17]. Then, Rao [19] introduced the idea of a Γ-semiring which is a generalization of a Γ-ring as well as of a semiring. Later, Dehkori and Davvaz [8, 9, 10] introduced and studied the concept of a Γ-semihyperring as a generalization of a semiring, of a Γ-semiring and of a semihyperring.

Henriksen [15] defined a more restricted class of ideals in a semiring, which is called k-ideals. Then, Sen and Adhikari [21, 22] studied on k-ideals of semirings. Akram and Dudek [2] introduced the notion of an intuitionistic fuzzy left k-ideal in a semiring. Also, Dheena and Mohanraaj [11] studied the concept of an intuitionistic fuzzy k-ideal of a semiring. The concept of a k-hyperideal in a semihyperring was studied by Ameri and Hedayati [3, 13, 14], where it has been discussed only the addition as a hyperoperation. Omidi and Davvaz [18] introduced the notion of a k-hyperideal on an ordered semihyperring. In 2013, Ersoy and Davvaz [12] introduced the notion of an intuitionistic fuzzy Γ-hyperideal of a Γ-semihyperring and investigated some of its properties.

In this work, we introduce the notion of an intuitionistic fuzzy k-Γ-hyperideal of a Γ-semihyperring and study some basic of its properties.

2 Preliminaries

Let H be a nonempty set, and $\mathcal{P}^*(H)$ be the set of all nonempty subsets of H. A *hyperoperation* on H is a mapping $\circ : H \times H \to \mathcal{P}^*(H)$ (e.g., [5, 6, 7, 25]). The hyperstructure $(H, \circ)$ is called a *hypergroupoid*. If $x \in H$ and $A, B \in \mathcal{P}^*(H)$, then we denote

$$A \circ B = \bigcup_{a\in A, b\in B} a \circ b, A \circ \{x\} = A \circ x, \{x\} \circ B = x \circ B.$$

A hypergroupoid $(H, \circ)$ is called a *semihypergroup* if for all $x, y, z \in H$, we have $(x \circ y) \circ z = x \circ (y \circ z)$. That is, $\bigcup_{u\in x\circ y} u \circ z = \bigcup_{v\in y\circ z} x \circ v$. A semihypergroup $(H, \circ)$ is called *commutative* if $x \circ y = y \circ x$, for all $x, y \in H$.

Let $(S, +)$ be a commutative semihypergroup, and $(\Gamma, +)$ be a commutative semigroup. Then S is called a Γ-*semihyperring* (see, [8, 9, 10]) if there exists a map $S \times \Gamma \times S \to \mathcal{P}^*(S)$ (the image of (a, α, b) is denoted by $a\alpha b$, for all $a, b \in S$ and $\alpha \in \Gamma$) which, for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$, satisfying the following conditions:

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$;

(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$;

(iii) $x(\alpha + \beta)z = x\alpha z + x\beta z$;

(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

A nonempty subset I of a Γ-semihyperring S is called a *left* (resp. *right*) Γ-*hyperideal* of S if it satisfies (i) $x+y \subseteq I$ and (ii) $s\alpha x \subseteq I$ (resp. $x\alpha s \subseteq I$), for all $x, y \in I, s \in S$ and $\alpha \in \Gamma$. We call I a Γ-*hyperideal* of a Γ-semihyperring S if it is both a left and a right Γ-hyperideal of S.

A left (resp. right) Γ-hyperideal I of a Γ-semihyperring S is called a *left* (resp. *right*) k-Γ-*hyperideal* of S if for any $a \in I, x \in S, x + a \subseteq I$ implies $x \in I$. We note that I is called a k-Γ-*hyperideal* of a Γ-semihyperring S if it is both a left and a right k-Γ-hyperideal of S.

Example 2.1. *Let $S = \{a, b, c, d\}$ and $\Gamma = \{\alpha, \beta, \gamma\}$ with the hyperoperation $\oplus$ and $x\delta y$, for every $x, y \in S$ and $\delta \in \Gamma$ are defined as follows:*

$\oplus$	a	b	c	d
a	$\{a\}$	$\{a,b\}$	$\{a,b,c\}$	S
b	$\{a,b\}$	$\{a,b\}$	$\{a,b,c\}$	S
c	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	S
d	S	S	S	S

α	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
c	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
d	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$

β	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
c	$\{a\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,b,c\}$
d	$\{a\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,b,c\}$

γ	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
c	$\{a\}$	$\{a,b\}$	$\{a,b,c\}$	$\{a,b,c\}$
d	$\{a\}$	$\{a,b\}$	$\{a,b,c\}$	S

Define the operation $+$ on Γ by

$+$	α	β	γ
α	α	α	α
β	α	β	β
γ	α	β	γ

Now, $(\Gamma, +)$ is a commutative semigroup and then S is a Γ-semihyperring. We can show that $\{a, b\}$ is a k-Γ-hyperideal of S.

Obviously, every k-Γ-hyperideal of a Γ-semihyperring S is a Γ-hyperideal of S, but the converse is not true as the following example shows.

Example 2.2. *Let $S = \{a, b, c, d\}$ and $\Gamma = \{\alpha, \beta, \gamma\}$ with the hyperoperation $\oplus$ and $x\delta y$, for every $x, y \in S$ and $\delta \in \Gamma$ are defined as follows:*

$\oplus$	a	b	c	d
a	$\{a\}$	$\{b\}$	$\{c,d\}$	$\{d\}$
b	$\{b\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$
c	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$
d	$\{d\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$

α	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
d	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$

β	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{b\}$	$\{c,d\}$	$\{c,d\}$
c	$\{a\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$
d	$\{a\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$

γ	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$
c	$\{a\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$
d	$\{a\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$

Define the operation $+$ on Γ by

$+$	α	β	γ
α	α	α	α
β	α	β	γ
γ	α	γ	γ

Now, $(\Gamma, +)$ is a commutative semigroup and then S is a Γ-semihyperring. It easy to see that $\{a, c, d\}$ is a Γ-hyperideal of S, but it is not a k-Γ-hyperideal of S, since $b \oplus c = \{c, d\} \subseteq \{a, c, d\}$ but $b \notin \{a, c, d\}$.

Now, we review the notion of fuzzy sets defined by Zadeh [26]. Let X be a nonempty set. A *fuzzy set* of X is a mapping $\mu : X \to [0, 1]$. Let μ be a fuzzy set of X. The set $U(\mu; t) = \{x \in X \mid \mu(x) \geq t\}$ is called an *upper level set* of μ, and the set $L(\mu; t) = \{x \in X \mid \mu(x) \leq t\}$ is called a *lower level set* of μ, where $t \in [0,1]$. The *complement* of μ denoted by μ^c, is the fuzzy set of X defined by $\mu^c(x) = 1 - \mu(x)$, for all $x \in X$. The intersection and the union of two fuzzy sets μ and λ of X, denoted by $\mu \cap \lambda$ and $\mu \cup \lambda$, resp., are defined by letting $x \in X$,

$$(\mu \cap \lambda)(x) = \min\{\mu(x), \lambda(x)\} \text{ and } (\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}.$$

Atanassov [4] introduced the concept of an intuitionistic fuzzy set, which is extension of a fuzzy set. An *intuitionistic fuzzy set* A in a nonempty set X is defined as the form $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$ where $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the *degree of membership* and the *degree of nonmembership*, resp., of each $x \in X$ to the set A and also $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$, for all $x \in X$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ instead of the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$.

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic fuzzy sets in a nonempty set X. We define

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$, for all $x \in X$,

(ii) $A \cap B = (\mu_A \cap \mu_B, \lambda_A \cup \lambda_B)$.

An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ of a Γ-semihyperring S is called an *intuitionistic fuzzy left* (resp. *right*) *Γ-hyperideal* (see, [1, 12]) of S which, for all $x, y \in S$ and $\alpha \in \Gamma$, if it satisfies the following axiom:

(i) $\inf\limits_{z \in x+y} \mu_A(z) \geq \min\{\mu_A(x), \mu_A(y)\}$;

(ii) $\inf\limits_{z \in x\alpha y} \mu_A(z) \geq \mu_A(y)$ (resp. $\inf\limits_{z \in x\alpha y} \mu_A(z) \geq \mu_A(x)$);

(iii) $\sup\limits_{z \in x+y} \lambda(z) \leq \max\{\lambda(x), \lambda(y)\}$;

(iv) $\sup\limits_{z \in x\alpha y} \lambda(z) \leq \lambda(y)$ (resp. $\sup\limits_{z \in x\alpha y} \lambda(z) \leq \lambda(x)$),

We call that $A = (\mu_A, \lambda_A)$ is an *intuitionistic fuzzy Γ-hyperideal* of S if it is both an intuitionistic fuzzy left Γ-hyperideal and an intuitionistic fuzzy right Γ-hyperideal of S.

3 Intuitionistic Fuzzy k-Γ-Hyperideals

In this section, we introduce the concept of intuitionistic fuzzy k-Γ-hyperideals of Γ-semihyperrings and investigate some of their fundamental properties.

Definition 3.1. *An intuitionistic fuzzy left (resp. right) Γ-hyperideal $A = (\mu_A, \lambda_A)$ of a Γ-semihyperring S is said to be an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S if for any $x, y \in S$,*

$$\text{(i)} \quad \mu_A(x) \geq \min\{\inf_{z \in x+y} \mu_A(z), \mu_A(y)\}$$

and

$$\text{(ii)} \quad \lambda_A(x) \leq \max\{\sup_{z \in x+y} \lambda_A(z), \lambda_A(y)\}.$$

$A = (\mu_A, \lambda_A)$ is called an intuitionistic fuzzy k-Γ-hyperideal of a Γ-semihyperring S if it is both an intuitionistic fuzzy left k-Γ-hyperideal and an intuitionistic fuzzy right k-Γ-hyperideal of S.

Example 3.2. *In Example 2.1, we define an intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ of S by for every $x \in S$,*

$$\mu_A(x) = \begin{cases} 0.7 & \text{if } x \in \{a, b\}, \\ 0.2 & \text{otherwise} \end{cases} \quad \text{and} \quad \lambda_A(x) = \begin{cases} 0.2 & \text{if } x \in \{a, b\}, \\ 0.7 & \text{otherwise.} \end{cases}$$

By routine calculations, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy k-Γ-hyperideal of S.

We note that every intuitionistic fuzzy k-Γ-hyperideal is an intuitionistic fuzzy Γ-hyperideal. But every intuitionistic fuzzy Γ-hyperideal need not to be an intuitionistic fuzzy k-Γ-hyperideal as the following example shows.

Example 3.3. *In Example 2.2, we define an intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ of S by for every $x \in S$,*

$$\mu_A(x) = \begin{cases} 0.8 & \text{if } x \in \{a, c, d\}, \\ 0.1 & \text{otherwise} \end{cases} \quad \text{and} \quad \lambda_A(x) = \begin{cases} 0.1 & \text{if } x \in \{a, c, d\}, \\ 0.8 & \text{otherwise,} \end{cases}$$

By routine computations, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy Γ-hyperideal of S, but it is not an intuitionistic fuzzy k-Γ-hyperideal, because $\mu_A(b) < \min\{\inf_{z \in b \oplus c} \mu_A(z), \mu_A(c)\}$ and $\lambda_A(b) > \max\{\sup_{z \in b \oplus c} \lambda_A(z), \lambda_A(c)\}$.

Throughout this section, we will prove the following theorems for intuitionistic fuzzy left k-Γ-hyperideals, for intuitionistic fuzzy right k-Γ-hyperideals, one can prove similarly.

Theorem 3.4. *Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of a Γ-semihyperring S. Then A is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S if and only if the subsets $U(\mu_A; t)$ and $L(\lambda_A; s)$ are left (resp. right) k-Γ-hyperideals of S for all $s, t \in [0, 1]$, whenever they are nonempty.*

Proof. Assume that A is an intuitionistic fuzzy left k-Γ-hyperideal of S. Then A is also an intuitionistic fuzzy left Γ-hyperideal of S. By Theorem 16 in [12], $U(\mu_A; t)$ and $L(\lambda_A; s)$ are left Γ-hyperideals of S for all $s, t \in [0, 1]$. Let $a \in U(\mu_A; t)$ and $x \in S$ with $x+a \subseteq U(\mu_A; t)$. Then $\mu_A(a) \geq t$ and $\mu_A(z) \geq t$, for all $z \in x + a$. This implies that $\inf\limits_{z \in x+a} \mu_A(z) \geq t$. By assumption, we have $\mu_A(x) \geq \{\inf\limits_{z \in x+a} \mu_A(z), \mu_A(a)\} \geq t$, i.e., $x \in U(\mu_A; t)$. Hence, $U(\mu_A; t)$ is a left k-Γ-hyperideal of S. Similarly, we can show that $L(\lambda_A; s)$ is a left k-Γ-hyperideal of S.

Conversely, assume that all nonempty level sets $U(\mu_A; t)$ and $L(\lambda_A; s)$ are left k-Γ-hyperideals of S. Then $U(\mu_A; t)$ and $L(\lambda_A; s)$ are also left Γ-hyperideals of S. By Theorem 16 in [12], we have that A is an intuitionistic fuzzy left Γ-hyperideal of S. Let $x, y \in S$, $\min\{\inf\limits_{z \in x+y} \mu_A(z), \mu_A(y)\} = t_0$ and $\max\{\sup\limits_{z \in x+y} \lambda_A(z), \lambda_A(y)\} = s_0$. Thus, $\inf\limits_{z \in x+y} \mu_A(z) \geq t_0$ and $\mu_A(y) \geq t_0$. So, $y \in U(\mu_A; t_0)$ and $x + y \subseteq U(\mu_A; t_0)$. By assumption, we have $x \in U(\mu_A; t_0)$. This implies that $\mu_A(x) \geq t_0 = \min\{\inf\limits_{z \in x+y} \mu_A(z), \mu_A(y)\}$. Similarly, we can show that $\lambda_A(x) \leq \max\{\sup\limits_{z \in x+y} \lambda(z), \lambda(y)\}$. Therefore, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. □

Corollary 3.5. *Let I be a nonempty subset of a Γ-semihyperring S. We define fuzzy sets μ_A and λ_A as follows:*

$$\mu_A(x) = \begin{cases} t_0 & \text{if } x \in I; \\ t_1 & \text{if } x \notin I \end{cases} \quad \text{and} \quad \lambda_A(x) = \begin{cases} s_0 & \text{if } x \in I; \\ s_1 & \text{if } x \notin I, \end{cases}$$

where $0 \leq t_1 < t_0$, $0 \leq s_0 < s_1$ and $t_i + s_i \leq 1$ for $i = 0, 1$. Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S if and only if I is a left (resp. right) k-Γ-hyperideal of S. Moreover, $U(\mu_A; t_0) = I = L(\lambda_A; s_0)$.

Theorem 3.6. *Let S be a Γ-semihyperring and $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of S. Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S if and only if $\Box A = (\mu_A, \mu_A^c)$ and $\triangle\ A = (\lambda_A^c, \lambda_A)$ are intuitionistic fuzzy left (resp. right) k-Γ-hyperideals of S.*

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. For any $x, y \in S$ and $\alpha \in \Gamma$, we consider

$$\begin{aligned}
\sup_{z \in x+y} \mu_A^c(z) &= \sup_{z \in x+y} (1 - \mu_A(z)) = 1 - \inf_{z \in x+y} \mu_A(z) \leq 1 - \min\{\mu_A(x), \mu_A(y)\} \\
&= \max\{1 - \mu_A(x), 1 - \mu_A(y)\} = \max\{\mu_A^c(x), \mu_A^c(y)\}, \\
\sup_{z \in x\alpha y} \mu_A^c(z) &= \sup_{z \in x\alpha y} (1 - \mu_A(z)) = 1 - \inf_{z \in x\alpha y} \mu_A(z) \leq 1 - \mu_A(y) = \mu_A^c(y) \ \text{ and} \\
\mu_A^c(x) &= 1 - \mu_A(x) \leq 1 - \min\{\inf_{z \in x+y} \mu_A(z), \mu_A(y)\} \\
&= \max\{1 - \inf_{z \in x+y} \mu_A(z), 1 - \mu_A(y)\} = \max\{\sup_{z \in x+y} \mu_A^c(z), \mu_A^c(y)\}.
\end{aligned}$$

Similarly, we can show that $\inf_{z \in x+y} \lambda_A^c(z) \geq \min\{\lambda_A^c(x), \lambda_A^c(y)\}$, $\inf_{z \in x\alpha y} \lambda_A^c(z) \geq \lambda_A^c(y)$ and $\lambda_A^c(x) \geq \min\{\inf_{z \in x+y} \lambda_A^c(z), \lambda_A^c(y)\}$. Hence, $\Box A = (\mu_A, \mu_A^c)$ and $\triangle\ A = (\lambda_A^c, \lambda_A)$ are intuitionistic fuzzy left k-Γ-hyperideals of S. The proof of the sufficiency part is similar to the necessity part. $\Box$

Theorem 3.7. *If $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are intuitionistic fuzzy left (resp. right) k-Γ-hyperideals of a Γ-semihyperring S, then $A \cap B$ is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S.*

Proof. Assume that A and B are intuitionistic fuzzy left k-Γ-hyperideals of S. Clearly, $A \cap B$ is an intuitionistic fuzzy set of S. Let $x, y \in S$ and $\alpha \in \Gamma$.

We have

$$\begin{aligned}
\min\{(\mu_A \cap \mu_B)(x), (\mu_A \cap \mu_B)(y)\}& \\
&= \min\{\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}\} \\
&= \min\{\min\{\mu_A(x), \mu_A(y)\}, \min\{\mu_B(x), \mu_B(y)\}\} \\
&\leq \min\{\inf_{z\in x+y} \mu_A(z), \inf_{z\in x+y} \mu_B(z)\} = \inf_{z\in x+y} (\mu_A \cap \mu_B)(z), \\
(\mu_A \cap \mu_B)(y) &= \min\{\mu_A(y), \mu_B(y)\} \leq \min\{\inf_{z\in x\alpha y} \mu_A(z), \inf_{z\in x\alpha y} \mu_B(z)\} \\
&= \inf_{z\in x\alpha y} (\mu_A \cap \mu_B)(z) \text{ and} \\
(\mu_A \cap \mu_B)(x) &= \min\{\mu_A(x), \mu_B(x)\} \\
&\geq \min\{\min\{\inf_{z\in x+y} \mu_A(z), \mu_A(y)\}, \min\{\inf_{z\in x+y} \mu_B(z), \mu_B(y)\}\} \\
&= \min\{\min\{\inf_{z\in x+y} \mu_A(z), \inf_{z\in x+y} \mu_B(z)\}, \min\{\mu_A(y), \mu_B(y)\}\} \\
&= \min\{\inf_{z\in x+y} (\mu_A \cap \mu_B)(z), (\mu_A \cap \mu_B)(y)\}.
\end{aligned}$$

Moreover, we have

$$\begin{aligned}
\max\{(\lambda_A \cup \lambda_B)(x), (\lambda_A \cup \lambda_B)(y)\}& \\
&= \max\{\max\{\lambda_A(x), \lambda_B(x)\}, \max\{\lambda_A(y), \lambda_B(y)\}\} \\
&= \max\{\max\{\lambda_A(x), \lambda_A(y)\}, \max\{\lambda_B(x), \lambda_B(y)\}\} \\
&\geq \max\{\sup_{z\in x+y} \lambda_A(z), \sup_{z\in x+y} \lambda_B(z)\} = \sup_{z\in x+y} (\lambda_A \cup \lambda_B)(z), \\
(\lambda_A \cup \lambda_B)(y) &\geq \max\{\sup_{z\in x\alpha y} \lambda_A(z), \sup_{z\in x\alpha y} \lambda_B(z)\} = \sup_{z\in x\alpha y} (\lambda_A \cup \lambda_B)(z) \text{ and} \\
(\lambda_A \cup \lambda_B)(x) &= \max\{\lambda_A(x), \lambda_B(x)\} \\
&\leq \max\{\max\{\sup_{z\in x+y} \lambda_A(z), \lambda_A(y)\}, \max\{\sup_{z\in x+y} \lambda_B(z), \lambda_B(y)\}\} \\
&= \max\{\max\{\sup_{z\in x+y} \lambda_A(z), \sup_{z\in x+y} \lambda_B(z)\}, \max\{\lambda_A(y), \lambda_B(y)\}\} \\
&= \max\{\sup_{z\in x+y} (\lambda_A \cup \lambda_B)(z), (\lambda_A \cup \lambda_B)(y)\}.
\end{aligned}$$

Therefore, $A \cap B$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. □

Let μ be a fuzzy set of a nonempty set X and $t \in [0, 1 - \sup_{x\in X} \mu(x)]$. The mapping $\mu^T : X \to [0, 1]$ is called a *fuzzy translation* [23] of μ if $\mu^T(x) = \mu(x) + t$, for all $x \in X$.

Theorem 3.8. *Let S be a Γ-semihyperring, μ_A and λ_A be fuzzy sets of S and $t \in [0, \frac{1}{2}(1 - \sup_{s \in S}\{\mu_A(s) + \lambda_A(s)\})]$. Suppose that μ_A^T and λ_A^T are fuzzy translations of μ_A and λ_A with respect to t, resp. Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S if and only if $A^T = (\mu_A^T, \lambda_A^T)$ is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S.*

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. Let $a \in S$ and choose $t = \frac{1}{2}(1 - \sup_{s \in S}\{\mu_A(s) + \lambda_A(s)\})$. We have

$$\begin{aligned}
\mu_A^T(a) + \lambda_A^T(a) &= \mu_A(a) + \lambda_A(a) + 2t \\
&= \mu_A(a) + \lambda_A(a) + 1 - \sup_{s \in S}\{\mu_A(s) + \lambda_A(s)\} \\
&\leq \mu_A(a) + \lambda_A(a) + 1 - (\mu_A(a) + \lambda_A(a)) = 1.
\end{aligned}$$

Let $x, y \in S$ and $\alpha \in \Gamma$. We have

$$\begin{aligned}
\inf_{z \in x+y} \mu_A^T(z) &= \inf_{z \in x+y} \mu_A(z) + t \geq \min\{\mu_A(x), \mu_A(y)\} + t \\
&= \min\{\mu_A(x) + t, \mu_A(y) + t\} = \min\{\mu_A^T(x), \mu_A^T(y)\}, \\
\inf_{z \in x\alpha y} \mu_A^T(z) &= \inf_{z \in x\alpha y} \mu_A(z) + t \geq \mu_A(y) + t = \mu_A^T(y) \quad \text{and} \\
\mu_A^T(x) &= \mu_A(x) + t \geq \min\{\inf_{z \in x+y} \mu_A(z), \mu_A(y)\} + t \\
&= \min\{\inf_{z \in x+y} \mu_A(z) + t, \mu_A(y) + t\} = \min\{\inf_{z \in x+y} \mu_A^T(z), \mu_A^T(y)\}.
\end{aligned}$$

Moreover, we have

$$\begin{aligned}
\sup_{z \in x+y} \lambda_A^T(z) &= \sup_{z \in x+y} \lambda_A(z) + t \leq \max\{\lambda_A(x), \lambda_A(y)\} + t \\
&= \max\{\lambda_A(x) + t, \lambda_A(y) + t\} = \max\{\lambda_A^T(x), \lambda_A^T(y)\}, \\
\sup_{z \in x\alpha y} \lambda_A^T(z) &= \sup_{z \in x\alpha y} \lambda_A(z) + t \leq \lambda_A(y) + t = \lambda_A^T(y) \quad \text{and} \\
\lambda_A^T(x) &= \lambda_A(x) + t \leq \max\{\sup_{z \in x+y} \lambda_A(z), \lambda_A(y)\} + t \\
&= \max\{\sup_{z \in x+y} \lambda_A(z) + t, \lambda_A(y) + t\} = \max\{\sup_{z \in x+y} \lambda_A^T(z), \lambda_A^T(y)\}.
\end{aligned}$$

Hence, $A^T = (\mu_A^T, \lambda_A^T)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S.

Conversely, assume that $A^T = (\mu_A^T, \lambda_A^T)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. Let $x, y \in S$ and $\alpha \in \Gamma$. We have

$$\begin{aligned}
\inf_{z\in x+y} \mu_A(z) + t &= \inf_{z\in x+y} \mu_A^T(z) \geq \min\{\mu_A^T(x), \mu_A^T(y)\} \\
&= \min\{\mu_A(x)+t, \mu_A(y)+t\} = \min\{\mu_A(x), \mu_A(y)\} + t, \\
\inf_{z\in x\alpha y} \mu_A(z) + t &= \inf_{z\in x\alpha y} \mu_A^T(z) \geq \mu_A^T(y) = \mu_A(y) + t \ \text{ and} \\
\mu_A(x) + t &= \mu_A^T(x) \geq \min\{\inf_{z\in x+y} \mu_A^T(z), \mu_A^T(y)\} \\
&= \min\{\inf_{z\in x+y} \mu_A(z) + t, \mu_A(y) + t\} \\
&= \min\{\inf_{z\in x+y} \mu_A(z), \mu_A(y)\} + t.
\end{aligned}$$

This implies that $\inf_{z\in x+y} \mu_A(z) \geq \min\{\mu_A(x), \mu_A(y)\}$, $\inf_{z\in x\alpha y} \mu_A(z) \geq \mu_A(y)$ and $\mu_A(x) \geq \min\{\inf_{z\in x+y} \mu_A(z), \mu_A(y)\}$ since $t \geq 0$. Moreover, we have

$$\begin{aligned}
\sup_{z\in x+y} \lambda_A(z) + t &= \sup_{z\in x+y} \lambda_A^T(z) \leq \max\{\lambda_A^T(x), \lambda_A^T(y)\} \\
&= \max\{\lambda_A(x)+t, \lambda_A(y)+t\} = \max\{\lambda_A(x), \lambda_A(y)\} + t, \\
\sup_{z\in x\alpha y} \lambda_A(z) + t &= \sup_{z\in x\alpha y} \lambda_A^T(z) \leq \lambda_A^T(y) = \lambda_A(y) + t \ \text{ and} \\
\lambda_A(x) + t &= \lambda_A^T(x) \leq \max\{\sup_{z\in x+y} \lambda_A^T(z), \lambda_A^T(y)\} \\
&= \max\{\sup_{z\in x+y} \lambda_A(z) + t, \lambda_A(y) + t\} \\
&= \max\{\sup_{z\in x+y} \lambda_A(z), \lambda_A(y)\} + t.
\end{aligned}$$

Since $t \geq 0$, we have $\sup_{z\in x+y} \lambda_A(z) \leq \max\{\lambda_A(x), \lambda_A(y)\}$, $\sup_{z\in x\alpha y} \lambda_A(z) \leq \lambda_A(y)$ and $\lambda_A(x) \leq \max\{\sup_{z\in x+y} \lambda_A(z), \lambda_A(y)\}$. Therefore, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. □

Let μ be a fuzzy set of a nonempty set X and $m \in [0,1]$. The mapping $\mu^M : X \to [0,1]$ is called a *fuzzy multiplication* [23] of μ if $\mu^M(x) = m\mu(x)$, for all $x \in X$.

Theorem 3.9. *Let S be a Γ-semihyperring, $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of S and $m \in (0,1]$. Suppose that μ_A^M and λ_A^M are fuzzy multiplications of μ_A and λ_A, resp., where $\mu_A^M(x) = m\mu_A(x)$ and $\lambda_A^M(x) = m\lambda_A(x)$, for all $x \in S$. Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left (resp. right)*

k-Γ-hyperideal of S if and only if $A^M = (\mu_A^M, \lambda_A^M)$ is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S.

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. Clearly, $\mu_A^M(a) + \lambda_A^M(a) \leq 1$, for all $a \in S$. Let $x, y \in S$ and $\alpha \in \Gamma$. We have

$$\begin{aligned}
\inf_{z\in x+y} \mu_A^M(z) &= \inf_{z\in x+y} m\mu_A(z) \geq m\min\{\mu_A(x), \mu_A(y)\} \\
&= \min\{m\mu_A(x), m\mu_A(y)\} = \min\{\mu_A^M(x), \mu_A^M(y)\}, \\
\inf_{z\in x\alpha y} \mu_A^M(z) &= \inf_{z\in x\alpha y} m\mu_A(z) \geq m\mu_A(y) = \mu_A^M(y) \quad \text{and} \\
\mu_A^M(x) &= m\mu_A(x) \geq m\min\{\inf_{z\in x+y} \mu_A(z), \mu_A(y)\} \\
&= \min\{\inf_{z\in x+y} m\mu_A(z), m\mu_A(y)\} = \min\{\inf_{z\in x+y} \mu_A^M(z), \mu_A^M(y)\}.
\end{aligned}$$

Moreover, we have

$$\begin{aligned}
\sup_{z\in x+y} \lambda_A^M(z) &= \sup_{z\in x+y} m\lambda_A(z) \leq m\max\{\lambda_A(x), \lambda_A(y)\} \\
&= \max\{m\lambda_A(x), m\lambda_A(y)\} = \max\{\lambda_A^M(x), \lambda_A^M(y)\}, \\
\sup_{z\in x\alpha y} \lambda_A^M(z) &= \sup_{z\in x\alpha y} m\lambda_A(z) \leq m\lambda_A(y) = \lambda_A^M(y) \quad \text{and}
\end{aligned}$$

$$\begin{aligned}
\lambda_A^M(x) &= m\lambda_A(x) \leq m\max\{\sup_{z\in x+y} \lambda_A(z), \lambda_A(y)\} \\
&= \max\{\sup_{z\in x+y} m\lambda_A(z), m\lambda_A(y)\} = \max\{\sup_{z\in x+y} \lambda_A^M(z), \lambda_A^M(y)\}.
\end{aligned}$$

Thus, $A^M = (\mu_A^M, \lambda_A^M)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S.

Conversely, assume that $A^M = (\mu_A^M, \lambda_A^M)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. Let $x, y \in S$ and $\alpha \in \Gamma$. we have

$$\begin{aligned}
m\inf_{z\in x+y} \mu_A(z) &= \inf_{z\in x+y} \mu_A^M(z) \geq \min\{\mu_A^M(x), \mu_A^M(y)\} \\
&= \min\{m\mu_A(x), m\mu_A(y)\} = m\min\{\mu_A(x), \mu_A(y)\}, \\
m\inf_{z\in x\alpha y} \mu_A(z) &= \inf_{z\in x\alpha y} \mu_A^M(z) \geq \mu_A^M(y) = m\mu_A(y) \quad \text{and} \\
m\mu_A(x) &= \mu_A^M(x) \geq \min\{\inf_{z\in x+y} \mu_A^M(z), \mu_A^M(y)\} \\
&= \min\{\inf_{z\in x+y} m\mu_A(z), m\mu_A(y)\} = m\min\{\inf_{z\in x+y} \mu_A(z), \mu_A(y)\}.
\end{aligned}$$

Since $m > 0$, we have $\inf\limits_{z\in x+y} \mu_A(z) \geq \min\{\mu_A(x), \mu_A(y)\}$, $\inf\limits_{z\in x\alpha y} \mu_A(z) \geq \mu_A(y)$ and $\mu_A(x) \geq \min\{\inf\limits_{z\in x+y} \mu_A(z), \mu_A(y)\}$. Moreover, we have

$$\begin{aligned}
m \sup_{z\in x+y} \lambda_A(z) &= \sup_{z\in x+y} \lambda_A^M(z) \leq \max\{\lambda_A^M(x), \lambda_A^M(y)\} \\
&= \max\{m\lambda_A(x), m\lambda_A(y)\} = m \max\{\lambda_A(x), \lambda_A(y)\}, \\
m \sup_{z\in x\alpha y} \lambda_A(z) &= \sup_{z\in x\alpha y} \lambda_A^M(z) \leq \lambda_A^M(y) = m\lambda_A(y) \quad \text{and} \\
m\lambda_A(x) &= \lambda_A^M(x) \leq \max\{\sup_{z\in x+y} \lambda_A^M(z), \lambda_A^M(y)\} \\
&= \max\{\sup_{z\in x+y} m\lambda_A(z), m\lambda_A(y)\} = m \max\{\sup_{z\in x+y} \lambda_A(z), \lambda_A(y)\}.
\end{aligned}$$

That is, $\sup\limits_{z\in x+y} \lambda_A(z) \leq \max\{\lambda_A(x), \lambda_A(y)\}$, $\sup\limits_{z\in x\alpha y} \lambda_A(z) \leq \lambda_A(y)$ and $\lambda_A(x) \leq \max\{\sup\limits_{z\in x+y} \lambda_A(z), \lambda_A(y)\}$ because $m > 0$. Therefore, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left k-Γ-hyperideal of S. □

Let μ be a fuzzy set of a nonempty set X, $m \in [0, 1]$ and $t \in [0, 1 - \sup\limits_{x\in X} \mu(x)]$. The mapping $\mu^{MT} : X \to [0, 1]$ is called a *fuzzy magnified translation* [20] of μ if $\mu^{MT}(x) = m\mu(x) + t$, for all $x \in X$. Then the following corollary is immediately done by Theorem 3.8 and Theorem 3.9.

Corollary 3.10. *Let S be a Γ-semihyperring, $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of S, $t \in [0, \frac{1}{2}(1-\sup\limits_{s\in S}\{\mu_A(s)+\lambda_A(s)\})]$ and $m \in (0, 1]$. Suppose that μ_A^{MT} and λ_A^{MT} are fuzzy magnified translation of μ_A and λ_A, with respect to t and m, resp. Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S if and only if $A^{MT} = (\mu_A^{MT}, \lambda_A^{MT})$ is an intuitionistic fuzzy left (resp. right) k-Γ-hyperideal of S.*

4 k-Noetherian and k-Artinian Γ-semihyperrings

In this section, we apply the concepts of Noetherian and Artinian Γ-semihyperrings in [12, 9], to define the notion of a k-Noetherian and k-Artinian Γ-semihyperrings and study some of their properties.

Let S be a Γ-semihyperring. Then S is called *Noetherian* (resp. *Artinian*) [9] if S satisfies the ascending (resp. descending) chain condition on

Γ-hyperideals, that is, for any Γ-hyperideals $I_1, I_2, I_3, \ldots$ of S, with

$$I_1 \subseteq I_2 \subseteq I_3 \subseteq \cdots \subseteq I_i \cdots \quad (\text{resp. } I_1 \supseteq I_2 \supseteq I_3 \supseteq \cdots \supseteq I_i \cdots),$$

there exists $n \in \mathbb{N}$ such that $I_i = I_{i+1}$, for all $i \geq n$.

Definition 4.1. *A Γ-semihyperring S is called k-Noetherian (resp. k-Artinian) if S satisfies the ascending (resp. descending) chain condition on k-Γ-hyperideals.*

Remark 4.2. *Every Noetherian (resp. Artinian) Γ-semihyperring is a k-Noetherian (resp. k-Artinian) Γ-semihyperring.*

Theorem 4.3. *If $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy k-Γ-hyperideal of a Γ-semihyperring S with the finite image, then S is k-Noetherian.*

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy k-Γ-hyperideal of a Γ-semihyperring S with the finite image. Suppose that S is not k-Noetherian. Then there exists an ascending chain condition on k-Γ-hyperideals of S, that is, $I_0 \subseteq I_1 \subseteq I_2 \subseteq I_3 \subseteq \cdots$. We define the intuitionistic fuzzy set A by

$$\mu_A(x) = \begin{cases} \frac{1}{n+2} & \text{if } x \in I_{n+1} - I_n; \\ 0 & \text{if } x \in S - \bigcup_{n=0}^{\infty} I_n; \\ 1 & \text{if } x \in I_0 \end{cases} \quad \text{and} \quad \lambda_A(x) = \begin{cases} \frac{n+1}{n+2} & \text{if } x \in I_{n+1} - I_n; \\ 1 & \text{if } x \in S - \bigcup_{n=0}^{\infty} I_n; \\ 0 & \text{if } x \in I_0, \end{cases}$$

for all $x \in S$. It is easy to show that A is an intuitionistic fuzzy k-Γ-hyperideal of S. This is a contradiction because $I_0 \subseteq I_1 \subseteq I_2 \subseteq I_3 \subseteq \cdots$ infinitely ascending chain of k-Γ-hyperideals of S. □

Theorem 4.4. *If $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy k-Γ-hyperideal of a Γ-semihyperring S with the finite image, then S is k-Artinian.*

Proof. The proof is similar to Theorem 27 in [12]. □

Theorem 4.5. *A Γ-semihyperring S is k-Noetherian if and only if the set of values of intuitionistic fuzzy k-Γ-hyperideals of S is a well-ordered subset of $[0, 1]$.*

Proof. Assume that S is k-Noetherian. Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy k-Γ-hyperideal of S. Suppose that the set of values of A is not a well-ordered subset of $[0, 1]$. Then there exists a infinite decreasing sequence

$\{t_n\}_{n=1}^{\infty}$ such that $\mu_A(x) = t_n$ and $\lambda_A(x) \leq 1 - t_n$, for some $x \in S$. Let $I_n = \{x \in S \mid \mu_A(x) \geq t_n\}$ and $J_n = \{x \in S \mid \lambda_A(x) \leq 1 - t_n\}$. By Theorem 3.4, I_n and J_n are k-Γ-hyperideals of S, for all $n \in \mathbb{N}$. Moreover, $I_1 \subset I_2 \subset I_3 \subset \cdots$ and $J_1 \subset J_2 \subset J_3 \subset \cdots$ are strictly infinite ascending chains of k-Γ-hyperideals of S. Thus, we get a contradiction.

Conversely, assume that the set of values of intuitionistic fuzzy k-Γ-hyperideals of S is a well-ordered subset of $[0, 1]$. Since every intuitionistic fuzzy k-Γ-hyperideal of S is an intuitionistic fuzzy Γ-hyperideal of S and by Theorem 28 in [12], we have that S is Northerian. By Remark 4.2, S is also k-Northerian. □

Theorem 4.6. *A Γ-semihyperring S is both k-Noetherian and k-Artinian if and only if every intuitionistic fuzzy k-Γ-hyperideal of S has a finite number of values.*

Proof. Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy k-Γ-hyperideal of S. Suppose that $\text{Im}(\mu_A)$ and $\text{Im}(\lambda_A)$ are infinite. By Theorem 3.4, $U(\mu_A; t_i)$ and $L(\lambda_A; s_j)$, for $i, j \in \mathbb{N}$, are k-Γ-hyperideals of S. Since S is k-Noetherian and Theorem 4.5, we have that $\text{Im}(\mu_A)$ and $\text{Im}(\lambda_A)$ are well-ordered subsets of $[0, 1]$. Thus, we can divide to be two cases, as follows.

Case 1. Assume that $t_1 < t_2 < t_3 < \cdots$ is an increasing sequence in $\text{Im}(\mu_A)$ and $s_1 > s_2 > s_3 > \cdots$ is a decreasing sequence in $\text{Im}(\lambda_A)$. This implies that $U(\mu_A; t_1) \supset U(\mu_A; t_2) \supset U(\mu_A; t_3) \supset \cdots$ and $L(\lambda_A; s_1) \supset L(\lambda_A; s_2) \supset L(\lambda_A; s_3) \supset \cdots$ are absolutely descending chains of k-Γ-hyperideals of S. Since S is k-Artinian, there exist $i, j \in \mathbb{N}$ such that $U(\mu_A; t_i) = U(\mu_A; t_{i+n})$ and $L(\lambda_A; s_j) = L(\lambda_A; s_{j+m})$, where $n, m \in \mathbb{N}$. It follows that $t_i = t_{i+n}$ and $s_j = s_{j+m}$. This is a contradiction.

Cases 2. Assume that $t_1 > t_2 > t_3 > \cdots$ is a decreasing sequence in $\text{Im}(\mu_A)$ and $s_1 < s_2 < s_3 < \cdots$ is an increasing sequence in $\text{Im}(\lambda_A)$. We obtain that $U(\mu_A; t_1) \subset U(\mu_A; t_2) \subset U(\mu_A; t_3) \subset \cdots$ and $L(\lambda_A; s_1) \subset L(\lambda_A; s_2) \subset L(\lambda_A; s_3) \subset \cdots$ are exactly ascending chains of k-Γ-hyperideals of S. Since S is k-Noetherian, there exist $i, j \in \mathbb{N}$ such that $U(\mu_A; t_i) = U(\mu_A; t_{i+n})$ and $L(\lambda_A; s_j) = L(\lambda_A; s_{j+m})$, where $n, m \in \mathbb{N}$. It follows that $t_i = t_{i+n}$ and $s_j = s_{j+m}$. We have a contradiction.

Conversely, it follows by Theorem 4.3 and Theorem 4.4. □

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