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A modern modification of Gjonbalaj-Salihu cornice determinant, transformation to semi-diagonal determinant

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Abstract

In this paper we will present a modern way of transforming cornice determinants to semi-diagonal determinants. This method is based on Gjonbalaj-Salihu's method to reduce the order of determinants from $n \times n$ to $(n-4) \times (n-4)$, but this new method presented has some advantages comparing to Gjonbalaj-Salihu's method of cornice determinants calculation, both those methods decrease the order of determinants for four orders, but this new modification uses only one semi-diagonal multiplication and reduces order of determinants for four orders, instead of Gjonbalaj-Salihu's method which uses four different multiplication

1 Gjonbalaj-Salihu method to calculate cornice determinants

Definition 1.1. *Every determinant having the second row and $(n-1)$th row, as well as the second column and $(n-1)$th column with elements equal to zero, except the first and last element of these rows/columns, is called the cornice determinant.*

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$$|A_{n\times n}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} \\ a_{21} & 0 & 0 & \cdots & 0 & 0 & a_{2n} \\ a_{31} & 0 & a_{33} & \cdots & a_{3,n-2} & 0 & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{n-2,1} & 0 & a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 & a_{n-2,n} \\ a_{n-1,1} & 0 & 0 & \cdots & 0 & 0 & a_{n-1,n} \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{n,n-2} & a_{n,n-1} & a_{nn} \end{vmatrix}, \quad n \geq 5 \quad (1)$$

Theorem 1.2. [5] *Every cornice determinant $|A_{n\times n}|$, $n \times n$, $(n \geq 5)$ can be computed by reducing the order of the determinant by four.*

$$|A_{n\times n}| = (a_{12}a_{21}a_{n,n-1}a_{n-1,n} - a_{12}a_{2,n}a_{n,n-1}a_{n-1,1} - a_{21}a_{n2}a_{n-1,n}a_{1,n-1} \quad (2)$$

$$+a_{1,n-1}a_{2,n}a_{n2}a_{n-1,1}) \cdot |A_{(n-4)\times(n-4)}|,$$

where

$$|A_{(n-4)\times(n-4)}| = \begin{vmatrix} a_{33} & \cdots & a_{3,n-2} \\ \vdots & \ddots & \vdots \\ a_{n-2,3} & \cdots & a_{n-2,n-2} \end{vmatrix} \quad (3)$$

In illustrative form:

$$|A_{n\times n}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} \\ a_{21} & 0 & 0 & \cdots & 0 & 0 & a_{2n} \\ a_{31} & 0 & a_{33} & \cdots & a_{3,n-2} & 0 & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{n-2,1} & 0 & a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 & a_{n-2,n} \\ a_{n-1,1} & 0 & 0 & \cdots & 0 & 0 & a_{n-1,n} \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{n,n-2} & a_{n,n-1} & a_{nn} \end{vmatrix} =$$

$$= (a_{12}a_{21}a_{n,n-1}a_{n-1,n} - a_{12}a_{2,n}a_{n,n-1}a_{n-1,1} - a_{21}a_{n2}a_{n-1,n}a_{1,n-1} + \quad (4)$$

$$+a_{1,n-1}a_{2,n}a_{n2}a_{n-1,1}) \cdot \begin{vmatrix} a_{33} & \cdots & a_{3,n-2} \\ \vdots & \ddots & \vdots \\ a_{n-2,3} & \cdots & a_{n-2,n-2} \end{vmatrix}$$

2 Semi-diagonal method to calculate determinants of nth order $(n \geq 5)$

Definition 2.1. *Every determinant having the first row elements equal to zero expect first element, nth row elements equal to zero expect last element, second column elements equal to zero expect second element and $(n-1)$th column elements equal to zero expect $(n-1)$th element, we will call semi-diagonal determinant.*

$$|A_{n\times n}| = \begin{vmatrix} a_{11} & 0 & 0 & \cdots & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2,n-2} & 0 & a_{2n} \\ a_{31} & 0 & a_{33} & \cdots & a_{3,n-2} & 0 & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{n-2,1} & 0 & a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 & a_{n-2,n} \\ a_{n-1,1} & 0 & a_{n-1,3} & \cdots & a_{n-1,n-2} & a_{n-1,n-1} & a_{n-1,n} \\ 0 & 0 & 0 & \cdots & 0 & 0 & a_{nn} \end{vmatrix}, \quad (n \geq 5) \tag{5}$$

Theorem 2.2. *Every cornice determinant can be transformed into the semi-diagonal determinant and the result can be calculated using formula below:*

$$|A_{n\times n}| = a_{12} \cdot a_{21} \cdot b_{n-1,n-1} \cdot b_{nn} \cdot |A_{(n-4)(n-4)}|, \tag{6}$$

where

$$|A_{(n-4)\times(n-4)}| = \begin{vmatrix} a_{33} & \cdots & a_{3,n-2} \\ \vdots & \ddots & \vdots \\ a_{n-2,3} & \cdots & a_{n-2,n-2} \end{vmatrix}$$

$$b_{n-1,n-1} = a_{n,n-1} - \frac{a_{1,n-1} \cdot a_{n2}}{a_{12}}$$

$$b_{nn} = a_{n-1,n} - \frac{a_{2,n} \cdot a_{n-1,1}}{a_{21}}$$

Proof: From equation (1), exchange the first row with the second row and the nth row with the $(n-1)$th row, we will have:

$$|A_{n\times n}| = \begin{vmatrix} a_{21} & 0 & 0 & \cdots & 0 & 0 & a_{2n} \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} \\ a_{31} & 0 & a_{33} & \cdots & a_{3,n-2} & 0 & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{n-2,1} & 0 & a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 & a_{n-2,n} \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{n,n-2} & a_{n,n-1} & a_{nn} \\ a_{n-1,1} & 0 & 0 & \cdots & 0 & 0 & a_{n-1,n} \end{vmatrix}, \quad (n \geq 5) \tag{7}$$

Multiply the first row by the $(-\frac{a_{n-1,1}}{a_{21}})$ and then add to the last row. Next multiply the second row by $(-\frac{a_{n,2}}{a_{12}})$ and then add to the $(n-1)$th row. We get:

$$|A_{n\times n}| = \begin{vmatrix} a_{21} & 0 & 0 & \cdots & 0 & 0 & a_{2n} \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} \\ a_{31} & 0 & a_{33} & \cdots & a_{3,n-2} & 0 & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{n-2,1} & 0 & a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 & a_{n-2,n} \\ b_{n-1,1} & 0 & b_{n-3,3} & \cdots & b_{n-1,n-2} & b_{n-1,n-1} & b_{n-1,n} \\ 0 & 0 & 0 & \cdots & 0 & 0 & b_{n,n} \end{vmatrix} \tag{8}$$

Multiply the first column by $(-\frac{a_{2,n}}{a_{21}})$ and then add to the last column. Next multiply the second column by $(-\frac{a_{1,n-1}}{a_{12}})$ and then add to the $(n-1)$th column. We now get:

$$|A_{n\times n}| = \begin{vmatrix} a_{21} & 0 & 0 & \cdots & 0 & 0 & 0 \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1,n-2} & 0 & b_{1n} \\ a_{31} & 0 & a_{33} & \cdots & a_{3,n-2} & 0 & b_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{n-2,1} & 0 & a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 & b_{n-2,n} \\ b_{n-1,1} & 0 & b_{n-3,3} & \cdots & b_{n-1,n-2} & b_{n-1,n-1} & b^{*}_{n-1,n} \\ 0 & 0 & 0 & \cdots & 0 & 0 & b_{n,n} \end{vmatrix} \tag{9}$$

Now:

$$|A_{n\times n}| = a_{21}\cdot \begin{vmatrix} a_{12} & a_{13} & \cdots & a_{1,n-2} & 0 & b_{1n} \\ 0 & a_{33} & \cdots & a_{3,n-2} & 0 & b_{3n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 & b_{n-2,n} \\ 0 & b_{n-3,3} & \cdots & b_{n-1,n-2} & b_{n-1,n-1} & b^*_{n-1,n} \\ 0 & 0 & \cdots & 0 & 0 & b_{n,n} \end{vmatrix} = \qquad (10)$$

$$= a_{21}\cdot a_{12}\cdot \begin{vmatrix} a_{33} & \cdots & a_{3,n-2} & 0 & b_{3n} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 & b_{n-2,n} \\ b_{n-3,3} & \cdots & b_{n-1,n-2} & b_{n-1,n-1} & b^*_{n-1,n} \\ 0 & \cdots & 0 & 0 & b_{n,n} \end{vmatrix} =$$

$$= a_{21}\cdot a_{12}\cdot b_{nn}\cdot \begin{vmatrix} a_{33} & \cdots & a_{3,n-2} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ a_{n-2,3} & \cdots & a_{n-2,n-2} & 0 \\ b_{n-3,3} & \cdots & b_{n-1,n-2} & b_{n-1,n-1} \end{vmatrix} =$$

$$= a_{21}\cdot a_{12}\cdot b_{nn}\cdot b_{n-1,n-1}\cdot \begin{vmatrix} a_{33} & \cdots & a_{3,n-2} \\ \vdots & \ddots & \vdots \\ a_{n-2,3} & \cdots & a_{n-2,n-2} \end{vmatrix}$$

The MATLAB function to calculate cornice determinants using semi-diagonal method is presented below:

```
function d = det − SemiDiagonal(A)
[m, n] = size(A);
if m ∼= n
disp('Matrix A is not square')
d = 0;
return
end
B(n − 1, n − 1) = A(n, n − 1) − A(1, n − 1) ∗ A(n, 2)/A(1, 2);
B(n, n) = A(n − 1, n) − A(2, n) ∗ A(n − 1, 1)/A(2, 1);
if n == 1
d = A;
return
elseif n == 2
```

$d = A(1,1) * A(2,2) - A(1,2) * A(2,1);$
return
elseif $n == 3$
$d = det(A);$
return
elseif $n == 4$
$d = det(A);$
return
end
$d0 = det(A(3 : n-2, 3 : n-2));$
if $d0 == 0$
$d = 0;$
else
$d = A(1,2) * A(2,1) * B(n-1, n-1) * B(n,n) * d0;$
end

References

[1] S. Barnard, J. M. Child, Higher Algebra, London Macmillan LTD, New York, St. Martins Press, 1959.

[2] P. Cohn, Elements of Linear Algebra, CRC Press, 1994, 69.

[3] H. Eves, An Introduction to the History of Mathematics, Saunders College Publishing, 1990, 405-493.

[4] W. L. Ferrar, Algebra, A Text-Book of Determinants, Matrices, and Algebraic Forms, Second edition, Oxford, 1957.

[5] Q. Gjonbalaj, A. Salihu, Computing the determinants of n x n (n = 5) matrices by reducing the order of the determinant by four, Applied. Mathematics E-Notes, 2010, 151–158.

[6] R. F. Scott, The theory of determinants and their applications, Ithaca, New York: Cornell University Library, Cambridge University Press, 1904, 3–5.