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Evaluation of proximity points for a class of neuron's network functions based on local fractional calculus

[1]**Rabha W. Ibrahim**, [2]**Maslina Darus**

[1]Faculty of Computer Science and Information Technology
University of Malaya, Malaysia

[2] School of Mathematical Sciences, Faculty of Science and Technology
Universiti Kebangsaan Malaysia
43600 UKM Bangi, Selangor, Malaysia

email: rabhaibrahim@yahoo.com, maslina@ukm.edu.my

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Abstract

In mathematical modeling of Artificial Neural Networks (ANN), a neuron's network function formulates as an arrangement of other functions. It represents a composition of other functions by using arrows between variables. In this paper, we introduce a new class of activation functions based on the concept of the univex function. This type of functions satisfies the convexity property. This property converges to the outcome of ANN. Moreover, we study the activation function by using the concept of approximation theory. Finally, we lay new connections of the nonlinear weighted sum depending on the fractional power. The simulation is introduced to maximize the utility function in a fractional cloud computing system.

1 Introduction

ANN is known as a highly linked system of fundamental processors named neurons. It represents a model entitled the multi-layered ANN and denoted

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by ML-ANN. ML- ANN contains many layers, one input layer, one or more hidden layers and one output layer. Every layer acts various number of neurons and every neuron in a layer links to the neurons in the neighboring layer with various weights. Signs movement into the input layer passes over the hidden layers and attains at the output layer. Every neuron obtains signs from the neurons of the previous layer linearly weighted (or non-linearly) by the intersect values between neurons. The neuron then yields its output sign by fleeting the summed sign over the activation function (see [1]).

Convexity shows a dynamic role in numerous characteristics of mathematical phenomena (linear and nonlinear) counting optimality conditions and duality theorems. It has been generalized in many ways, one of these is the concept of the univexity. Univex functions were presented and considered by Bector et al. [2]. Rueda et al. [3] achieved optimality and duality outcomes for several mathematical programs by uniting the concepts of type-I and univex functions. The univex function method (UFM) has been created by designing series of periodic signals. This periodicity has been provided a rough prediction of the capacity at the certain period. The variation between the computation and the actual load has been formulated as a stochastic process. One can get accurate information by using the analysis of this random signal. These methods have been used for the analysis of this random sign containing some process. Usually, UFM effected well even if there is a quick variation in the environmental or sociological variables. Therefore, we suggest employing this class of functions to define the activation functions in ML-ANN. Comparing with the time series method and others, if there is any variation in those variables, the time series method is no longer beneficial. We aim to apply the method of best proximity points. This method constructs nonlinear contractions probabilistic metric spaces [4]. Moreover, we aim to consider the concept of local fractional calculus (fractal calculus) [5]. This concept plays a significant role in nonlinear analysis. We suggest the difference fractional operators because the ML-ANN considers a discrete construction. These operators are useful in applied mathematics and science [6, 7, 8]. The author introduced different classes of fractional ML-ANN (see [9]).

Fixed point theory has a significant role in numerous areas of mathematics such as differential and integral equations and optimization. This theory mainly deals with the fixed point equation $Tx = x$, where T is a nonlinear operator. The result of this equation is called a fixed point of the operator T. It is not necessary that the equation has a result for every nonlinear operator

T. If T has no result, in this case, one can obtain a point x which is closest to Tx such that the distance between Tx and x is least as compared to other points. Such a point is christened the best proximity point of T. The idea of best proximity point was introduced by Fan [10] for normed spaces. We use this idea to activate the fractional ML-ANN.

2 Processing

We need the following concepts in the sequel:

2.1 Local fractional calculus

Local fractional derivative of $\phi(\chi)$ of order $0 < \alpha \leq 1$ is specified by (see [11])

$$D^{\alpha}\,\phi(\chi) = \frac{d^{\alpha}\,\phi(\chi)}{\chi^{\alpha}}\Big|_{\chi=\chi_0} = \lim_{\chi\to\chi_0}\frac{d^{\alpha}[\phi(\chi)-\phi(\chi_0)]}{[d(\chi-\chi_0)]^{\alpha}},$$

where the expression $d^{\alpha}[\phi(\chi)-\phi(\chi_0)]/[d(\chi-\chi_0)]^{\alpha}$ is the Riemann-Liouville fractional derivative given by

$$\frac{d^{\alpha}\phi(\chi)}{d\chi^{\alpha}} = \frac{1}{\Gamma(1-\alpha)}\frac{d}{d\chi}\int_0^{\chi}\frac{\phi(t)}{(\chi-t)^{\alpha}}\,dt;$$

corresponding to the integral operator

$$(I^{\alpha}\phi)(\chi) = \frac{1}{\Gamma(\alpha)}\int_0^{\chi}(\chi-t)^{\alpha-1}\phi(t)\,dt.$$

This operator is well-defined and it is represented to the classical fractional calculus.

The local fractional derivative utilizing the fractal geometry is defined by the formula [7]

$$D^{\alpha}\,\phi(\chi) = \frac{d^{\alpha}\,\phi(\chi)}{\chi^{\alpha}}\Big|_{\chi=\chi_0} = \lim_{\chi\to\chi_0}\frac{\Delta^{\alpha}[\phi(\chi)-\phi(\chi_0)]}{[(\chi-\chi_0)]^{\alpha}},$$

where

$$\Delta^{\alpha}[\phi(\chi)-\phi(\chi_0)] \cong \Gamma(\alpha+1)[\phi(\chi)-\phi(\chi_0)].$$

Hence, if ϕ is a contraction mapping of $\ell > 0$ contraction constant, then we have

$$\lim_{\chi\to\chi_0}\frac{\Delta^{\alpha}[\phi(\chi)-\phi(\chi_0)]}{[(\chi-\chi_0)]^{\alpha}} = \ell\Gamma(\alpha+1)[\chi-\chi_0]^{1-\alpha}.$$

Moreover, if ϕ satisfies

$$|\phi(\chi) - \phi(\eta)| \leq \ell |\chi - \eta|^\alpha,$$

then we have

$$\lim_{\chi \to \chi_0} \frac{\Delta^\alpha[\phi(\chi) - \phi(\chi_0)]}{[(\chi - \chi_0)]^\alpha} = \ell \Gamma(\alpha + 1)|\chi - \chi_0|.$$

A function ϕ is said to be local fractional continuous at χ_0 if for all $\varepsilon > 0$ there is κ such that

$$|\phi(\chi) - \phi(\chi_0)| < \varepsilon^\alpha$$

provide $|\chi - \chi_0| < \kappa$. We denote the space of all local fractional continuous functions by C_α. For $\phi \in \mathrm{C}_\alpha$, the local fractional integral is defined by

$$I^\alpha_{[a,b]}\, \phi(\varsigma) = \frac{1}{\Gamma(1+\alpha)} \int_a^b \phi(\varsigma)\, (d\varsigma)^\alpha,$$

where (see [8])

$$(d\varsigma)^\alpha = d^\alpha \varsigma = \frac{\varsigma^{1-\alpha}}{\Gamma(2-\alpha)}\, d\varsigma^\alpha.$$

2.2 Local fractional univex function

A differentiable function ψ is called univex at χ_0 in its domain if there is a function $\phi : \mathbb{R} \to \mathbb{R}$ such that

$$\psi'(\chi) \leq \phi(\psi(\chi) - \psi(\chi_0)).$$

In [12, 13], Ibrahim introduced a generalization of the univex function in term of tempered fractional calculus. Here, we generalize the univex function by applying the idea of the local fractional differential operator. Let $J =: [a, b]$, $\xi \in C_\alpha[J]$ and $\psi : C_\alpha \to \mathbb{R}, \phi : \mathbb{R} \to \mathbb{R}$.

Definition 2.1. *A differentiable function ψ is said to be a local fractional univex function of order $\alpha \in (0, 1]$ in the direction of $\xi \in C_\alpha$ if for all $\chi \in \Omega$, we have*

$$D^\alpha\, \psi(\chi) \leq \phi\Big(\psi(\chi) - \psi(\xi)\Big), \quad \alpha \in (0, 1].$$

It is clear that when $\alpha = 1$ Definition 2.1 reduces to the original definition of the univex function [2]. A good example of the univexity is that every convex function is a univex function (well-known examples of convex functions involve the quadratic function x^2 and the exponential function e^x.) The advantage of using fractional operators is that can act on multi-dimensional Euclidean spaces. Therefore, it can be employed in a nonlinear multi-objective problems. In this paper, we deal with the following problem:

$$\begin{aligned} &Minimize \quad \Psi(\chi) \\ &subject\,to \quad \Theta(\chi) \leq 0, \end{aligned} \tag{2.1}$$

Definition 2.2. *A point $\xi \in \Lambda := \{\chi\ inC_\alpha : \Theta(\chi) \leq 0\}$ is called an efficient solution of (1), if there exists no $\chi \in \Lambda$, such that $\Psi(\chi) \leq \Psi(\xi)$. And it is called a weak efficient solution if $\Psi(\chi) < \Psi(\xi)$.*

Definition 2.3. *The couple (Ψ, Θ) is called (α)-type local fractional univex at $\xi \in C_\alpha$ if for all $\chi \in \Lambda$ such that*

$$D^\alpha \Psi(\chi) \leq \phi_1\Big(\Psi(\chi) - \Psi(\xi)\Big)$$

and

$$D^\alpha \Theta(\chi) \leq -\phi_2\Big(\Theta(\chi) - \Theta(\xi)\Big),$$

where ϕ_1 and ϕ_2 are real-valued functions.

2.3 Convergence analysis

Consider the function $\Psi(\chi)$ as an activation function. Also, consider the proposed ML-ANN with a satisfactorily large value of different parameters. Its convergence property is delivered by means of the α-type local fractional univex function. Usually, the researchers use the Lyapunov method and non-smooth analysis [14].

Proposition 2.4. *Let ξ be an initial solution of the problem* (2.1) *such that*
(A) $\Theta(\xi) = 0$*;*
(B) $D^\alpha \Psi(\chi) + D^\alpha \Theta(\chi) \geq 0$;
(C) The couple (Ψ, Θ) is of α-type local fractional univex at $\xi \in C_\alpha$;
(D) $u \leq 0 \in \mathbb{R} \Rightarrow \phi_1(u) \leq 0$ and $v \geq 0 \in \mathbb{R} \Rightarrow \phi_2(v) \geq 0$.
Then ξ is a solution of (2.1)*.*

Proof. Suppose that ξ is not an efficient solution of (2.1), then there exists $\chi \in \Lambda$ such that $\Psi(\chi) \leq \Psi(\xi)$. By the assumptions (A) and (D), we have

$$\phi_1(\Psi(\chi) - \Psi(\xi)) \leq 0, \quad and \quad \phi_2(\Theta(\xi)) \geq 0. \tag{2.2}$$

In view of the assumption (C), we get

$$D^{\alpha}\,\Psi(\chi) \le \phi_1\Big(\Psi(\chi) - \Psi(\xi)\Big)$$

and

$$D^{\alpha}\,\Theta(\chi) \le\ -\phi_2\Big(\Theta(\chi)\Big),$$

Summing the above inequalities, we conclude that

$$D^{\alpha}\,\Psi(\chi) + D^{\alpha}\,\Theta(\chi) < 0,$$

which contradicts the assumption (B). Hence, ξ is an efficient solution of (2.1). This completes the proof. □

Proposition 2.5. *If the following conditions are satisfied:*
(A) ξ *is a weakly efficient solution of* (2.1)*;*
(B) Θ *is continuous in* ξ*;*
(C) The functions Ψ *and* Θ *are* α*- local fractional univex functions of order in the direction of* $\xi \in \Lambda$*. Moreover, for some* $\bar{\chi} \in \Lambda$*, we have* $\Theta(\bar{\chi}) < 0$ *(*$\bar{\chi}$ *is an equilibrium point of system (1)).*
Then there are two constants $c_1 \ge 0$ *and* $c_2 \ge 0$ *such that*

$$c_1 D^{\alpha}\,\Psi(\chi) + c_2 D^{\alpha}\Theta(\chi) \ge 0.$$

Proof. Our aim is to show that the system

$$D^{\alpha}\,\Psi(\chi) < 0, \quad D^{\alpha}\,\Theta(\chi) < 0,$$

has no solution for $\chi \in C_{\alpha}[a,b]$. Let the system have a solution $y \in C_{\alpha}[a,b]$. By the assumption (A), we have

$$\Psi(\xi + \epsilon_1 y) < \Psi(\xi) \quad and \quad \Theta(\xi + \epsilon_2 y) < \Theta(\xi),$$

for sufficient small arbitrary constants ϵ_1, $\epsilon_2 > 0$. Now, we let $\bar{\chi} := \xi + \epsilon_2 y$; which implies that $\bar{\chi} \in \Lambda \cap N_{\epsilon_2}(\xi)$. Thus by (B) and (C), we have $\Theta(\xi+\epsilon_2 y) = \Theta(\bar{\chi}) < 0$; which contradicts (A), where ξ is a weak solution. Therefore, the above inequalities are non-negative. Hence, in view of (C) these are two constants c_1 and c_2 satisfy the inequality

$$c_1 D^{\alpha}\,\Psi(\chi) + c_2 D^{\alpha}\,\Theta(\chi) \ge 0,$$

with the property $\Theta(\xi) = 0$. This completes the proof. □

Proposition 2.6. *If* $\Psi(\chi)$ *has a finite minimum, then the state vector of the neural network* (2.1) *is stable in the sense of* α*-local fractional univex functions. Furthermore, it is globally convergent to an equilibrium point with any positive* c_1 *and* c_2*.*

2.4 Approximate points

In this section, we deal with the univex function as an activation function. The wight of the ML-ANN is determined together with the function. The equilibrium points of the system are calculated by the approximate points of (2.1). Our study is based on the theory of best proximity point.

Definition 2.7. *Let the couple* (Ψ, Θ) *be* α*-type local fractional univex function. If*

$$D^{\alpha}\,\Psi(\chi) = \phi_1\Big(\Psi(\chi) - \Psi(\xi^*)\Big)$$

and

$$D^{\alpha}\,\Theta(\chi) = \, -\phi_2\Big(\Theta(\chi) - \Theta(\xi^*)\Big),$$

then ξ^* *is called a best proximity point of* Ψ.

Proposition 2.8. *Let* $\Psi(x)$, $\chi \in X$ *be a* α*-type local fractional univex function. If the following assumptions are satisfied:*
(A) ϕ_1, ϕ_2 *are linear contraction mappings;*
(B) $|x - \xi^*| < \rho < \frac{1}{\ell_1 \ell_2}$ *for some positive constants* ℓ_1 *and* ℓ_2*;*
(C) $D^{\alpha}\,\Psi(\chi) \geq 1$ *and* $D^{\alpha}\,\Theta(\chi) \geq 1$.
Then Ψ *has a best proximity point.*

Proof. Since $\Psi(\chi)$, $\chi \in C_{\alpha}[a, b]$ is a α-type local fractional univex function, then it is a contraction mapping. Assume that $D^{\alpha}\,\Psi(\chi) < \phi_1\Big(\Psi(\chi) - \Psi(\xi^*)\Big)$ then we have

$$\begin{aligned} D^{\alpha}\,\Psi(\chi) &< \phi_1\Big(\Psi(x) - \Psi(\xi^*)\Big) \\ &= \phi_1\big(\Psi(\chi)\big) - \phi_1\big(\Psi(\xi^*)\big) \\ &\leq \, |\phi_1\big(\Psi(\chi)\big) - \phi_1\big(\Psi(\xi^*)\big)| \\ &\leq \, \ell_1 |\Psi(\chi) - \Psi(\xi^*)| \\ &\leq \, \ell_1 \ell_2 |\chi - \xi^*| \leq \, \rho \ell_1 \ell_2 < 1, \end{aligned} \tag{2.3}$$

which contradicts (C). Thus, we conclude that

$$D^{\alpha}\,\Psi(\chi) = \phi_1\Big(\Psi(\chi) - \Psi(\xi^*)\Big)$$

and similarly, we have

$$D^{\alpha}\,\Theta(\chi) = \, -\phi_2\Big(\Theta(\chi) - \Theta(\xi^*)\Big)$$

This completes the proof. □

3 Simulation

Our aim is to introduce a process of ML-ANN over the set of data X. The activation function is suggested to be a univex-function of fractional order. The model signifies the main features, activities of the selected activation function. It symbolizes the system itself, whereas the simulation characterizes the operation of the system over X. Since we use optimization, in this case the simulations of physical processes are often employed in conjunction with evolutionary computation to optimize control strategies. We consider a fractional cloud computing system constructing in (2.1). The function Ψ is refereeing to be the cost function of the system. our aim is to minimize it. Each neuron is assuming to be the agent. All agents connected to receive the same information from the cloud.

3.1 Architecture of ML-ANN

The total of $S-$ sets of training data is expected to be offered. Input sets of the form $\{X_1, X_2, ..., X_S\}$ are levied on the superior layer. The ML-ANN domesticated to reply to the conformable target vectors, $\{T_1, T_2, ..., T_S\}$ on the bottom layer. The domesticating resumes until a definite stop-criterion fulfilled. Usually, practicing is stopped when the average error between the desired and actual outputs of the neural network over the S practicing information collections is less than a predetermined threshold. The practicing is determined by different elements; such as complexity, structure of the network, data and the practicing parameters set.

3.2 Training of ML-ANN

In this simulation, the fractional Delta rule (FDR) is utilized to practice ML- ANN. This rule is based on the definition of fractional difference gradient. An output vector is created by giving an input pattern to the network. According to the difference between the formed and target outputs, the network's weights ω_{ij} are accustomed to decrease the output error. The error at the output layer propagates backward to the hidden layer, until it touches the input layer. The FDR is also named fractional error back propagation algorithm. The output from X_i (O_i), is linked to the input of X_j through the interconnection weight ω_{ij}. If the neuron X_k is one of the input neurons, then the state is as follows:

$$O_k = \Psi(\sum_i \omega_{ik} O_i), \tag{3.4}$$

where Ψ is α-type univex function. Let the target be T. Then the error at the output neuron can be defined as

$$E_{\alpha,k} = \alpha |T_k - O_k|^\alpha, \quad \alpha \in (0,1), \tag{3.5}$$

where O_k is the output of the neuron k. When $\alpha = 1/2$, we obtain the usual error. While the total error can be calculated by the sum

$$\mathfrak{E}_\alpha = \frac{\sum_{k=1}^n E_{\alpha,k}}{100}. \tag{3.6}$$

The changing in the weight is determined from the equation

$$\Delta^\alpha \omega_{ik} = D^\alpha E_\alpha, \tag{3.7}$$

where $D^\alpha E_\alpha$ is the fractional derivative with respect to O_k. Once the neural network is training, it is producing very fast output for a specific input information.

3.3 Tasting

Suppose 5-agent system . All of them receive the same information. Suppose $\mathfrak{L} = \{L_1, ..., L_5\}$ is the load set of the 5-agent system, where

$$L_s = \sum_1^{24} L(h), s = 1, ..., 5. \tag{3.8}$$

The cost is determine by

$$\mathfrak{C} = L(h) * U,$$

where U is a unit per hour. In this simulation, we assume the target value of the cost function is $T_k = 0.5$. The first view is the functional view: the input x switched into a 5-dimensional vector ψ, which finally switched into Ψ. This construction is most commonly encountered in the context of optimization. Suppose that ψ is the Heaviside step function

$$\psi(x) := \frac{d}{dx} \max\{x, 0\}, \tag{3.9}$$

Table 1: Fractional multi-objective function (cost function), $\rho = 1$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\mu$	[13]
0.1	1.033	1.040	1.030	1.028	1.027	0.072	0.0023
0.2	1.066	1.08	1.06	1.056	1.055	0.097	-
0.3	1.099	1.12	1.090	1.048	1.06	0.20	-

Table 2: Fractional multi-objective function (cost function), $\rho = 1/2$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\alpha$	[13]
0.1	0.83	0.9	0.8	0.78	0.777	0.0025	0.002
0.2	0.566	0.58	0.56	0.556	0.54	0.00002	-
0.3	0.6	0.65	0.6	0.58	0.56	0.00015	-

while the function Ψ is the (α, ρ, ω)-type univex function. Hence, we have

$$\mathfrak{D}^\alpha \psi(x) = \frac{\psi(x)(x-\xi)^{-\alpha}}{\Gamma(2-\alpha)}, \quad \alpha \in (0,1), x_i \in (0,1], \, x > \xi. \tag{3.10}$$

The initial weight is 1/5 for each agent. Table 1 shows the requested information for each agent daily, with initial random load $x = (0.5, 0.3, 0.7, 0.9, 1)$, $\xi = 0.1$. Then for $(x - \xi) = (0.4, 0.2, 0.6, 0.8, 0.9)$, we have

$$\begin{aligned} \mathfrak{D}^{0.25} \psi(x) &= (0.33, 0.40, 0.30, 0.28, 0.277) \\ \mathfrak{D}^{0.5} \psi(x) &= (0.88, 1.25, 0.722, 0.626, 0.59) \\ \mathfrak{D}^{0.75} \psi(x) &= (1.6, 2.708, 1.18, 0.95, 0.87) \end{aligned} \tag{3.11}$$

3.3.1 Case I; $\mu = 0.25$

The topology of ML-ANN is as follows:
Input neurons: $L_l(h), L_2(h), ..., L_5(h)$
Hidden neurons: 5 hidden neurons
Output neuron : $L(h)$

To satisfy the conditions of Proposition 2.4, we let $\rho < 1$. Thus we obtain the following results:

Table 3: Fractional multi-objective function (cost function), $\rho = 1/3$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\alpha$,	[13]
0.1	0.73	0.8	0.7	0.68	0.677	0.0025	0.002
0.2	0.466	0.48	0.46	0.456	0.44	0.00002	-
0.3	0.5	0.55	0.5	0.48	0.46	0.0001	-

Table 4: Fractional multi-objective function (cost function), $\rho = 1$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\alpha$	[13]
0.1	1.088	1.125	1.072	1.0626	1.059	0.0018	0.004
0.2	1.17	1.25	1.14	1.12	1.1	0.001	-
0.3	1.2	1.3	1.19	1.18	1.17	0.002	-

It is clearly that the minimum error is in the case $\rho = 1/3, \omega = 0.2$. Note that when $\rho \leq 1/4$, the error interval is $[0.01, 0.1]$. Therefore, the best case to minimize the cost function when $\rho = 1/3, \omega = 0.2$.

3.3.2 Case II; $\mu = 0.5$

Obviously, the minimum error is in the case $\rho = 1/3, \omega = 0.3$. This case minimizes the cost function.

Table 5: Fractional multi-objective function (cost function), $\rho = 1/2$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\alpha$	[13]
0.1	0.588	0.6	0.57	0.566	0.56	0.0018	0.0048
0.2	0.7	0.75	0.715	0.714	0.7	0.005	-
0.3	0.7	0.695	0.694	0.692	0.69	0.0051	-

Table 6: Fractional multi-objective function (cost function), $\rho = 1/3$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\alpha$	[13]
0.1	0.388	0.425	0.372	0.3626	0.359	0.0015	0.0049
0.2	0.476	0.55	0.444	0.4	0.39	0.0004	-
0.3	0.564	0.675	0.55	0.51	0.499	0.00031	-

Table 7: Fractional multi-objective function (cost function), $\rho = 1$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\alpha$	[13]
0.1	1.16	1.2708	1.118	1.095	1.087	0.0038	0.0071
0.2	1.32	1.54	1.23	1.18	1.16	0.0047	-
0.3	1.48	1.7	1.38	1.34	1.32	0.0074	-

Table 8: Fractional multi-objective function (cost function), $\rho = 1/2$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\alpha$	[13]
0.1	0.66	0.77	0.6	0.59	0.56	0.0077	0.007
0.2	0.82	1.01	0.77	0.66	0.6	0.012	-
0.3	1.0	1.2	0.88	0.85	0.8	0.0127	-

Table 9: Fractional multi-objective function (cost function), $\rho = 1/3$

ω	agent 1	agent 2	agent 3	agent 4	agent 5	$\mathfrak{E}_\alpha$	[13]
0.1	0.46	0.578	0.4	0.395	0.387	0.00304	0.007
0.2	0.62	0.88	0.55	0.44	0.4	0.0056	-
0.3	0.78	1.118	0.654	0.585	0.56	0.0068	-

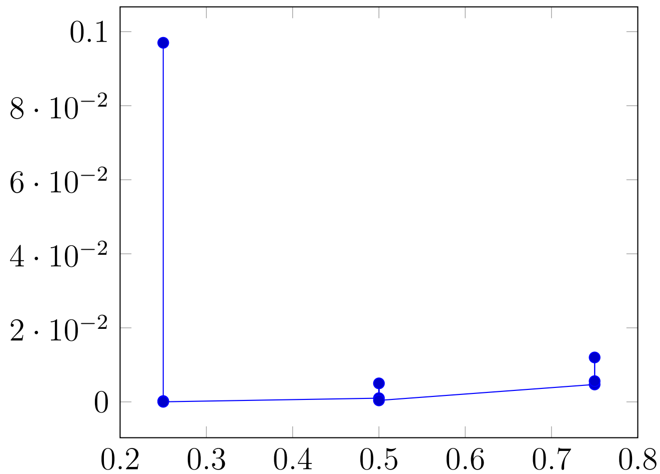

Figure 1: The total error with respect to the fractional value α and fixed $\omega = 0.2$

3.3.3 Case III; $\mu = 0.75$

Obviously, the minimum error is in the case $\rho = 1/3, \omega = 0.1$. This case minimizes the cost function.

4 Discussion and conclusion

A fractional multi-objective function is established based on the coarse-grained calculus. The function is defined by using a fractional univex function. This function is suggested to be some kind of activation function to solve a control problem by using the ANN. The simulation was showing the accuracy and efficiency of the algorithm. The algorithm was involving a set of parameters. These parameters are illustrated from the fractional calculus as well as the univex function with the wight of the ANN. The total error is evaluated by the functional $\mathfrak{E}_\alpha$. Fig.2 showed the total error in the case $\omega = 0.2$. We conclude that the fractional ML-ANN has been proved the outcome faster than the ordinary one. The number of iterations was decreasing into the half (we used the approximation of solution instead off the Lyapunov method). Tables 1-9 showed the total error of the method. We realize that Table 2 and 3 imply the best outcome.

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