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Prediction algorithm of the receiving antenna location for quasi-optical circuit components

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Abstract

Quasi-optical beam forming structures are very interesting and involve different kinds of methods for their analysis. In order to optimize the cost function of our target solution, it is important to develop a predictive system to determine where the receiving antenna should be located for more efficient propagation of electromagnetic waves. Solving this antenna problem with the mathematical theory of diffraction is a complicated task due to the involved integrals with hyper-singular singularities in their kernels. This has raised the objective to develop a discrete mathematical model which could solve those difficulties.

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1 Introduction

Power splitter/combiner devices are widely used in microwave and millimeter wave systems. The diffraction model used to design terahertz parallel-plate wave-guide power splitters is an easy and efficient tool. The design problem is to achieve electromagnetic wavefront transformation with high efficiency so that a broad beam is split into a finite number of equal power beams. Metallic waveguides are important components of many technologies with practical applications such as radar antenna feeds, waveguide slot antenna arrays, horn antennas, microwave filters, and other various passive circuit components.

In the former TUHH MEMSTIC project signal splitting and combining was a key circuit function for a circuit technology in the THz range which is exclusively using semiconductor devices in oscillators, amplifiers, multipliers and mixers. That is because any semiconductor device can handle just a low power level at such high frequencies, so that usually several semiconductor devices have to be combined to realize a useful nonlinear or active component. The letters ME in MEMSTIC mean Multiple Element and the remaining ones Multiple Substrate Terahertz Integrated Circuits. Hence it is obvious that a strong signal should be split into several or many low-level ones before entering e.g. a multiple element amplifier. Then the individual amplified signals must still be combined into a sum signal before they can proceed further in a circuit. Hence a key component is a signal splitter or combiner, which should be loss-free in the ideal case. This means that all individual signals are summed up with an efficiency of 100 % and without any Ohmic losses in the passive device.

The publications [1] - [8] represent the latest and complete descriptions of what had till now been done and of how it has been done. Paper [7] is dealing with modeling and analysis of a metallic grating in a 2D arrangement. The [1-3], [5, 6] papers describe original MEMSTIC work. Paper [1] presents the basic ideas of MEMSTIC. For illustration, a 3D power combiner (i.e. the reciprocal device to a splitter) has been designed which consists of a dielectric phase grating placed in the near-field zone of the feeding, infinitely wide antenna array. The task of the phase grating is to transform the impinging wave in a sum plane wave without losing power. Paper [2] describes generalization of the methods of paper [1] and, in particular, points out the relationships of these methods to the Talbot effect of (quasi-) optics.

2 Problem statement

The basic structure which shall be the starting point is an arrangement of several passive components within a parallel-plate waveguide. These components are located at several reference planes: In case of a splitter, a horn antenna is placed in the input reference plane, while the output reference plane contains N horn antennas to couple to the N beams which have to be generated by a splitting element, which is placed e.g. in an intermediate reference plane.

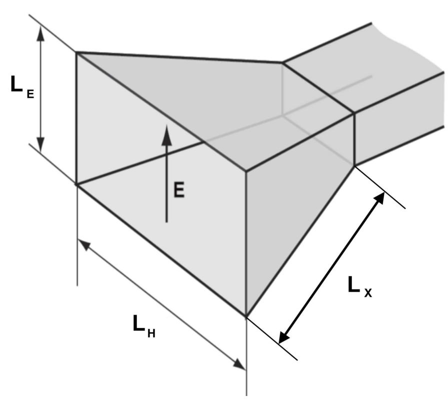

Figure 1: General structure of horn antenna and its parameters

This is the simplest arrangement for a splitter. (A combiner is just the same structure with interchanged input and output reference planes.) In the case described the splitting element transmits the signal(s) and is called a grating which can be metallic or dielectric. The minimal arrangement for a splitter (or combiner) then consists of the input and output horn antennas and one relief mirror.

In Fig. 1 we can see a rectangular horn antenna with its main parameters. Its characteristic is the directional pattern from which follows a lot of auxiliary parameters such as the directivity and the side lobe level. An antenna is a reciprocal component. In transmission state, it radiates the energy from a source as electromagnetic wave into free space, in reception state, it collects some of the power of an electromagnetic wave in order to produce an electric current at its terminals, which is then applied to a receiver to be amplified. In case of splitting, the signal of a transmission antenna is radiated on a reflector, split and then coupled to the various receiving antennas. In case of combining, just the opposite. Fig. 2 is reciprocal to a 1:5 splitter.

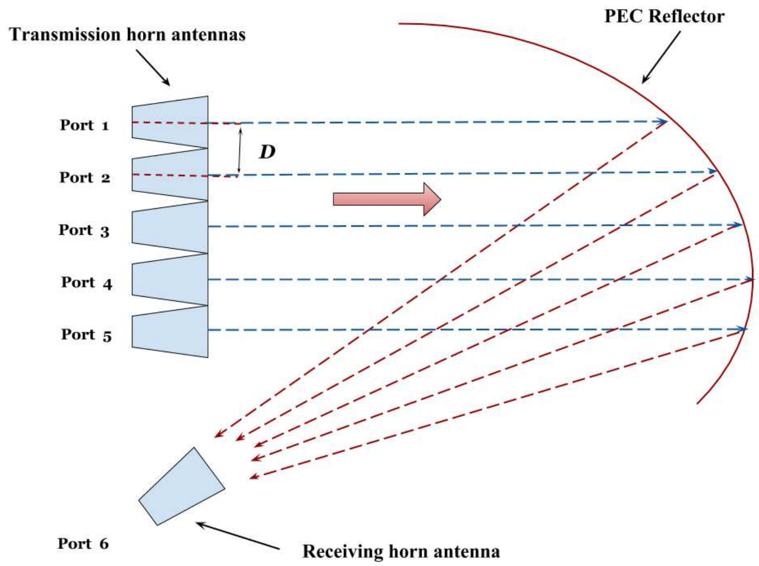

Figure 2: Scheme of quasi-optical structure with 5:1 combiner

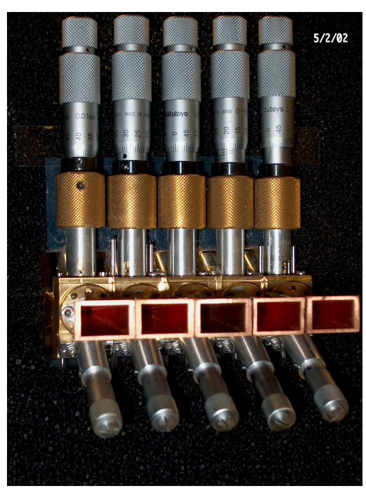

Figure 3: Quasi-optical structures

In the input reference plane there are 5 transmission horn antennas Fig. 3. These components are located in one plane. In the other plane, we place a receiving horn antenna. Its coupling to the 5 beams is performed by a combining element, which in its simplest form is a reflector. We have chosen here a Perfect Electric Conductor (PEC) reflector where the shape is defined as a parabolic type.

3 Mathematical model

A mathematical model based on hypersingular integral equations (HSIEs) of the considered structure is generally represented by the following formulas:

$$\begin{cases} BF_1^N(\xi) - \frac{1}{\pi} \int\limits_{St^{(N)}} \frac{F_1^N(\eta)}{(\eta-\xi)^2} d\eta + \frac{\kappa^2}{2\pi} \int\limits_{St^{(N)}} \ln|\eta - \xi| \, F_1^N(\eta) d\eta + \\ \frac{1}{\pi} \int\limits_{St^{(N)}} Q_1^N(\eta,\xi) F_1^N(\eta) d\eta = f_1^N(\xi), \; \xi \in St^{(N)}, \\ F_2^N(\xi) - \frac{B}{\pi} \int\limits_{St^{(N)}} \ln|\eta - \xi| \, F_2^N(\eta) d\eta + \frac{1}{\pi} \int\limits_{St^N} Q_2^N(\eta,\xi) F_2^N(\eta) d\eta = f_2^N(\xi), \; \xi \in St^{(N)}, \end{cases} \tag{3.1}$$

where

$$K(\eta,\xi) = \frac{\kappa^4}{4} \int\limits_{-\infty}^{\infty} \frac{\exp^{i\lambda(\xi-\eta)}}{\gamma(\lambda)(|\lambda|+\gamma(\lambda))^2} d\lambda, \;\; KQ(\xi,\eta) = H_0^{(1)}(\kappa \mid \eta - \xi \mid) - \frac{2i}{\pi} \ln \mid \eta - \xi \mid,$$

$$Q_1(\eta,\xi) = K(\eta,\xi) - \frac{\kappa^2 i\pi}{4} KQ(\eta,\xi), \;\; Q_2(\eta,\xi) = B\frac{i\pi}{2} KQ(\eta,\xi),$$

$$f_1^N(\xi) = 2 \left.\frac{\partial u_{inc}^N(\xi,\zeta)}{\partial \zeta}\right|_{\zeta=0}, \;\; f_2^N(\xi) = -2Bu_{inc}^N(\xi,+0), \;\; u_{inc}^N(\xi,\zeta) = e^{i\kappa(\xi\sin\alpha - \zeta\cos\alpha)}. \tag{3.2}$$

Term $Q_2^N(\eta,\xi)$ in equation (3.2) shows a logarithmic singularity when $\eta = \xi$. For this reason we need to take advantage and to expand it into a series of Hankel function using Bessel functions of the first kind. Thus, the remainders of the series have been obtained which do not show a singularity.

The solutions of the HSIEs (3.1) of the second kind were proposed in [7] where the regularization method for these IEs had been applied. The theorem of existence and uniqueness of this type of IEs has also been proved there.

4 Prediction algorithm for location

The solution of (3.1) gives us the power of system at the specific location at time t_0. Let's define $\{x_i,\ y_i,\ z_i\},\ i = 0,..,7$, the locations of each port 1-6 and 7 is location of PEC reflector. We need to find $\{x_{i,j},\ y_{i,j},\ z_{i,j}\},\ i = 0,..,7.$, where j is time-point for optimal predicted location. We will use a technique with the quadratic performance for control systems [8] in order to predict the efficient location for each point in our system. The singular-value decomposition (SVD) as a factorization allows us to extract the major pattern of behavior system and translate it into propagation direction for whole system. We consider the method of spectral projections of the type and Riesz method based on SVD coordinate system. The analytical solution and optimal control of discrete descriptor systems were given in [8].

Algorithm 1: Prediction of location

Input: $\{x_{i,0},\ y_{i,0},\ z_{i,0}\},\ i = 0,..,7,\ J(u_{t_0}) \ \to max.$

Output: $\{x_{i,p},\ y_{i,p},\ z_{i,p}\},\ i = 0,..,7.$

for $i = 0,..,7$ **do**

$$\frac{1}{2}\begin{pmatrix} x_{7,0} \\ y_{7,0} \\ z_{7,0} \end{pmatrix}^T \cdot A^T \cdot P_{N+1} + \frac{1}{2}\sum_{j=0}^{N}\left[\begin{pmatrix} x_{i,j+1} \\ y_{i,j+1} \\ z_{i,j+1} \end{pmatrix},\ u_{t,j+1}^T\right] \cdot \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \cdot \begin{pmatrix} x_{i,j+1} \\ y_{i,j+1} \\ z_{i,j+1} \\ u_{i,j+1} \end{pmatrix};$$

$S_{m\times r},\ Q_{m\times m},\ R_{r\times r},\quad rank(A) = k \leq m$;

for $j = 0,..,N$ **do**

$c_{i,j} = \{x_{i,j},\ y_{i,j},\ z_{i,j}\}$;

$$\begin{pmatrix} \Sigma^2 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} c_{i,j+1} \\ \gamma_{i,j+1} \end{pmatrix} + B \cdot \begin{pmatrix} c_{i,0} \\ \gamma_{i,0} \end{pmatrix} = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \cdot u_j\ ;$$

end

$$\begin{pmatrix} x_{i,j+1} \\ y_{i,j+1} \\ z_{i,j+1} \\ u_{i,j+1} \end{pmatrix} = \begin{pmatrix} I \\ P_{11} \cdot \Sigma^2 \\ -\left(\overline{R} + \overline{B}^T \cdot P_{11,j+1} \cdot \overline{B}\right) \end{pmatrix} \cdot \begin{pmatrix} x_{i,0} \\ y_{i,0} \\ z_{i,0} \\ u_{i,0} \end{pmatrix}.$$

end

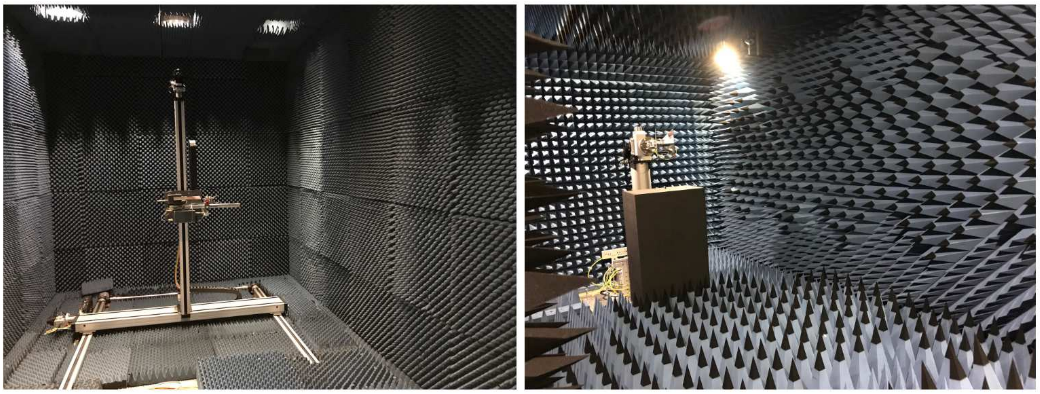

Figure 4: Antenna Laboratories at Kiel University and Hamburg University of Technology

Outlook

The mathematical and computer models have been developed during research 2017-2018 years funded by DAAD Germany Academy of Science for short research grants. The computational approach for quasi-optical circuit components based on hypersingular integral equations will be applied at the antenna laboratories. The guidelines for that work will be investigated and developed through the measurements of prototype components designed and produced by the workshop of the institute. For the experiments the antenna laboratories of both universities (see Fig. 4) will be available. They differ from each other in the frequency range for which they have been designed.

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