International Journal of Mathematics and
Computer Science, **14**(2019), no. 4, 775–786

On The Diophantine Equation $(132k)^x + (4355k)^y = (4357k)^z$

Abdulrahman Balfaqih, Hailiza Kamarulhaili

School of Mathematical Sciences
Universiti Sains Malaysia
11800, Penang, Malaysia

email: mathsfriend417154@hotmail.com, hailiza@usm.my

(Received April 4, 2019, Accepted May 29, 2019)

Abstract

The Jeśmanowicz's conjecture written in 1956 states that for any primitive Pythagorean triple (a, b, c) with $a^2+b^2 = c^2$ and any positive integer k, the only solution of equation $(ak)^x+(bk)^y = (ck)^z$ in positive integers is $(x, y, z) = (2, 2, 2)$. In this paper, we show that the special Diophantine equation $(132k)^x + (4355k)^y = (4357k)^z$ has the only positive integer solution $(x, y, z) = (2, 2, 2)$ for every positive integer k.

1 Introduction

In 1956, Sierpińśki [6] showed that the only positive integer solution of the Diophantine Equation

$$(ak)^x + (bk)^y = (ck)^z \tag{1.1}$$

is $(x, y, z) = (2, 2, 2)$, for $k = 1$ and $(a, b, c) = (3, 4, 5)$, and Jeśmano wicz [2] proved that the conjecture is true when $k = 1$ and $(a, b, c) \in \{(5, 12, 13), (7, 24, 25), (9, 40, 41), (11, 60, 61)\}$. Jeśmanowicz also conjectured that the Diophantine equation (1.1) has the only positive integer solution $(x, y, z) = (2, 2, 2)$ for any positive integer k. There are many special cases

Key words and phrases: Jeśmanowicz's conjecture, Diophantine equation, Pythagorean triple.
AMS (MOS) Subject Classification: 11D61.
ISSN 1814-0432, 2019, http://ijmcs.future-in-tech.net

of Jeśmanowicz's conjecture solved for $k = 1$. In 2012, Yang and Tang [11] proved that the only solution of the Diophantine Equation

$$(8k)^x + (15k)^y = (17k)^z \tag{1.2}$$

is $(x, y, z) = (2, 2, 2)$, for $k \geqslant 1$. Several authors had shown that Jeśmanowicz's conjecture is true for $n \in \{2, 3, 4, 8\}$ where $(a, b, c) = (4n, 4n^2 - 1, 4n^2 + 1)$, see [9] and [12]. Yang and Jianxin [12] proved that the only solution of

$$(12k)^x + (35k)^y = (37k)^z \tag{1.3}$$

is $(x, y, z) = (2, 2, 2)$ for $k \geqslant 1$. In 2015, Ma and Wu [5] proved that the only solution of the Diophantine Equation

$$((4n^2 - 1)k)^x + (4nk)^y = ((4n^2 + 1)k)^z \tag{1.4}$$

is $(x, y, z) = (2, 2, 2)$ when $P(4n^2 - 1)|k$, (where $P(m)$ denote the product of distinct prime of m). They showed that if k is a positive integer, and $P(k) \nmid (4n^2-1)$, then the only solution for the equation (1.4) is $(x, y, z) = (2, 2, 2)$, in this case they considered $n = p^m$, p prime and $m \geqslant 0$ with $p \equiv -1(\text{mod}4)$. In 2017, Gökhan Soydan, Musa Demirci, Ismail Naci Cangul, and Alain Togbé [7] considered(1.1) with $(a, b, c) = (20, 99, 101)$ and they proved the Diophantine equation

$$(20k)^x + (99k)^y = (101k)^z \tag{1.5}$$

has only the solution $(x, y, z) = (2, 2, 2)$. In this paper, we consider the case $n = 33$ and for $(a, b, c) = (4n, 4n^2 - 1, 4n^2 + 1)$ for (1.1). For other results, see for instance [10], [8], [3] and [1]. Our main result is the following theorem.

Theorem 1.1. *The only positive integer solution of the Diophantine equation*

$$(132k)^x + (4355k)^y = (4357k)^z \tag{1.6}$$

is $(x, y, z) = (2, 2, 2)$*, for every positive integer* k.

2 Proof Of Theorem 1.1

In this section, we begin with three useful results as follows:

Lemma 2.1. *(see [3]) If* (x, y, z) *is a solution of* (1.1) *with* $(x, y, z) \neq (2, 2, 2)$ *, then* x, y *and* z *are distinct.*

Lemma 2.2. *(see [4]) The only positive integer solution of the Diophantine equation* $(2n^2-1)^x + (4n)^y = (4n^2+1)^z$ *is* $(x, y, z) = (2, 2, 2)$.

Lemma 2.3. *(see [1]) If* $z \geqslant \max\{x, y\}$*, then the Diophantine equation* $a^x + b^y = c^z$ *where* a, b *and* c *are any positive integers (not necessarily relatively prime) such that* $a^2 + b^2 = c^2$*, has no solution other than* $x = y = z = 2$.

Proof. (**Theorem 1.1**)
When $k = 1$ the equation (1.6) becomes

$$(132)^x + (4355)^y = (4357)^z \tag{2.7}$$

from lemma 2.2, the Diophantine equation (2.7) has the only positive integer solution $(x, y, z) = (2, 2, 2)$. Suppose that (1.6) has at least another solution $(x, y, z) \neq (2, 2, 2)$ then, by lemma 2.3 we have $z < \max\{x, y\}$ and from lemma 2.1, we have $x \neq y, y \neq z$ and $x \neq z$. Thus, we consider two cases as follows.

Case 1 If $(x < y)$, then we obtain two subcases $z < x < y$ and $x < z < y$.

Subcase 1.1 If $(z < x < y)$ then, rewrite equation (1.6) as

$$k^{x-z}(132^x + 4355^y k^{y-z}) = 4357^z \tag{2.8}$$

So if $(k, 4357) = 1$, then $x = z$, where $k \geqslant 2$, which is a contradiction. And if $(k, 4357) = 4357$, then we can write $k = 4357^m n_1$, where $m \geqslant 1$ and $4357 \nmid n_1$, So rewrite equation (2.8) as

$$4357^{m(x-z)} {n_1}^{x-z}(132^x + 4355^y 4357^{m(y-x)} {n_1}^{y-x}) = 4357^z \tag{2.9}$$

thus $n_1^{x-z} \big| \, 4357^z$ and so $n_1 = 1$. Therefore (2.9), becomes

$$132^x + 4355^y 4357^{m(y-x)} = 4357^{z-m(z-x)} \tag{2.10}$$

implies $4357 | \, 132^x$ and this is impossible.

Subcase 1.2 If $(x < z < y)$ then, rewrite (1.6) as

$$132^x + 4355^y k^{y-x} = 4357^z k^{z-x} \tag{2.11}$$

So if $(k, 132) = 1$, then $x = z$, where $k \geqslant 2$, which is a contradiction. And if $(k, 132) > 1$, then we can write $k = 2^r 3^s 11^q n_1$, where $r + s + q \geqslant 1$ and $(66, n_1) = 1$, So rewrite (2.11) as

$$132^x = 2^{r(z-x)} 3^{s(z-x)} 11^{q(z-x)} {n_1}^{z-x} \left[\, 4357^z - 4355^y 2^{r(y-z)} 3^{s(y-z)} 11^{q(y-z)} {n_1}^{y-z} \right] \tag{2.12}$$

Then we get seven cases as the following :

1. If $k = 2^r n_1$, where $r \geqslant 1$, $s = q = 0$ and $(60, n_1) = 1$, then (2.12) becomes

$$132^x = 2^{r(z-x)} {n_1}^{z-x} \left[\, 4357^z - 4355^y 2^{r(y-z)} {n_1}^{y-z} \right] \tag{2.13}$$

thus $2x = r(z-x)$ and $33^x = {n_1}^{z-x} \left[4357^z - 4355^y 2^{r(y-z)} {n_1}^{y-z} \right]$, hence $n_1 = 1$ and

$$4357^z - 33^x = 4355^y 2^{r(y-z)} \tag{2.14}$$

where $(60, n_1) = 1$, So

$$4357^z - 33^x \equiv 2^z - 7^x \equiv 0 (\text{mod} 13) \tag{2.15}$$

Thus $z \equiv 0(\text{mod}2)$ and $x \equiv 0(\text{mod}2)$ or $z \equiv 1(\text{mod}2)$ and $x \equiv 1(\text{mod } 2)$. Thus, if $z = 2z_1$ and $x = 2x_1$, $z_1, x_1 > 0$. Hence (2.14) becomes

$$(4357^{z_1} - 33^{x_1})(4357^{z_1} + 33^{x_1}) = 4355^y 2^{r(y-z)}. \tag{2.16}$$

So,

$$67^y | \, 4357^{z_1} - 33^{x_1} \; \textit{or} \; \; 67^y | \, 4357^{z_1} + 33^{x_1}, \tag{2.17}$$

where $(4357^{z_1} - 33^{x_1}, 4357^{z_1} + 33^{x_1}) = 2$. But

$$\begin{aligned} 67^y > 67^z = 4489^{z_1} &> (4357 + 33)^{z_1} \\ &> 4357^{z_1} + 33^{z_1} \\ &> 4357^{z_1} + 33^{x_1} \\ &> 4357^{z_1} - 33^{x_1} \end{aligned}$$

and this contradicts (2.17). Also, if $z = 2z_1 + 1$ and $x = 2x_1 + 1$, $z_1, x_1 > 0$ then from equation (2.14) we obtain

$$4357^z - 33^x \equiv 0 (\text{mod} 4324)$$

But,

$$4355^y 2^{(y-z)} \not\equiv 0 \text{ mod } (4324)$$

2. If $k = 3^s n_1$ where $s \geqslant 1$, $r = q = 0$ and $(60, n_1) = 1$, then, (2.12) becomes

$$132^x = 3^{s(z-x)} {n_1}^{z-x} \left[\; 4357^z - 4355^y 3^{s(y-z)} {n_1}^{y-z} \right] \tag{2.18}$$

Thus $x = s(z-x)$ and $44^x = {n_1}^{z-x} \left[4357^z - 4355^y 3^{s(y-z)} {n_1}^{y-z} \right]$, hence $n_1 = 1$ and

$$4357^z - 44^x = 4355^y 3^{s(y-z)}, \tag{2.19}$$

where $(60, n_1) = 1$, So, $4357^z - 44^x \equiv 2^z - 44^x \equiv 0(\text{mod}67)$. Thus $z \equiv 0(\text{mod}2)$ and $x \equiv 0(\text{mod}2)$ or $z \equiv 1(\text{mod}2)$ and $x \equiv 1(\text{mod}2)$. Thus, If $z = 2z_1$ and $x = 2x_1$, $z_1, x_1 > 0$. Hence (2.19) becomes

$$(4357^{z_1} - 44^{x_1})(4357^{z_1} + 44^{x_1}) = 4355^y 3^{s(y-z)}. \tag{2.20}$$

So,

$$67^y | \, 4357^{z_1} - 44^{x_1} \; or \; 67^y | \, 4357^{z_1} + 44^{x_1} \tag{2.21}$$

where $(4357^{z_1} - 44^{x_1}, 4357^{z_1} + 44^{x_1}) = 1$. But

$$\begin{aligned} 67^y > 67^z = 4489^{z_1} &> (4357 + 44)^{z_1} \\ &> 4357^{z_1} + 44^{z_1} \\ &> 4357^{z_1} + 44^{x_1} \\ &> 4357^{z_1} - 44^{x_1} \end{aligned}$$

and this contradicts (2.21). Also if $z = 2z_1 + 1$ and $x = 2x_1 + 1$, $z_1, x_1 > 0$ then from equation (2.19) we obtain

$$4357^z - 44^x \equiv 0(\text{mod}4313)$$

But

$$4355^y 3^{s(y-z)} \not\equiv 0 \text{ mod } (4313)$$

.

3. If $k = 11^q n_1$ where $q \geqslant 1$, $r = s = 0$ and $(66, n_1) = 1$ then, from (2.12) we get the equation

$$132^x = 11^{q(z-x)} {n_1}^{z-x} \left[4357^z - 4355^y 11^{q(y-z)} {n_1}^{y-z} \right], \tag{2.22}$$

thus, $n_1 = 1$. Therefore

$$4357^z - 12^x = 4355^y 11^{q(y-z)}. \tag{2.23}$$

Since, $4357^z - 12^x \equiv 2^z - 12^x \equiv 0(\text{mod}67)$, hence $z \equiv 0(\text{mod}2)$ and $x \equiv 0(\text{mod}2)$ or $z \equiv 1(\text{mod}2)$ and $x \equiv 1(\text{mod}2)$. Thus if $z = 2z_1$ and $z = 2x_1$, $z_1, x_1 > 0$ then equation (2.23) becomes

$$(4357^{z_1} - 12^{x_1})(4357^{z_1} + 12^{x_1}) = 4355^y 11^{q(y-z)} \tag{2.24}$$

So

$$67^y | \, 4357^{z_1} - 12^{x_1} \ \textit{or} \ \ 67^y | \, 4357^{z_1} + 12^{x_1} \tag{2.25}$$

where $(4357^{z_1} - 12^{x_1}, 4357^{z_1} + 12^{x_1}) = 1$. But

$$\begin{aligned} 67^y > 67^z = 4489^{z_1} &> (4357 + 12)^{z_1} \\ &> 4357^{z_1} + 12^{z_1} \\ &> 4357^{z_1} + 12^{x_1} \\ &> 4357^{z_1} - 12^{x_1} \end{aligned}$$

and this contradicts (2.25). Also if $z = 2z_1 + 1$ and $x = 2x_1 + 1$, $z_1, x_1 > 0$ then from equation (2.23) we obtain

$$4357^z - 12^x \equiv 0(\text{mod}4345)$$

But

$$4355^y 11^{q(y-z)} \not\equiv 0 \bmod (4345)$$

.

4. If $k = 2^r 3^s n_1$ where $r \geqslant 1, s \geqslant 1, q = 0$ and $(66, n_1) = 1$ then, from (2.12) we get the equation

$$132^x = 2^{r(z-x)} 3^{s(z-x)} {n_1}^{z-x} \left[4357^z - 4355^y 2^{r(y-z)} 3^{s(y-z)} {n_1}^{y-z}\right]. \tag{2.26}$$

Thus $2x = r(z-x), x = s(z-x)$ and

$$11^x = {n_1}^{z-x} \left[4357^z - 4355^y 2^{r(y-z)} 3^{s(y-z)} {n_1}^{y-z}\right] \tag{2.27}$$

Since, $(66, n_1) = 1$ then, $n_1 = 1$. Therefore,

$$4357^z - 11^x = 4355^y 2^{r(y-z)} 3^{s(y-z)} \tag{2.28}$$

Since, $4357^z - 11^x \equiv 2^z - 11^x \equiv 0(\text{mod}67)$, hence $z \equiv 0(\text{mod}2)$ and $x \equiv 0(\text{mod}2)$ or $z \equiv 1(\text{mod}2)$ and $x \equiv 1(\text{mod}2)$. Thus if $z = 2z_1$ and $z = 2x_1$, $z_1, x_1 > 0$ then equation (2.28) becomes

$$(4357^{z_1} - 11^{x_1})(4357^{z_1} + 11^{x_1}) = 4355^y 2^{r(y-z)} 3^{s(y-z)} \tag{2.29}$$

So,

$$67^y | \, 4357^{z_1} - 11^{x_1} \ \text{or} \ 67^y | \, 4357^{z_1} + 11^{x_1} \tag{2.30}$$

where $(4357^{z_1} - 11^{x_1}, 4357^{z_1} + 11^{x_1}) = 2$. But

$$\begin{aligned} 67^y > 67^z = 4489^{z_1} &> (4357 + 11)^{z_1} \\ &> 4357^{z_1} + 11^{z_1} \\ &> 4357^{z_1} + 11^{x_1} \\ &> 4357^{z_1} - 11^{x_1} \end{aligned}$$

and this contradicts (2.30). Also if $z = 2z_1 + 1$ and $x = 2x_1 + 1$, $z_1, x_1 > 0$ then from equation (2.28) we obtain,

$$4357^z - 11^x \equiv 0(\text{mod}4346)$$

But

$$4355^y 2^{r(y-z)} 3^{s(y-z)} \not\equiv 0 \text{ mod } (4346)$$

.

5. If $k = 2^r 11^q n_1$ where $r \geqslant 1, q \geqslant 1, s = 0$ and $(66, n_1) = 1$ then, from (2.12) we get the equation

$$132^x = 2^{r(z-x)} 11^{q(z-x)} {n_1}^{z-x} \left[4357^z - 4355^y 2^{r(y-z)} 11^{q(y-z)} {n_1}^{y-z}\right], \tag{2.31}$$

Thus $2x = r(z - x), x = q(z - x)$ and

$$3^x = {n_1}^{z-x} \left[4357^z - 4355^y 2^{r(y-z)} 11^{q(y-z)} {n_1}^{y-z}\right] \tag{2.32}$$

Since, $(66, n_1) = 1$ then, $n_1 = 1$. Therefore,

$$4357^z - 3^x = 4355^y 2^{r(y-z)} 11^{q(y-z)} \tag{2.33}$$

Since, $4357^z - 3^x \equiv 2^z - 3^x \equiv 0(\text{mod}67)$, hence, $z \equiv 0(\text{mod}2)$ and $x \equiv 0(\text{mod}2)$ or $z \equiv 1(\text{mod}2)$ and $x \equiv 1(\text{mod}2)$. Thus, if $z = 2z_1$ and $z = 2x_1$, $z_1, x_1 > 0$ then equation (2.33) becomes

$$(4357^{z_1} - 3^{x_1})(4357^{z_1} + 3^{x_1}) = 4355^y 2^{r(y-z)} 11^{q(y-z)} \tag{2.34}$$

So,

$$67^y | \, 4357^{z_1} - 3^{x_1} \ \ or \ \ 67^y | \, 4357^{z_1} + 3^{x_1} \tag{2.35}$$

where $(4357^{z_1} - 3^{x_1}, 4357^{z_1} + 3^{x_1}) = 2$. But

$$\begin{aligned} 67^y > 67^z = 4489^{z_1} &> (4357+3)^{z_1} \\ &> 4357^{z_1} + 3^{z_1} \\ &> 4357^{z_1} + 3^{x_1} \\ &> 4357^{z_1} - 3^{x_1} \end{aligned}$$

and this contradicts (2.35). Also if $z = 2z_1 + 1$ and $x = 2x_1 + 1$, $z_1, x_1 > 0$ then from equation (2.33) we obtain

$$4357^z - 3^x \equiv 0(\text{mod}4354)$$

But

$$4355^y 2^{r(y-z)} 11^{q(y-z)} \not\equiv 0 \text{ mod } (4354)$$

.

6. If $k = 3^s 11^q n_1$ where $s \geqslant 1, q \geqslant 1, r = 0$ and $(66, n_1) = 1$ then, from (2.12) we get the equation

$$132^x = 3^{s(z-x)} 11^{q(z-x)} {n_1}^{z-x} \left[4357^z - 4355^y 3^{s(y-z)} 11^{q(y-z)} {n_1}^{y-z}\right], \tag{2.36}$$

Thus $x = s(z-x) = q(z-x)$ and

$$4^x = {n_1}^{z-x} \left[4357^z - 4355^y 3^{s(y-z)} 11^{q(y-z)} {n_1}^{y-z}\right] \tag{2.37}$$

Since, $(66, n_1) = 1$ then, $n_1 = 1$. Therefore,

$$4357^z - 4^x = 4355^y 3^{s(y-z)} 11^{q(y-z)} \tag{2.38}$$

Since, $4357^z - 4^x \equiv 2^z - 4^x \equiv 0(\text{mod}67)$, hence $z \equiv 0(\text{mod}2)$. Thus $z = 2z_1, z_1 > 0$ then equation (2.38) becomes

$$(4357^{z_1} - 2^{x_1})(4357^{z_1} + 2^{x_1}) = 4355^y 3^{s(y-z)} 11^{q(y-z)} \tag{2.39}$$

So,

$$67^y | \, 4357^{z_1} - 3^{x_1} \; or \; 67^y | \, 4357^{z_1} + 2^{x_1} \tag{2.40}$$

where $(4357^{z_1} - 2^{x_1}, 4357^{z_1} + 2^{x_1}) = 1$. But

$$\begin{aligned} 67^y > 67^z = 4489^{z_1} &> (4357 + 4)^{z_1} \\ &> 4357^{z_1} + 4^{z_1} \\ &> 4357^{z_1} + 4^{x_1} \\ &> 4357^{z_1} - 4^{x_1} \end{aligned}$$

and this contradicts (2.40).

7. If $k = 2^r 3^s 11^q n_1$ where $s \geqslant 1, s \geqslant 1, q \geqslant 1$, and $(66, n_1) = 1$ then, from (2.12) we get the equation

$$n_1{}^{z-x}\left[4357^z - 4355^y 2^{r(y-z)} 11^{q(y-z)} 3^{s(y-z)} n_1{}^{y-z}\right] = 1 \tag{2.41}$$

Since, $x \neq z$ then $n_1 = 1$. Therefore

$$4357^z - 1 = 4355^y 2^{r(y-z)} 11^{q(y-z)} 3^{s(y-z)} \tag{2.42}$$

Since $4357^z - 1 \equiv 2^z - 1(\text{mod}5)$ hence $z \equiv 0(\text{mod}2)$. Thus $z = 2z_1, z_1 > 0$. But $4357^2 \equiv 1(\text{mod}2179)$ implies $4357^z - 1 \equiv 0(\text{mod}\ 2179)$. Then from (2.42) we obtain

$$4355^y 2^{r(y-z)} 11^{q(y-z)} 3^{s(y-z)} \equiv 0(\text{mod}2179),$$

which is impossible. This completes the proof for the first case.

Case 2 If $(x > y)$, then we obtain two subcases $z < y < x$ and $y < z < x$.

Subcase 2.1 If $(z < y < x)$ then, rewrite equation (1.6) as

$$k^{y-z}(132^x k^{x-y} + 4355^y) = 4357^z \tag{2.43}$$

So if $(k, 4357) = 1$, then $y = z$, where $k \geqslant 2$, which is a contradiction. And if $(k, 4357) = 4357$, then we can write $k = 4357^m n_1$, where $m \geqslant 1$ and $4357 \nmid n_1$, So rewrite equation (2.43) as

$$4357^{m(y-z)} n_1{}^{y-z}(132^x 4357^{m(x-y)} n_1{}^{x-y} + 4355^y) = 4357^z \tag{2.44}$$

Since,

$$(n_1, 4357) = (132^x 4357^{m(x-y)} {n_1}^{x-y} + 4355^y, 4357) = 1$$

hence,

$$ {n_1}^{y-z}(132^x 4357^{m(x-y)} {n_1}^{x-y} + 4355^y) = 1$$

which is impossible.

Subcase 2.2 If $(y < z < x)$ then, rewrite (1.6) as

$$k^{z-y}(4357^z - 132^x k^{x-z}) = 4355^y \tag{2.45}$$

So if $(k, 4355) = 1$, then $y = z$, where $k \geqslant 2$, which is a contradiction. And if $(k, 4355) > 1$, then we can write $k = 5^r 13^q 67^q n_1$, where $r + q + q \geqslant 1$ and $(4355, n_1) = 1$, So rewrite (2.45) as

$$5^{r(z-y)} 13^{p(z-y)} 67^{q(z-y)} {n_1}^{z-y}(4357^z - 132^x 5^{r(x-z)} 13^{p(x-z)} 67^{q(x-z)} {n_1}^{x-z}) = 4355^y \tag{2.46}$$

Since,

$$(n_1, 4355) = {n_1}^{z-y}(4357^z - 132^x 5^{r(x-z)} 13^{p(x-z)} 67^{q(x-z)} {n_1}^{x-z}) = 1$$

Then,

$${n_1}^{z-y}(4357^z - 132^x 5^{r(x-z)} 13^{p(x-z)} 67^{q(x-z)} {n_1}^{x-z}) = 1$$

and $r(z-y) = p(z-y) = q(z-y) = y$ then, $r = p = q$ also $n_1 = 1$. Thus equation (2.46) becomes

$$4357^z - 1 = 132^x 4355^{r(x-z)} \tag{2.47}$$

Since $4357^z - 1 \equiv 2^z - 1(\text{mod}5)$ hence $z \equiv 0(\text{mod}2)$. But $4357^2 \equiv 1(\text{mod}\ 2179)$ implies $4357^z - 1 \equiv 0(\text{mod}2179)$, so from (2.47) we obtain

$$132^x 4355^{r(x-z)} \equiv 0(\text{mod}2179),$$

which is impossible. Thus, completes the proof for the second case and then **this completes the proof of theorem** (1.1**).** □

3 conclusion

We have obtained a new Pythagorean triple for Jeśmanowicz's conjecture and proved that the special Diophantine equation $(132k)^x + (4355k)^y = (4357k)^z$ has the only positive integer solution $(x, y, z) = (2, 2, 2)$ for every positive integer k.

Acknowledgments

This work is supported by the Research University (RU) funding, account number 1001 /PMATHS/ 811337, Universiti Sains Malaysia.

References

[1] Moujie Deng, G. L. Cohen, On the conjecture of jesmanowicz concerning Pythagorean triples, Bulletin of the Australian Mathematical Society, **57,** (1998), no. 3, 515–524.

[2] L. Jeśmanowicz, Several remarks on Pythagorean numbers, Wiadom. Mat. (2) **1,** (1955/1956), 196–202.

[3] Maohua Le, A note on Jeśmanowicz'conjecture concerning Pythagorean triples, Bulletin of the Australian Mathematical Society, **59,** (1999), no. 3, 477–480.

[4] Wen-twan Lu, On the Pythagorean numbers $4n^2 - 1, 4n$ and $4n^2 + 1$, Acta Sci. Natur. Univ. Szechuan,(1959), 2: 39–42. (in Chinese)

[5] Mi-Mi Ma, Jian-Dong Wu, On the Diophantine equation $(an)^x + (bn)^y = (cn)^z$, Bulletin of the Korean Mathematical Society, **52,** no. 4, (2015), 1133–1138.

[6] W. Sierpiński, On the equation $3^x + 4^y = 5^z$, Wiadom. Mat. (2) **1,** (1955/1956), 194–195.

[7] Soydan, Göokhan Soydan, Musa Demirci,Ismail Naci Cangul, Alain Togbé, On the conjecture of Jeśmanowicz, International Journal of Applied Mathematics and Statistics, **56,** (2017), no. 6, 46–72.

[8] Cuifang Sun, Zhi Cheng, On Jeśmanowicz' conjecture concerning pythagorean triples, Journal of Mathematical Research with Applications, **35,** no. 2, (2015), 143–148.

[9] Min Tang, Jian-Xin Weng, Jeśmanowicz' conjecture with Fermat numbers, Taiwanese J. Math., **18,** no. 3, (2014), 925–930.

[10] Min Tang, Zhi-Juan Yang, Jeśmanowicz' conjecture revisited, Bulletin of the Australian Mathematical Society, **88,** no. 3, (2013), 486–491.

[11] Zhi-Juan Yang, Min Tang, On the Diophantine equation $(8n)^x + (15n)^y = (17n)^z$, Bulletin of the Australian Mathematical Society, **86,** no. 2, (2012), 348–352.

[12] Zhi-Juan Yang, W. Jianxin, On the Diophantine equation $(12n)^x + (35n)^y = (37n)^z$, Pure and App. Math. (Chinese), **28,** (2012), 698–704.