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Algebraic Properties of ω-Q-fuzzy subgroups

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Abstract

In this paper, we first define ω-Q-fuzzy sets and then explain the idea of ω-Q-fuzzy subgroups and prove that the product of two ω-Q-fuzzy subgroups is an ω-Q-fuzzy subgroup. We also study the ω-Q-fuzzy normal subgroups and discover their several algebraic properties.

1 Introduction

Zadeh [19] launched the study of fuzzy sets in 1965. In the view of group theory, Rosenfield [11] invented the theory of fuzzy groups using the idea of a fuzzy set in 1971. Liu [2] described the invariant subgroups in 1982. Mukherjee and Bhttacharya [7] initiated the concept of fuzzy cosets in 1984.

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Afterwards, Biwas [3] introduced anti-fuzzy subgroups in 1990. Sidky and Mishref [18] introduced the λ-cycle and λ abelian fuzzy subgroups and described some features of fuzzy cosets and S-fuzzy subgroups in 1991. Gupta and Qi [4] reviewed the theory of t-norms and t-conorms and also defined classical t-operators in 1991. Naseem Ajmal [1] explored the inner structure of fuzzy subgroups with the sup property in 1996. Ray [10] developed the aspects of the product of two fuzzy subsets and fuzzy subgroups in 1999. Solairaju [13] described a new structure and construction of Q-fuzzy groups in 2009. Later on, Muthuraj et al. [8] proposed the study of a lower level subset of anti-Q-fuzzy subgroups in 2010. Kang and Kul Hur [6] described the notion of interval-valued fuzzy subgroups in 2010. Sharma [14] considered α-anti fuzzy subgroups and depicted their several algebraic properties in 2012. Sharma [15] defined the α-fuzzy subgroup and demonstrated their various algebraic properties in 2013. The concept of Q-fuzzy normal subgroups and Q-fuzzy normalizer were established by Priya et al. [9] in 2013. Umer Shuaib et al. [16] described the feature of o-fuzzy subgroups in 2018. Umer Shuaib et al. [17] depicted the various algebraic facts of 0-anti fuzzy subgroups in 2018. In addition, more recent developments on Q-fuzzy subgroups may be viewed in [5,12].
This paper is arranged as follows. Section 2 contains the elementary definitions and related results which are thoroughly crucial to this research paper. In section 3, we define ω-Q-fuzzy set with respect to Q-fuzzy set and describe the algebraic structure of ω-Q-fuzzy set. In section 4, we define the ω-QFS, ω-QFNS and the addition theory of ω-QFS of a group. This constitutes the ω-Q-fuzzy version of group theory. We also study ω-Q-fuzzy cosets, ω-QFNS and their algebraic structure.

2 Preliminaries

We recall the elementary notion of fuzzy sets which play a key role for our further analysis.

Definition (2.1) [19]: A fuzzy set A of a non empty set P is a function

$$A : P \longrightarrow [0,\ 1].$$

Definition (2.2) [11]: A fuzzy subset A of a group H is called a fuzzy subgroup if $A(m^{-1}n) \geq \min\{A(m), A(n)\}$ for all $m, n \in H$.

Definition (2.3) [13]: Let P and Q be two non empty sets. A Q- fuzzy subset A of set P is a function

$$A : P \times Q \longrightarrow [0,\ 1].$$

Definition (2.4) [13]: A Q-fuzzy subset A of a group H is called a QFS if $A(m^{-1}n, q) \geq \min\{A(m, q), A(n, q)\}$ for all $m, n \in H$ and $q \in Q$.

Definition (2.5) [9]: A QFS of a group H is said to be a QFNS if $A(mn, q) = A(nm, q)\}$ for all $m, n \in H$ and $q \in Q$.

Definition (2.6) [9]: Let A be QFS of a group H. For arbitrary $m \in H$, the fuzzy set mA of H is called a Q-fuzzy left coset of A if $(mA)(n, q) = A(m^{-1}n, q)$ for all $n \in H$ and $q \in Q$.

Similarly, Am is called the Q-fuzzy right coset of A if $(Am)(n, q) = A(nm^{-1}, q)$ for all $n \in H$ and $q \in Q$.

Definition (2.7) [4]: The t-norm is a function that maps in the pair of number in the unit interval into the unit interval, such as: $t : [0, 1] \times [0, 1] \longrightarrow [0,\ 1]$. The following axiom hold for all $n_1, n_2, n_3, n_4 \in [0, 1]$.

1. $t(n_1, n_2) = t(n_2, n_1)$
2. $t(n_1, t(n_2, n_3)) = t(t(n_1, n_2), n_3)$
3. $t(n_1, 1) = t(1, n_1) = 1$
4. If $n_1 \leq n_3$ and $n_2 \leq n_4$ then $t(n_1, n_2) \leq t(n_3, n_4)$.

Definition (2.8) [4]: Let $t_p : [0, 1] \times [0, 1] \longrightarrow [0,\ 1]$ be the algebraic product t-norm on $[0, 1]$ defined by $t_p\{n_1, n_2\} = n_1 n_2$,$0 \leq n_1 \leq, 0 \leq n_2 \leq 1$.

Clearly, algebraic product t-norm admits all the axioms of t-norm.

3 ω-Q-fuzzy subsets and their properties

Definition (3.1) Let P and Q be any two nonempty sets and A be a Q-fuzzy subset of a set P and $\omega \in [0, 1]$. The fuzzy set A^ω of P is called the ω-Q-fuzzy subset of P (w.r.t Q-fuzzy set A) and defined by

$A^{\omega}(m,q) = t_p(A(m,q),\omega)$, for all $m \in P$ and $q \in Q$.

Remark (3.2): Obviously, $A^0(m,q) = 0$ and $A^1(m,q) = A(m,q)$.

Theorem (3.3): Let A_1 and A_2 be any two arbitrary Q-fuzzy subsets of P. Then $(A_1 \cap A_2)^{\omega} = A_1^{\omega} \cap A_2^{\omega}$.
Proof: $(A_1 \cap A_2)^{\omega}(m_1,q) = t_p\{(A_1 \cap A_2)(m_1,q),\omega\} = t_p\{\min\{A_1(m_1,q), A_2(m_1,q),\omega\}\} = \min\{t_p\{A_1(m_1,q),\omega\}, t_p\{A_2(m_1,q),\omega\}\} = \min\{A_1^{\omega}(m_1,q), A_2^{\omega}(m_1,q)\} = (A_1^{\omega} \cap A_2^{\omega})(m_1,q)$, for all $m_1 \in P$. This implies that $(A_1 \cap A_2)^{\omega} = A_1^{\omega} \cap A_2^{\omega}$.

4 ω-Q-fuzzy subgroups

In this section, we clarify the idea of ω-QFS and ω-QFNS. We show that each QFS(QFNS) is also ω-QFS(QFNS). The opinion of ω-Q-fuzzy coset is discussed in this section.

Definition (4.1): A Q-fuzzy subset of a group H is called ω-QFS, and $\omega \in [0,1]$, if

1. $A^{\omega}(mn,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\}$ for all $m,n \in H$ and $q \in Q$
2. $A^{\omega}(m^{-1},q) = A^{\omega}(m,q)$.

Theorem (4.2): If $A : H \times Q \longrightarrow [0,1]$ is a ω-QFS of a group H, then

1. $A^{\omega}(mn^{-1},q) = A^{\omega}(e,q)$ which implies that $A^{\omega}(m,q) = A^{\omega}(n,q)$ for all $m,n \in H$ and $q \in Q$.
2. $A^{\omega}(m,q) \leq A^{\omega}(e,q)$, for all $m \in H$ and $q \in Q$ where e is the identity element of H.

Proof: (1) $A^{\omega}(m,q) = A^{\omega}(mn^{-1}n,q) \geq \min\{A^{\omega}(mn^{-1},q), A^{\omega}(n,q)\} = \min\{A^{\omega}(e,q), A^{\omega}(n,q),\} = A^{\omega}(n,q)$.
Hence , $A^{\omega}(m,q) \geq A^{\omega}(n,q)$.
Similarly, $A^{\omega}(n,q) \geq A^{\omega}(m,q)$.
This implies that $A^{\omega}(n,q) = A^{\omega}(m,q)$ for all $m,n \in H$
(2) $A^{\omega}(e,q) = A^{\omega}(mm^{-1},q) \geq \min\{A^{\omega}(m,q), A^{\omega}(m^{-1},q)\} = \min\{A^{\omega}(m,q), A^{\omega}(m,q)\} = A^{\omega}(m,q)$.
Hence, $A^{\omega}(m,q) \leq A^{\omega}(e,q)$, for all $m \in H$ and $q \in Q$.

Theorem (4.3): Every QFS of a group H is a ω-QFS of H.

Proof: Assume that A is a QFS of a group H and for all $m, n \in H$ and $a \in Q$. Consider, $A^{\omega}(mn, q) = t_p\{A(mn, q), \omega\} \geq t_p\{\min\{A(m, q), A(n, q), \omega\} = \min\{t_p\{A(m, q), \omega\}, t_p\{A(n, q), \omega\}\} = \min\{A^{\omega}(m, q), A^{\omega}(n, q)\}$.
$A^{\omega}(mn, q) \geq \min\{A^{\omega}(m, q), A^{\omega}(n, q)\}$.
Further $A^{\omega}(m^{-1}, q) = t_p\{A(m^{-1}, q), \omega\} = t_p\{A(m^{-1}, q), \omega\} = A^{\omega}(m, q)$
$A^{\omega}(m, q) = A^{\omega}(m^{-1}, q)$.
Consequently, A is ω-QFS of H.
The converse of the above theorem may not be true in general.
Note (4.4): We take $Q = \{q\}$ in all examples.
Example (4.5): Let $H = \{e, m, n, mn\}$, where $m^2 = n^2 = e$ and $mn = nm$ be a group and $Q = \{q\}$. Let the Q-fuzzy set A of H described by

$$A(x, q) = \begin{cases} 0.3 & \text{if } x = e \\ 0.5 & \text{if } x = m \text{ or } n \\ 0.4 & \text{if } x = mn \end{cases}$$

Take $\omega = 0$. Then
$A^{\omega}(x, q) = t_p\{A(x, q), \omega\} = t_p\{A(x, q), 0\} = 0$,for all $x \in H$.

This implies that $A^{\omega}(mn, q) \geq \min A^{\omega}(m, q), A^{\omega}(n, q)$. Further, we have $m^{-1} = m, n^{-1} = n$ and $(mn)^{-1} = mn$. Hence, $A^{\omega}(x^{-1}, q) = A^{\omega}(x, q)$,for all $x \in G$.

Consequently, A is ω-QFS of of H and obviously A is not QFS of H.

Theorem (4.6): The intersection of two ω-QFSs of a group H is also ω-QFS of H.
Proof: Let A and B be two ω-QFSs of a group H and for all $m_1, m_2 \in H$ and $q \in Q$.
$(A \cap B)^{\omega}(m_1 m_2, q) = (A^{\omega} \cap B^{\omega})(m_1 m_2, q) = \min\{A^{\omega}(m_1 m_2, q), B^{\omega}(m_1 m_2, q)\}$
$\geq \min\{\min\{A^{\omega}(m_1, q), A^{\omega}(m_2, q)\}, \{B^{\omega}(m_1, q), B^{\omega}(m_2, q)\}\}$
$= \min\{\min\{A^{\omega}(m_1, q), B^{\omega}(m_1, q), A^{\omega}(m_2, q), B^{\omega}(m_2, q)\}\}$
$= \min\{(A \cap B)^{\omega}(m_1, q), (A \cap B)^{\omega}(m_2, q)\}$
Thus, $(A \cap B)^{\omega}(m_1 m_2, q) \geq \min\{(A \cap B)^{\omega}(m_1, q), (A \cap B)^{\omega}(m_2, q)\}$.
Moreover,
$(A \cap B)^{\omega}(m_1^{-1}, q) = (A^{\omega} \cap B^{\omega})(m_1^{-1}, q) = \min\{(A^{\omega}(m_1^{-1}, q), B^{\omega}(m_1^{-1}, q)\} = \min\{(A^{\omega}(m, q), B^{\omega}(m, q)\}$.

Hence $(A \cap B)^{\omega}(m_1^{-1}, q) = (A \cap B)^{\omega}(m_1, q)$.
Consequently, $(A \cap B)$ is ω-QFA of H.

Corollary (4.7): The intersection of any family of ω-QFSs of a group H is also ω-QFS of H.

Remark (4.8): Union of two ω-QFSs of a group G need not be ω-QFS OF H.

Example (4.9): Let $Z = \{0, \pm 1, \pm 2, \ldots\}$ be a group under addition and $q \in Q$. Define the two Q-fuzzy subsets A and B of Z as follows

$$A(m,q) = \begin{cases} 0.6 & \text{if } m \in 3Z \\ 0.04 & \text{otherwise} \end{cases} \quad \text{and} \quad B(m,q) = \begin{cases} 0.4 & \text{if } m \in 2Z \\ 0.06 & \text{otherwise} \end{cases}$$

Taking $\omega = 1$, one can easily see that A and B are 1-fuzzy subgroup of Z. Now,

$$(A \cup B)(m,q) = \max\{A(m,q), B(m,q)\}$$

Therefore,

$$A \cup B = \begin{cases} 0.6 & \text{if } m \in 3Z \\ 0.4 & \text{if } m \in 2Z - 3Z \\ 0.06 & \text{if } m \notin 2Z \text{ or } m \notin 3Z \end{cases}$$

Take $m = 21$ and $n = 4$.
Then, $(A \cup B)(m,q) = 0.6$ and $(A \cup B)(n,q) = 0.4$
But $(A \cup B)(m-n,q) = 0.04$. In addition, $\min\{(A \cup B)(m,q), (A \cup B)(n,q)\} = \min\{0.6, 0.4\} = 0.4$,
Clearly, $(A \cup B)(m-n,q) < \min\{(A \cup B)(m,q), (A \cup B)(n,q)\}$.

Consequently, $A \cup B$ is not 1-QFS of Z. Hence, the union of two ω-QFSs of Z is not ω-QFS of Z.

Definition (4.10): Let A and B two ω-QFSs of group H_1 and H_2 respectively. then product of A and B is defined as $A^\omega \times B^\omega((m_1,n_1),q) = \min\{A^\omega(m_1,q), B^\omega(n_1,q)\}$, for all $m_1 \in H_1$, $n_1 \in H_2$ and $q \in Q$.

Theorem (4.11): Let A and B two ω-QFSs of subgroups H_1 and H_2 respectively. Then $A^\omega \times B^\omega$ is ω-QFS of $H_1 \times H_2$.

Proof: Let $m_1, m_2 \in H_1$ and $n_1, n_2 \in H_2$ then $(m_1,n_1)(m_2,n_2) \in H_1 \times H_2$ and $q \in Q$. Now, $A^\omega \times B^\omega((m_1,n_1)(m_2^{-1},n_2^{-1}),q) = A^\omega \times B^\omega((m_1m_2^{-1}, n_1n_2^{-1},),q)$
$= \min\{A^\omega(m_1m_2^{-1},q), B^\omega(n_1n_2^{-1},q)\}$
$\geq \min\{\min\{A^\omega(m_1,q), A^\omega(m_2^{-1},q)\}, \{\min\{B^\omega(n_1,q), B^\omega(n_2^{-1},q)\}\}$
$\geq \min\{\min\{A^\omega(m_1,q), A^\omega(m_2,q)\}, \{\min\{B^\omega(n_1,q), B^\omega(n_2,q)\}\}$
$= \min\{\min\{A^\omega(m_1,q), B^\omega(n_1,q)\}, \{\min\{A^\omega(m_2,q), B^\omega(n_2,q)\}\} = \min\{A^\omega \times B^\omega((m_1,n_1),q), A^\omega \times B^\omega((m_2,n_2),q)\}$.
Hence, $A^\omega \times B^\omega((m_1,n_1)(m_2^{-1},n_2^{-1}),q) \geq \min\{A^\omega \times B^\omega((m_1,n_1),q), A^\omega \times$

$B^{\omega}((m_2, n_2), q)\}$.

Definition (4.12): Let A be a ω-QFS of a group H and $\omega \in [0, 1]$. For any $m \in H$ and $q \in Q$. The ω-Q-fuzzy left coset of A in H is represented by mA^{ω} as defined as:

$$(mA^{\omega})(h, q) = t_p\{A(m^{-1}h, q), \omega\},$$

for all $m, h \in G$ and $q \in Q$.
Similarly, we define the ω-fuzzy right coset of A in H denoted by $A^{\omega}m$ and defined as:

$$(A^{\omega}m)(h, q) = t_p\{A(hm^{-1}, q), \omega\},$$

for all $m, h \in G$ and $q \in Q$.

Remark (4.13): Let A be a ω-QFS of a group H and $\omega \in [0, 1]$. Then A is called ω-QFNS of H if and only if $mA^{\omega} = A^{\omega}m$, for all $m \in H$.

Theorem (4.14): Let A be a ω-QFNS of a group H. Then $A^{\omega}(n^{-1}mn, q) = A^{\omega}(m, q)$, or equivalently,
$A^{\omega}(mn, q) = A^{\omega}(nm, q)$, for all $m, n \in H$ and $q \in Q$.
Proof: Since A be ω-QFNS of a group H, $mA^{\omega} = A^{\omega}m, \forall\ m \in H$ and $q \in Q$. Thus $(mA^{\omega})(n^{-1}, q) = (A^{\omega}m)(n^{-1}, q) \implies t_p\{A(m^{-1}n^{-1}, q), \omega\} = t_p\{A(n^{-1}m^{-1}, q), \omega\}$ which implies that $A^{\omega}((nm)^{-1}, q) = A^{\omega}((mn)^{-1}, q)$ as A is ω-QFS of H so $A^{\omega}(g^{-1}, q) = A^{\omega}(g, q)$, $\forall g \in H$ and $q \in Q$. Consequently, $A^{\omega}(nm, q) = A^{\omega}(mn, q)$.

Theorem (4.15): Every QFNS of a group H is a ω-QFNS of H.
Proof: Let A be a QFNS of a group H. Then for all $m \in H$. We have $mA = Am$ which implies that $(mA)(h, q) = (mA)(h, q)$, for all $h \in H$ and $q \in Q$. So $A(m^{-1}h, q) = A(hm^{-1}, q)$. Hence $t_p\{A(m^{-1}h, q), \omega\} = t_p\{A(hm^{-1}, q), \omega\}$; i.e., $(mA^{\omega})(h, q) = (A^{\omega}m)(h, q)$. Therefore, $mA^{\omega} = A^{\omega}m$, for all $m \in H$.
Consequently, A is ω-QFNS of H. The converse of the above theorem may not to be true.

Example (4.16): Take the dihedral group of order six with finite presentation $H = D_3 = \langle m, n : m^3 = n^2 = e,\ nm = m^2n\rangle$ and $q \in Q$. Define the QFS of D_3 by

$$A(y, q) = \begin{cases} 0.3 & \text{if } y \in < n > \\ 0.05 & \text{otherwise} \end{cases}$$

Taking $\omega = 0$, we have $(yA^{\omega})(g,q) = t_p\{A(y^{-1}g,q),\omega\} = t_p\{A(y^{-1}g,q),0\}$ $= A(y^{-1}g,q) \times 0 = 0 = 0 \times A(gy^{-1},q) = t_p\{A(gy^{-1},q),0\} = A^{\omega}y(g,q)$. Hence, $yA^{\omega} = A^{\omega}y$,for all $y \in H$. This shows that A is 0-QFNS of H. Now, $A(m^2(mn)) = A(m^3n) = A(n) = 0.3$

$$A((mn)m^2,q) = A(m(nm)m,q) = A(m(m^2n)m,q) = A(m^3nm,q) = A(nm,q) = 0.05$$

This implies that A is not QFNS of H.

Definition (4.17): Let A be a ω-QFNS of a group H. We define a set $H_{A^{\omega}} = \{m \in H : A^{\omega}(m,q)\} = A^{\omega}(e,q)$, where $e \in H$ and $q \in Q$.

In what follows we explain why the set $H_{A^{\omega}}$ is, in fact, a normal subgroup of H.

Theorem (4.18): Let H be a ω-QFNS of a group H. Then $H_{A^{\omega}} \trianglelefteq H$.
Proof: Obviously, $H_{A^{\omega}} \neq \phi$, for $e \in H_{A^{\omega}}$.

Let $m,n \in H_{A^{\omega}}$ be any element and $q \in Q$. Then we have $A^{\omega}(mn^{-1}q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\} = \min\{A^{\omega}(e,q), A^{\omega}(e,q)\}$

This implies that $A^{\omega}(mn^{-1},q) \geq A^{\omega}(e,q)$ but $A^{\omega}(mn^{-1},q) \leq A^{\omega}(e,q)$. Therefore, $A^{\omega}(mn1,q) = A^{\omega}(e,q)$.
$\implies mn^{-1} \in H_{A^{\omega}}$ Hence, $H_{A^{\omega}}$ is a subgroup of H.

Moreover, let $m \in H_{A^{\omega}}$, $n \in H$ and $q \in Q$. We have $A^{\omega}(n^{-1}mn,q) = A^{\omega}(m,q) = A^{\omega}(e,q)$ This implies that $n^{-1}mn \in H_{A^{\omega}}$. Consequently, $H_{A^{\omega}} \trianglelefteq H$.

Theorem (4.19): Let A be a ω-QFNS of H. Then
(i) $mA^{\omega} = nA^{\omega}$ iff $m^{-1}n \in H_{A^{\omega}}$
(ii) $A^{\omega}m = A^{\omega}n$ iff $mn^{-1} \in H_{A^{\omega}}$
Proof: Suppose that $mA^{\omega} = nA^{\omega}$ for all $m,n \in H$ and $q \in Q$.
We have, $A^{\omega}(m^{-1}n,q) = t_p\{A(m^{-1}n,q),\omega\}$,
So, $(mA^{\omega})(n,q) = (nA^{\omega})(n,q) = t_p\{A(n^{-1}n,q),\omega\} = t_p\{A(e,q),\omega\} = A^{\omega}(e,q)$, which implies that $m^{-1}n \in H_{A^{\omega}}$. Conversely, let $m^{-1}n \in H_{A^{\omega}}$. Then $A^{\omega}(m^{-1}n,q) = A^{\omega}(e,q)$. For any element $r \in H_{A^{\omega}}$, $(xA^{\omega})(r,q) = t_p\{A(x^{-1}r,q),\omega\} = A^{\omega}(m^{-1}r,q) = A^{\omega}((m^{-1}n)(n^{-1}r),q) \geq \min\{A^{\omega}(m^{-1}n,q), A^{\omega}(n^{-1}r,q)\} = \min\{A^{\omega}(e,q), A^{\omega}(n^{-1}r,q)\} = A^{\omega}(n^{-1}r,q) = (nA^{\omega})(r,q)$. Interchanging the roles of m and n, we get $(mA^{\omega})(r,q) = (nA^{\omega}))(r,q)$, for all $r \in H$. Consequently, $(mA^{\omega}) = (nA^{\omega})$
(ii) The proof of this part is similar to part (i).

Theorem (4.20):

Let A be a ω-QFNS of a group H and m, n, s, t be any element in H. If $(mH^{\omega}) = (sA\omega)$ and $(nA^{\omega}) = (tA^{\omega})$, then $(mnA^{\omega}) = (stA^{\omega})$

Proof: Given that $mA^{\omega} = sA^{\omega}$ and $nA^{\omega} = tA^{\omega}$, we have $m^{-1}s, n^{-1}t \in H_{A^{\omega}}$. Consider, $(mn)^{-1}st = n^{-1}(m^{-1}s)(nn^{-1})t = (n^{-1}(m^{-1}s)n)(n^{-1}t) \in H_{A^{\omega}}$. This implies that $(mn)^{-1}st \in H_{A^{\omega}}$. Consequently, $(mnA^{\omega}) = (stA^{\omega})$.

5 Conclusion

In this paper, the idea of a ω-Q-fuzzy subset, ω-QFS and ω-fuzzy cosets of a given group have been delineated. The idea of ω-QFNS has been innovated and several related properties were mentioned.

References

[1] N. Ajmal, Fuzzy groups with sup property, Information Sciences **93**, no. 4, (1996), 247-264.

[2] W. J. Liu, Fuzzy invariant subgroups and fuzzy ideals. Fuzzy Sets and Systems, **8,** (1982), 133-139.

[3] R. Biswas, Fuzzy subgroups and anti-fuzzy subgroups, Fuzzy Sets and Systems, **35,** (1990), 121–124.

[4] M. M. Gupta, J. Qi, Theory of T-norms and fuzzy inference methods, Fuzzy Sets and Systems, **40,** no. 3, (1991), 431–450.

[5] R. Jahir Hussain, A Review on Q-fuzzy subgroup in Algebra, International Journal of Applied Engineering Research **14**, no. 3 (2019), 60–63.

[6] H. W. Kang, K. Hur, Interval-valued fuzzy subgroups and Rings, Honam Mathematical Journal **32**, (2010), 593–617.

[7] N. P. Mukherjee, P. Bhattacharya, Fuzzy normal subgroups and fuzzy cosets, Inform. Sci.,**34,** (1984), 225–239.

[8] R. Muthuraj, P. M. Sitharselvam, M. S. Muthuraman, Anti Q-Fuzzy Group and Its Lower Level Subgroups, International Journal of Computer Application, **3**, no. 3, (2010), 16–20.

[9] T. Priya, T. Ramachandran, K. T. Nagalakshmi, On Q -fuzzy Normal Subgroups, International Journal of Computer and Organization Trends, **3**, no. 6, (2013), 39–42.

[10] A. K. Ray, On product of fuzzy subgroups, Fuzzy Sets and Systems, **105,** (1999), 181–183.

[11] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl, **35,** (1971), 512–517.

[12] M. M. Salih, D. H. Ahmed, α-Q-Fuzzy Subgroups, Academic Journal of Nawoz University, **6**, no. 3, (2017), 26–31.

[13] A. Solairaju, R. Nagarajan, A new structure and construction of Q-fuzzy groups, Advances in Fuzzy Mathematics, **4**, no. 1, (2009), 23–29.

[14] P. K. Sharma, α-Anti Fuzzy subgroups, International Review of Fuzzy Mathematics, **7**, no. 2, (2012), 47–58.

[15] P. K. Sharma, α-Fuzzy subgroups, International Journal of Fuzzy Mathematics and Systems, **3,** no. 1, (2013), 47–59.

[16] U. Shuaib, M. Sheheryar, W. Asghar, On some characterization of o-fuzzy subgroups, International Journal of Mathematics and Computer Science, **13,** no. 2 (2018), 119–131.

[17] U. Shuaib, M. Sheheryar, W. Asghar, On some characterization of o-anti fuzzy subgroups, International Journal of Mathematics and Computer Science 14, no. 1, (2018), 215–230.

[18] F. I. Sidky, M. A. Mishref, Fuzzy cosets and cyclic and Abelian fuzzy subgroups, Fuzzy Sets and Systems, **43,** no. 2, (1991), 243–250.

[19] L. A. Zadeh, Fuzzy sets, Inform. and Control, **8,** (1965), 338–353.