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Fixed Point Theorems for Functions satisfying Implicit Relations in Generalized b-Metric spaces

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Abstract

The aim of this paper is to present some fixed point results for functions satisfying implicit relations in two complete and compact Generalized b-Metric spaces

1 Introduction and Preliminaries

Mustafa and Sims [10] introduced the concept of G-metric spaces as a generalization of a metric space. Since then, several interesting results for existence of fixed point in G-metric spaces have been obtained (see [2-4],[7], [10–11]).

On the other hand, V. Papa et al. proved some fixed point theorems for functions satisfying an implicit relations in metric spaces and G-metric spaces (see [13-18]).

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The notion of b-metric space was introduced by Czerwik [8]. Many authors studied fixed point theorems in b-metic spaces (see [1],[6],[9],[12],[19]). Recently, Aghajani et al. [5] generalized the concept of b-metic spaces by using the notions of b-metric spaces and G-metric spaces, which is called G_b-metric spaces and they discussed some properties and common fixed point results of G_b-metric.

The aim of this paper is to present some fixed point results for functions satisfying implicit relations in two complete and compact Generalized b-Metric spaces.

We recall some basic definitions and results following [5].

Definition 1.1. *Let X be a non empty set and $\delta \geq 1$ be a given real number. Suppose that a mapping $G : X \times X \times X \to \mathbb{R}^+$ satisfies the following axioms:*

(G$_b$1) $G(x, y, z) = 0$ if $x = y = z$,
(G$_b$2) $0 < G(x, x, y)$, for all x, $y \in X$ with $x \neq y$,
(G$_b$3) $G(x, x, y) \leq G(x, y, z)$, for all x, y, $z \in X$, with $z \neq y$,
(G$_b$4) $G(x, y, z) = G(p(x, z, y))$ where p is a permutation of x, y, z (symmetry),
(G$_b$5) $G(x, y, z) \leq \delta(G(x, a, a)+G(a, y, z))$, for all x, y, z, $a \in X$, (triangle inequality).

Then the function G is called a generalized b-metric (G$_b$-metric on X) and the pair (X, G) is called a generalized b-metric space.

Proposition 1.2. *Let (X, G) be a G_b-metric space. Then for any x, y, z and $a \in X$,*

(1) if $G(x, y, z) = 0$, then $x = y = z$,
(2) $G(x, y, z) \leq \delta(G(x, x, y) + G(x, x, z))$,
(3) $G(x, y, y) \leq 2\delta G(y, x, x)$,
(4) $G(x, y, z) \leq \delta(G(x, a, z) + G(a, y, z))$.

Definition 1.3. *Let (X, G) be a G_b-metric space, and (x_n) be a sequence of points of X. We say that (x_n) is G_b-convergent to x if for any $\varepsilon > 0$, there exists $n_0 \in N$ such that $G(x, x_n, x_m) < \varepsilon$, for all $n, m \geq n_0$.*

Proposition 1.4. *Let (X, G) be a G_b-metric space. Then the following are equivalent:*

(1) (x_n) is G-convergent to x,
(2) $G(x_n, x_n, x) \longrightarrow 0$, as $n \longrightarrow \infty$,
(3) $G(x_n, x, x) \longrightarrow 0$, as $n \longrightarrow \infty$,

Definition 1.5. *Let (X, G) be a G_b-metric space. A sequence (x_n) is called G_b-Cauchy if given $\varepsilon\ > 0$, there is $n_o \in N$ such that $G(x_n, x_m, x_l) < \varepsilon$, for all $n, m, l \geq n_o$.*

Proposition 1.6. *Let X be a G_b-metric space. Then the following are equivalent:*

(1) the sequence $\{x_n\}$ is G_b-Cauchy.

(2) for any $\varepsilon > 0$, there exists $n_0 \in N$ such that $G(x_n,\ x_m,\ x_m < \varepsilon$, for all $m,\ n \geq n_0$.

Definition 1.7. *Let (X, G) and (X^*, G^*) be G_b-metric space and let $h : (X, G) \longrightarrow (X^*, G^*)$ be a function. Then h is said to be $G_b -$ continuous at a point $a \in X$, if given $\varepsilon > 0$, there exists $\delta > 0$ such that $x, y \in X, G(a, x, y) < \delta$ implies $G^*(h(a),\ h(x),\ h(y)) < \varepsilon$.*

Definition 1.8. *A G_b-metric space (X, G) is said to be G_b-complete if every G_b-Cauchy sequence in (X, G) is G_b-convergent in (X, G).*

Definition 1.9. *A G_b-metric space (X, G) is said to be a compact G_b-metric space if it is G_b-complete and G_b-totally bounded.*

2 Main Results

Let π be the set of all upper semi-continuous in each coordinate variable $h : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ such that, if $h(\mu^2, \mu\nu, 0, 0) \leq 0$ or $h(\mu^2, 0, \mu\nu, 0) \leq 0$ or $h(\mu^2, 0, 0, \mu\nu) \leq 0$ for all $\mu, v \geq 0$, then $\mu \leq \frac{1}{4\delta^3} v$.

Example 2.1. $h(t_1, t_2, t_3, t_4) = t_1 - \frac{1}{4\delta^3} \max(t_2, t_3, t_4)$.

Example 2.2. $h(t_1, t_2, t_3, t_4) = t_1 - \frac{1}{4\delta^3} t_2 - \frac{1}{4\delta^3} \max(t_3, t_4)$.

Example 2.3. $h(t_1, t_2, t_3, t_4) = t_1 - \frac{1}{4\delta^3} t_2 - \frac{1}{4\delta^3} t_3 - \frac{1}{4\delta^3} t_4$.

Theorem 2.1. *Let (X, G_1) and (Y, G_2) be complete G_b- metric spaces, and $H : X \longrightarrow Y$, $K : Y \longrightarrow X$ be functions satisfying the following conditions:*

$$g(G_2^2(H\xi, HKr_1, HKr_2), G_2(r_1, HKr_1, HKr_2) G_2(r_1, r_2, H\xi\ ), G_2(r_1, r_2, H\xi) G_1(\xi, Kr_1, Kr_2), \tag{2.1}$$
$$G_1(\xi, Kr_1, Kr_2) G_2(r_1, HKr_1, HKr_2)) \leq 0$$

$$g(G_1^2(Kr_1, Kr_2, KH\xi\ ),\ G_1(\xi, \xi, KH\xi)G_1(\xi, Kr_1, Kr_2), G_1(\xi, Kr_1, Kr_2)G_2(r_1, r_2, H\xi), G_2(r_1, r_2, H\xi)G_1(\xi, \xi, KH\xi)) \leq 0 \tag{2.2}$$

for all ξ in X and r_1, r_2 in Y, $h \in \pi$. Then KH and HK have unique fixed points z in X and w in Y respectively. Moreover, $Hz = w$ and $Kw = z$.

Proof. Define the sequences (ξ_n) in X, and (r_n) in Y by $\xi_n = (KH)^n\xi, r_n = H(KH)^{n-1}\xi$, for $n = 1, 2, ...$ We suppose that $\xi_n \neq \xi_{n+1}$ and $r_n \neq r_{n+1}$ for all n. Applying the inequality (2.1) and property π, we have

$$g(G_2^2(H\xi_{n-1}, HKr_n, HKr_n), G_2(r_n, HKr_n, HKr_n)G_2(r_n, r_n, H\xi_{n-1}),$$

$$G_2(r_n, r_n, H\xi_{n-1})G_1(\xi_{n-1}, Kr_n, Kr_n), G_1(\xi_{n-1}, Kr_n, Kr_n)G_2(r_n, HKr_n, HKr_n)) \leq 0$$

$$g(G_2^2(r_n, r_{n+1}, r_{n+1}), 0, 0, G_1(\xi_{n-1}, \xi_n, \xi_n)G_2(r_n, r_{n+1}, r_{n+1})) \leq 0$$

and it follows that

$$G_2^2(r_n, r_{n+1}, r_{n+1}) \leq \frac{1}{4\delta^3}G_1(\xi_{n-1}, \xi_n, \xi_n)G_2(r_n, r_{n+1}, r_{n+1})$$

$$G_2(r_n, r_{n+1}, r_{n+1}) \leq \frac{1}{4\delta^3}G_1(\xi_{n-1}, \xi_n, \xi_n) \tag{2.3}$$

Similarly, applying the inequality (2.2),

$$g(G_1^2(Kr_n, Kr_n, KH\xi_n), G_1(\xi_n, \xi_n, \xi_{n+1})G_1(\xi_n, Kr_n, Kr_n),$$

$$G_1(\xi_n, Kr_n, Kr_n)G_2(r_n, r_n, H\xi_n), G_2(r_n, r_n, H\xi_n)G_1(\xi_n, \xi_n, \xi_{n+1})) \leq 0$$

$$g(G_1^2(\xi_n, \xi_n, \xi_{n+1}), G_1(\xi_n, \xi_n, \xi_{n+1})G_1(\xi_n, \xi_n, \xi_n), G_1(\xi_n, \xi_n,$$

$$\xi_n)G_2(r_n, r_n, r_{n+1}), G_2(r_n, r_n, r_{n+1})G_1(\xi_n, \xi_n, \xi_{n+1})) \leq 0$$

Using property π and the Proposition (1.2), we have

$$G_1^2(\xi_n, \xi_n, \xi_{n+1}) \leq \frac{1}{4\delta^3}G_2(r_n, r_n, r_{n+1})G_1(\xi_n, \xi_n, \xi_{n+1})$$

$$\frac{1}{2\delta}G_1(\xi_n, \xi_{n+1}, \xi_{n+1}) \leq G_1(\xi_n, \xi_n, \xi_{n+1}) \leq \frac{1}{4\delta^3}G_2(r_n, r_n, r_{n+1}) \leq \frac{1}{2\delta^2}G_2(r_n, r_{n+1}, r_{n+1})$$

$$G_1(\xi_n, \xi_{n+1}, \xi_{n+1}) \leq \frac{1}{\delta}G_2(r_n, r_{n+1}, r_{n+1}) \tag{2.4}$$

Now it follows from the inequalities (2.3) and (2.4) that

$$G_1(\xi_n, \xi_{n+1}, \xi_{n+1}) \leq \frac{1}{4\delta^4} G_1(\xi_{n-1}, \xi_n, \xi_n).$$

Hence, by induction we get

$$G_1(\xi_n, \xi_{n+1}, \xi_{n+1}) \leq (\frac{1}{4\delta^4})^n G_1(\xi, \xi_1, \xi_1), \text{ for } n = 1, 2, \cdots \tag{2.5}$$

So (ξ_n) and (r_n) are G_b-Cauchy sequences with limits z in X and w in Y. Using the inequality (2.1), we have

$$g(G_2^2(Hz, HKr_{n-1}, HKr_{n-1}), G_2(r_{n-1}, HKr_{n-1}, HKr_{n-1})G_2(r_{n-1}, r_{n-1}, Hz),$$

$$G_2(r_{n-1}, r_{n-1}, Hz)G_1(z, Kr_{n-1}, Kr_{n-1}), G_1(z, Kr_{n-1}, Kr_{n-1})G_2(r_{n-1}, HKr_{n-1}, HKr_{n-1})) \leq 0$$

$$g\ (G_2^2(Hz, r_n, r_n), G_2(r_{n-1}, r_n, r_n)G_2(r_{n-1}, r_{n-1}, Hz),$$

$$G_2(r_{n-1}, r_{n-1}, Hz)G_1(z, \xi_{n-1}, \xi_{n-1}), G_1(z, \xi_{n-1}, \xi_{n-1})G_2(r_{n-1}, r_n, r_n)) \leq 0.$$

$$g(G_2^2(Hz, w, w), 0, 0, 0) \leq 0,$$

it follows that $G_2(Hz, w, w) = 0$, hence $w = Hz$. Using the inequality (2.2), we have

$$g(G_1^2(Kw, Kw, KH\xi_{n-1}), G_1(\xi_{n-1}, \xi_{n-1}, KH\xi_{n-1})G_1(\xi_{n-1}, Kw, Kw),$$

$$G_1(\xi_{n-1}, Kw, Kw)G_2(w, w, H\xi_{n-1}), G_2(w, w, H\xi_{n-1})G_1(\xi_{n-1}, \xi_{n-1}, KH\xi_{n-1})) \leq 0.$$

$$g(G_1^2(Kw, Kw, \xi_n), G_1(\xi_{n-1}, \xi_{n-1}, KH\xi_{n-1})G_1(\xi_{n-1}, Kw, Kw),$$

$$G_1(\xi_{n-1}, Kw, Kw)G_2(w, w, H\xi_{n-1}), G_2(w, w, H\xi_{n-1})G_1(\xi_{n-1}, \xi_{n-1}, KH\xi_{n-1})) \leq 0.$$

Letting $n \longrightarrow \infty$, we get $g(G_1^2(Kw, Kw, \xi_n), 0, 0, 0) \leq 0$, and it follows that $z = Kw$. Thus $KHz = Kw = z, HKw = Hz = w$, and so KH and HK have a fixed points z and w respectively. Suppose that KH and HK have other fixed points z_1 in X and w_1 in Y respectively. Using property π and the inequality (2.1), we have

$$g(G_2^2(Hz, HKw_1, HKw_1), G_2(w_1, HKw_1, HKw_1)G_2(w_1, w_1, Hz),$$

$$G_2(w_1, w_1, Hz)G_1(z, Kw_1, Kw_1), G_1(z, Kw_1, Kw_1)G_2(w_1, HKw_1, HKw_1)) \leq 0$$

$$g(G_2^2(HKw, HKw_1, HKw_1), 0, G_2(w_1, w_1, w)G_1(Kw, Kw_1, Kw_1), 0) \leq 0,$$

$$g(G_2^2(w, w_1, w_1), 0, G_2(w_1, w_1, w)G_1(Kw, Kw_1, Kw_1), 0) \leq 0,$$

it follows that $G_2^2(w,w_1,w_1)\le \frac{1}{4\delta^3}G_1(Kw,Kw_1,Kw_1)G_2(w_1,w_1,w)$,

$$G_2(w,w_1,w_1)\le \frac{1}{4\delta^3}G_1(Kw,Kw_1,Kw_1). \tag{2.6}$$

Using the property π and the inequality (2.2), we have

$$g(G_1^2(KHKw,KHKw,KHKw_1),G_1(Kw_1,Kw_1,KHSw_1)G_1(Kw_1,KHKw,KHKw),$$
$$G_1(Kw_1,KHKw,KHKw)G_2(HKw,HKw,HKw_1)G_2$$
$$(HKw,HKw,HKw_1)G_1(Kw_1,Kw_1,KHKw_1))\le 0,$$
$$g(G_1^2(Kw,Kw,Kw_1),0,G_1(Kw_1,Kw,Kw)G_2(w,w,w_1),0)\le 0$$

which implies that $G_1^2(Kw,Kw,Kw_1)\le \frac{1}{4\delta^3}G_2(w,w,w_1)G_1(Kw,Kw,Kw_1)$,

$$G_1(Kw,Kw,Kw_1)\le \frac{1}{4\delta^3}G_2(w,w,w_1), \tag{2.7}$$

by using the Proposition (1.2), we get,

$$\frac{1}{2\delta}G_1(Kw,Kw_1,Kw_1)\le G_1(Kw,Kw,Kw_1)\le \frac{1}{4\delta^3}G_2(w,w,w_1)\le \frac{1}{2\delta^3}G_2(w,w_1,w_1)$$

$$G_1(Kw,Kw_1,Kw_1)\le \frac{1}{\delta^2}G_2(w,w_1,w_1). \tag{2.8}$$

Now it follows from the inequalities (2.6) and (2.8) that

$$G_2(w,w_1,w_1)\le \frac{1}{4\delta^3}G_1(Sw,Sw_1,Sw_1)<\frac{1}{4\delta^5}G_2(w,w_1,w_1)<G_2(w,w_1,w_1)$$

and so $w=w_1$. Hence w must be a unique fixed point of HK. Now $HKz_1=z_1$ implies $HKHz_1=Hz_1$ and so $Hz_1=w$. Thus $z=KHz=Kw=KHz_1=z_1$, z is a unique fixed point of KH. □

Now, we prove an analogous result for compact G-metric spaces.

Let π^* be the set of all functions $h:\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\to\mathbb{R}$ such that, if $h(\mu^2,\mu\nu,0,0)<0$ or $h(\mu^2,0,\mu\nu,0)<0$ or $h(\mu^2,0,0,\mu\nu)<0$ for all $\mu,v\ge 0$, then $\mu\le \frac{1}{2\delta}v$.

Theorem 2.2. *Let (X,G_1) and (Y,G_2) be compact G-metric spaces, and H be a continuous function of X into Y and let S be a continuous function of Y into X satisfying the following conditions:*

$$g(G_2^2(Hx,HSy_1,HSy_2),G_2(y_1,HSy_1,HSy_2)G_2(y_1,y_2,Hx),G_2(y_1,y_2,Hx)G_1(x,Sy_1,Sy_2), \tag{2.9}$$

$$G_1(x, Sy_1, Sy_2)G_2(y_1, HSy_1, HSy_2)) < 0, \textit{with } x \neq Sy_1, \textit{and} x \neq Sy_2$$

$$g(G_1^2(Sy_1, Sy_2, SHx), G_1(x, x, SHx)G_1(x, Sy_1, Sy_2), G_1(x, Sy_1, Sy_2)G_2(y_1, y_2, Hx),$$

$$G_2(y_1, y_2, Hx)G_1(x, x, SHx)) < 0, \textit{with } y_1 \neq Hx, y_2 \neq Hx. \tag{2.10}$$

for all x in X and y_1, y_2 in Y, $g \in \pi^*$. Then SH has a unique fixed point z in X and HS has a unique fixed point $w \in Y$. Moreover, $Hz = w$ and $Sw = z$.

Proof. Let $\psi : X \to R^+$ defined by $\psi(x) = G_1(x, SHx, SHx)$ be G-continuous on X. Since X is compact, there exists a point u in X such that $\psi(u) = G_1(u, SHu, SHu) = \min\{G_1(x, SHx, SHx); x \in X\}$. Now suppose that $Hu \neq HSHu$. Then $u \neq SHu$. Put $y_1 = y_2 = Hu, x = Sy = SHu$ in the inequality (2.10). We have

$$g(G_1^2(SHu, SHu, SHSHu), G_1(SHu, SHu, SHSHu)G_1(SHu, SHu, SHu),$$

$$G_1(SHu, SHu, SHu)G_2(Hu, Hu, HSHu), G_2(Hu, Hu, HSHu)G_1(SHu, SHu, SHSHu)) < 0$$

$$g(G_1^2(SHu, SHu, SHSHu), 0, 0, G_2(Hu, Hu, HSHu)G_1(SHu, SHu, SHSHu)) < 0.$$

Using the condition of π^* and Proposition(1.2) we have

$$G_1^2(SHu, SHu, SHSHu) < \frac{1}{2\delta}G_2(Hu, Hu, HSHu)G_1(SHu, SHu, SHSHu)$$

$$G_1(SHu, SHu, SHSHu) < \frac{1}{2\delta}G_2(Hu, Hu, HSHu) < G_2(Hu, HSHu, HSHu)$$

Put $y_1 = y_2 = Hu$, $x = u$ in the inequality (2.9). We have

$$g(G_2^2(Hu, HSHu, HSHu), G_2(Hu, HSHu, HSHu)G_2(Hu, Hu, Hu),$$

$$G_2(Hu, Hu, Hu)G_1(u, SHu, SHu), G_1(u, SHu, SHu)G_2(Hu, HSHu, HSHu)) < 0$$

$$g(G_2^2(Hu, HSHu, HSHu), 0, 0, G_1(u, SHu, SHu)G_2(Hu, HSHu, HSHu)) < 0$$

But, using condition π^*

$$G_2^2(Hu, HSHu, HSHu) < \frac{1}{2\delta}G_1(u, SHu, SHu)G_2(Hu, HSHu, HSHu),$$

$$G_2(Hu, HSHu, HSHu) < \frac{1}{2\delta}G_1(u, SHu, SHu)$$

$$\frac{1}{2\delta}G_1(SHu, SHSHu, SHSHu) \leq G_1(SHu, SHu, SHSHu\,) < \frac{1}{2\delta}G_1(u, SH, SHu)$$
$$G_1(SHu, SHSHu, SHSHu) < G_1(u, SHu, SHu).$$

Hence $\psi(SHu) < \psi(u)$, which is a contradiction. So $HSHu = Hu$. If $Hu = w$ and $Sw = z$, then $SH(SHu) = S(HSHu) = SHu = Sw = z$, and $w = Hu = HS(Hu) = H(SHu) = Hz$. Thus, $Sw = z$ is a fixed point of SH and $Hz = w$ is a fixed point of HS. □

Suppose that SH has a another fixed point z_1. Then applying the inequality (2.10), we have

$$g(G_1^2(SHz, SHz, SHz_1), G_1(z_1, z_1, SHz_1)G_1(z_1, z, z),$$
$$G_1(z_1, z, z)G_2(Hz, Hz, Hz_1), G_2(Hz, Hz, Hz_1)G_1(z_1, z_1, SHz_1)) < 0.$$
$$g(G_1^2(z, z, z_1)), G_1(z_1, z_1, SHz_1)G_1(z_1, z, z), G_1(z_1, z, z)G_2(Hz, Hz, Hz_1),$$
$$2(Hz, Hz, Hz_1)G_1(z_1, z_1, SHz_1)) < 0.$$

Using condition π^*

$$G_1^2(z, z, z_1) < \frac{1}{2\delta}G_2(Hz, Hz, Hz_1)G_1(z_1, z, z)$$
$$G_1(z, z, z_1) < \frac{1}{2\delta}G_2(Hz, Hz, Hz_1).$$

Applying the inequality (2.9) and using condition π^* we obtain,

$$g(G_2^2(Hz, HSHz_1, HSHz_1), G_2(H\ z_1, HSHz_1, HSHz_1)G_2(Hz_1, 1, Hz\ ),$$
$$G_2(Hz_1, Hz_1, Hz\ )G_1(z, SHz_1, SHz_1),$$
$$G_1(z, SHz_1, SHz_1)G_2(Hz_1, HSHz_1, HSHz_1)) < 0$$
$$g(G_2^2(Hz, Hz_1, Hz_1), 0, G_2(Hz_1, Hz_1, Hz)G_1(z, z_1, z_1), 0) < 0,$$
$$G_2^2(Hz, Hz_1, Hz_1) < \frac{1}{2\delta}G_1(z, z_1, z_1)G_2(Hz_1, Hz_1, Hz),$$
$$\frac{1}{2\delta}G_2(Hz_1, Hz, Hz) \leq G_2(Hz, Hz_1, Hz_1) < \frac{1}{2\delta}G_1(z, z_1, z_1).$$

It follows that $G_1(\ z,\ z, z_1) < \frac{1}{2\delta}G_1(\ z,\ z_1, z_1) \leq G_1(\ z,\ z, z_1)$, which is a contradiction and so z must be a unique fixed point. Similarly, w is a unique fixed point of HS.

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