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On Algebraic Attributes of ξ-Intuitionistic Fuzzy Subgroups

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Abstract

In this paper, we propose the notion of ξ-intuitionistic fuzzy subgroup and demonstrate the condition under which an intuitionistic fuzzy set is ξ-intuitionistic fuzzy subgroup. We additionally characterize the concept of ξ-intuitionistic fuzzy cosets and ξ-intuitionistic fuzzy normal subgroup and furthermore, investigate a portion of their important mathematical angles. Moreover, we broaden the investigation of this notion to build up the concept of an ξ-intuitionistic fuzzy homomorphism and establish some essential properties identified with this notion.

1 Introduction

Fuzzy logic can deal with issues like counts and thinking which are often not succinct and clear. This hypothesis comprises the idea of relative graded membership and it is propelled by the human musings and the capacity to see things in the psyche. It has a key job in various expert and logical fields. Because of the unpredictability of the executive's condition and choice issues

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themselves, leaders may give their evaluations or decisions to some specific degree, however, it is conceivable that they are not certain of their judgment. This also implies that there may exist some degree of hesitancy. It may reasonably be communicated in the area of intuitionistic fuzzy set rather than exact numerical values. Intuitionistic fuzzy data turns into an essential piece of multi-property basic leadership with intuitionistic fuzzy sets.

The concept of fuzzy set was first introduced by Zadeh [39] in 1965. Rosenfeld [32] further utilized this idea to define the notion of fuzzy groups in 1971. Das [11] introduced similar characterization of all fuzzy subgroups of finite cyclic groups and also presented the concept of level subgroups of fuzzy subgroups in 1981. Mukherjee and Bhattacharya [28] next utilized the same idea of fuzzy subgroups to initiate the notions of fuzzy normal subgroups and fuzzy cosets in 1984. Akgul [1] described the characterization of fuzzy subgroups of finite groups in 1988. Choudhury *et al.* [10] proposed the notion of a fuzzy homomorphism between any two fuzzy groups and investigated its effect on fuzzy subgroups in 1988. Mashour *et al.* [26] presented the study of fuzzy normal subgroups and demonstrated various fundamental properties in 1990. The concept of complex fuzzy set was introduced by Ramot et al. [29] in 2002. Ghorai and Pal [19] talked about certain properties of m-polar fuzzy graphs in 2016. Alsarahead [4] proceeded the idea of complex fuzzy subgroups in 2017.

Atanassov [6] presented the concept of intuitionistic fuzzy sets and drew many important results of this notion in 1986. He furthermore, contemplated on intuitionistic fuzzy set theory and its applications in [7]. Biswas [8] built up the intuitionistic fuzzification of subgroups of a group and described the notion of intuitionistic fuzzy subgroups in 1989. The idea of intuitionistic fuzzy optimization was developed by Angelov [5] in 1995. Gargov and Atanassov [18] proposed the operations of fuzzy logic in 1995. An introduction to fuzzy subspaces in intuitionistic fuzzy topological spaces was given by Coker [9] in 1997. De *et al.* [12] presented an application of intuitionistic fuzzy sets in medicinal diagnosis in 2001. Deschrijver *et al.* [13] initiated the study of fuzzy t-norm and t-conorm in 2004. Zhan and Tan [40] broadened this ideology over intuitionistic M-fuzzy groups in 2004. The concept of intuitionistic fuzzy cosets and some important properties of intuitionistic fuzzy normal subgroup of a group was presented by Hur *et al.* [20] in 2004. Fathi and Salleh [17] proposed the relationship among intuitionistic fuzzy groups in 2009. Marashdeh and Salleh [25] proceeded with the investigation of intuitionistic fuzzy groups by introducing the notion of intuitionistic fuzzy normal subgroups based on intuitionistic fuzzy space as a generalization of the fuzzy normal subgroups in 2010. Subramanian *et al.* [37] established the structures of intuitionistic Q-fuzzy quotient sublattices in terms of fuzzy lattice in 2011. Larimi [22] analyzed the homomorphism of intuitionistic (α, β)-fuzzy H_v submodule in 2011. Li and wang [24] presented (λ, α)-homomorphism and the (λ, α)-isomorphism between two intuitionistic fuzzy groups in 2011. Sharma [34] characterized the (α, β)-cut of intuitionistic fuzzy subgroups in 2011. He additionally introduced the notions of t-intuitionistic fuzzy sets and t-intuitionistic fuzzy subgroups in [35]. The idea of complex intuitionistic fuzzy set was presented by Alkouri and Salleh [3] in 2012. Mondal and Roy [27] examined the concept of intuitionistic fuzzy soft matrix theory and a few fundamental outcomes in 2013. Doda and Sharma [15] counted the number of intuitionistic fuzzy subgroups of finite abelian groups of different order in 2013. Rashmanlou *et al.* [30] studied intuitionistic fuzzy graphs with categorical properties of intuitionistic fuzzy group in 2015. Also, they developed various results of interval valued intuitionistic (S, T)-fuzzy graphs in [31]. Sahoo and pal [33] established many important properties of product on intuitionistic fuzzy graphs in 2015.

Sun and Liu [38] propounded the concepts of (λ, μ)-intuitionistic fuzzy subgroups and examined their algebraic aspects in 2016. Husban *et al.* [21] started the investigation of a complex intuitionistic fuzzy group in 2016. The idea of intuitionistic fuzzy graph structures as speculation of the intuitionistic fuzzy graph was presented by Akram and Akmal [2] in 2016. Ejegwa *et al.* [16] presented an application of intuitionistic fuzzy sets in electoral process in 2016. Dey and Roy [14] developed an approach for solving multi-objective structural design using basic T-norm and T-conorm over an intuitionistic fuzzy optimization skills in 2017. Lee *et al.* [23] defined intuitionistic quotient mapping and its some properties along with some types of continuous, closed and open mapping in 2017. Singh and Garg [36] formulated the symmetric triangular interval type-2 intuitionistic fuzzy sets with their applications in multi criteria decision making in 2018. Zhou and Xu [41] extended the study of intuitionistic fuzzy sets based on the hesitant fuzzy membership and their applications in basic leadership with hazard inclination in 2018.

This paper is shaped as: section 2 contains basic definitions of intuitionistic fuzzy subgroup and the related results which are useful to build up the consequent investigation of this paper. In section 3, we define the notions of ξ-intuitionistic fuzzy subgroup, ξ-intuitionistic fuzzy normal subgroup and prove some of their important properties. In section 4, we extend the study of this notion to introduce ξ-intuitionistic fuzzy homomorphism and establish some fundamental algebraic aspects of this notion.

2 Preliminaries

In this section, we list some basic notions and results from intuitionistic fuzzy theory which are mandatory to understand the novelty of this paper.

Definition 2.1. [6] Intuitionistic fuzzy set (IFS) is an extension of a classical fuzzy set which defines the degree of membership $\mu_A(x)$ and non-membership $\nu_A(x)$, for every $x \in E$. Where E is universe set. Each ordinary intuitionistic fuzzy set is written as:

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in E\}, \text{ and } 0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Definition 2.2. [8] An IFS A of a group G is called intuitionistic fuzzy subgroup ($IFSG$) if A admits the following conditions:

1. $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,
2. $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$,
3. $\mu_A(x^{-1}) = \mu_A(x)$,
4. $\nu_A(x^{-1}) = \nu_A(x)$, for all $x, y \in G$.

Definition 2.3. [20] Consider $IFSG$ A of a group G, for any $x \in G$ the IFS Ax of G is called intuitionistic fuzzy right coset of A in G as follows: $Ax(g) = (\mu_{Ax}(g), \nu_{Ax}(g))$ where $\mu_{Ax}(g) = \mu_A(gx^{-1})$ and $\nu_{Ax}(g) = \nu_A(gx^{-1})$, for all $g \in G$.

Definition 2.4. [1] Let A be $IFSG$ of a group G then A is called intuitionistic fuzzy normal subgroup ($IFSNG$) of a group G if $xA = Ax$, for all $x \in G$.

Definition 2.5. [34] Consider A be an intuitionistic fuzzy set over universe E and α, β be any two fixed positive numbers from the closed unit interval then cut set of A is defined as:

$$S_{\alpha,\beta}(A) = \{x | x \in E \text{ such that } \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\}, \text{where } \alpha + \beta \leq 1.$$

Theorem 2.6. [34] An IFS A of a group G is $IFSG$ if and only if each (α, β)-cut set of A is a subgroup of G.

Definition 2.7. [7] Consider any two $IFS's$ A and B of the universe E and we define the following operator as:
$A * B = \{< x, \frac{\mu_A(x)+\mu_B(x)}{2(\mu_A(x).\mu_B(x)+1)}, \frac{\nu_A(x)+\nu_B(x)}{2(\nu_A(x).\nu_B(x)+1)} > |x \in E\}$ is called convolution operator.

Definition 2.8. [35] Let A and B be any two $IFS's$ of sets X and Y respectively and ρ be a function from A to B. The image of A under ρ is defined as:

$\rho(A)(y) = (\mu_{\rho A}(y), \nu_{\rho A}(y))$, where

$$\mu_{\rho(A)}(y) = \begin{cases} \vee\{\mu_A(x) & x \in \rho^{-1}(y)\} \\ 0 & otherwise \end{cases}$$

and

$$\nu_{\rho(A)}(y) = \begin{cases} \wedge\{\nu_A(x) & x \in \rho^{-1}(y)\} \\ 1 & otherwise \end{cases}$$

preimage of B under ρ is defined as $\rho^{-1}(B)(x) = (\mu_{\rho^{-1}(B)}(x), \nu_{\rho^{-1}(B)}(x))$, where $\mu_{\rho^{-1}(B)}(x) = \mu_B(\rho(x))$ and $\nu_{\rho^{-1}(B)}(x) = \nu_B(\rho(x))$.

Remark **2.9.** For any $x \in X$, we have $\mu_{\rho(A)}(\rho(x)) \geq \mu_A(x)$ and $\nu_{\rho(A)}(\rho(x)) \leq \nu_A(x)$.

3 Algebraic Aspects of ξ-Intuitionistic Fuzzy Subgroups

In this part, we initiate the study of the notion of ξ-intuitionistic fuzzy subgroup defined on intuitionistic fuzzy set of G and establish many fundamental aspects of this phenomenon.

Definition 3.1. Let A be an IFS of set X and $\xi \in [0,1]$, then the IFS A^ξ is called ξ-intuitionistic fuzzy set $(\xi - IFS)$ of set X with respect to IFS A and is defined as $A^\xi = (\mu_{A^\xi}, \nu_{A^\xi})$ where $\mu_{A^\xi}(x) = \psi\{\mu_A(x), \xi\}$ and $\nu_{A^\xi}(x) = \hat{\psi}\{\nu_A(x), \xi\}$, for all $x \in G$ where ψ and $\hat{\psi}$ are convolution operators.

Remark **3.2.** It is easy to account that we get IFS for the possibility of $\xi = 1$.
The following result specifies that intersection of two $\xi - IFS's$ is also $\xi - IFS$.

Proposition 3.3. Intersection of any two $\xi - IFS's$ is also $\xi - IFS$.

Proof. For any two $\xi - IFS's$ A^ξ and B^ξ of a group G, we have

$$\begin{aligned}\mu_{(A\cap B)^\xi}(x) &= \psi\{\mu_{(A\cap B)}(x), \xi\}\\ &= \psi\{\min\{\mu_A(x), \mu_B(x)\}, \xi\}\\ &= \psi[\min\{\mu_A(x), \xi\}, \min\{\mu_B(x), \xi\}]\\ &= \min\{\mu_{A^\xi}(x), \mu_{B^\xi}(x)\}\\ &= \mu_{(A^\xi \cap B^\xi)}(x) \text{ for all } x \in X.\end{aligned}$$

Similarly, one can easily prove that $\nu_{(A\cap B)^\xi}(x) = \nu_{(A^\xi\cap B^\xi)}(x)$. Consequently, $(A \cap B)^\xi = A^\xi \cap B^\xi$.

□

This result specifies that union of two $\xi - IFS's$ is also $\xi - IFS$.

Proposition 3.4. Union of any two $\xi - IFS's$ is also $\xi - IFS$.

Proof. For any two $\xi - IFS's$ A^ξ and B^ξ of a group G, we have

$$\begin{aligned}\mu_{(A\cup B)^\xi}(x) &= \psi\{\mu_{(A\cup B)}(x), \xi\}\\ &= \psi\{\max\{\mu_A(x), \mu_B(x)\}, \xi\}\\ &= \psi[\max\{\mu_A(x), \xi\}, \max\{\mu_B(x), \xi\}]\\ &= \max\{\mu_{A^\xi}(x), \mu_{B^\xi}(x)\}\\ &= \mu_{(A^\xi \cup B^\xi)}(x).\end{aligned}$$

Similarly, one can easily prove that $\nu_{(A\cup B)^\xi}(x) = \nu_{(A^\xi\cup B^\xi)}(x)$. Consequently, $(A \cup B)^\xi = A^\xi \cup B^\xi$.

□

Definition 3.5. An $\xi - IFS$ A^ξ of a group G is called ξ-intuitionistic fuzzy subgroup ($\xi - IFSG$) if A^ξ admits the following conditions:

1. $\mu_{A^\xi}(xy) \geq \min\{\mu_{A^\xi}(x), \mu_{A^\xi}(y)\}$,
2. $\nu_{A^\xi}(xy) \leq \max\{\nu_{A^\xi}(x), \nu_{A^\xi}(y)\}$,
3. $\mu_{A^\xi}(x^{-1}) = \mu_{A^\xi}(x)$,

4. $\nu_{A^\xi}(x^{-1}) = \nu_{A^\xi}(x)$, for all $x, y \in G$.

The following result shows that every $IFSG$ is always $\xi - IFSG$.

Proposition 3.6. Every $IFSG$ is $\xi - IFSG$.

Proof. For any $IFSG$ A and elements x and y of a group G, we have

$$\begin{aligned}\mu_{A^\xi}(xy) &= \psi\{\mu_A(xy), \xi\} \\ &\geq \psi\{\min\{\mu_A(x), \mu_A(y)\}, \xi\} \\ &= \psi[\min\{\mu_A(x), \xi\}, \min\{\mu_A(y), \xi\}] \\ &= \min\{\mu_{A^\xi}(x), \mu_{A^\xi}(y)\}.\end{aligned}$$

Similarly, one can easily prove that $\nu_{A^\xi}(xy) \leq \max\{\nu_{A^\xi}(x), \nu_{A^\xi}(y)\}$.
Moreover,

$$\begin{aligned}\mu_{A^\xi}(x^{-1}) &= \psi\{\mu_A(x^{-1}), \xi\} \\ &= \psi\{\mu_A(x), \xi\} \\ &= \mu_{A^\xi}(x).\end{aligned}$$

Similarly, one can easily prove that $\nu_{A^\xi}(x^{-1}) = \nu_{A^\xi}(x)$.
This concludes the proof. □

The converse of above proposition need not be true; this algebraic fact may be viewed in the following example.

Example 3.7. Let $G = \{1, \alpha_1, \beta_1, \alpha_1\beta_1\}$ where, $\alpha_1^2 = \beta_1^2 = 1$ and $\alpha_1\beta_1 = \beta_1\alpha_1$ define the IFS:
$A = \{< 1, 0.3, 0.1 >, < \alpha_1, 0.2, 0.3 >, < \beta_1, 0.2, 0.3 >, < \alpha_1\beta_1, 0.1, 0.4 >\}$ of G.
Note that A is not $IFSG$ of G. We take $\xi = 0.05$, we get
$A^{0.05} = \{< 1, 0.2, 0.5 >, < \alpha_1, 0.1, 0.5 >, < \beta_1, 0.1, 0.5 >, < \alpha_1\beta_1, 0.1, 0.5 >\}$
$(0.2, 0.5)$-cut set of $0.05-IFS$ is given by $A^{0.05} = 1$ and $A^{0.95} = \{1, \alpha_1, \beta_1, \alpha_1\beta_1\}$.
$(0.1, 0.5)$-cut set of $0.05 - IFS$ is given by $A^{0.05} = \{1, \alpha_1, \beta_1, \alpha_1\beta_1\}$and $A^{0.95} = \{1, \alpha_1, \beta_1, \alpha_1\beta_1\}$.
Note that each of the cut set of $\xi - IFS$ is a subgroup of G. Hence, it is $\xi - IFS$.

The following result indicates that intersection of two $\xi - IFSG's$ is also $\xi - IFSG$.

Proposition 3.8. Intersection of two $\xi - IFSG's$ of G is also $\xi - IFSG$ of G.

Proof. For any two elements x and y of G, we have

$$\begin{aligned} \mu_{(A\cap B)^{\xi}}(xy) &= \psi\{\mu_{(A\cap B)}(xy), \xi\} \\ &= \psi\{\min\{\mu_A(xy), \mu_B(xy)\}, \xi\} \\ &= \min\{\psi\{\mu_A(xy), \xi\}, \psi\{\mu_B(xy), \xi\}\} \\ \text{which implies that} & \\ \min\{\mu_{A^{\xi}}(xy), \mu_{B^{\xi}}(xy)\} &\geq \min[\min\{\mu_{A^{\xi}}(x), \mu_{A^{\xi}}(y)\}, \min\{\mu_{B^{\xi}}(x), \mu_{B^{\xi}}(y)\}] \\ &= \min[\min\{\mu_{A^{\xi}}(x), \mu_{B^{\xi}}(x)\}, \min\{\mu_{A^{\xi}}(y), \mu_{B^{\xi}}(y)\}] \\ &= \min\{\mu_{(A\cap B)^{\xi}}(x), \mu_{(A\cap B)^{\xi}}(y)\}. \end{aligned}$$

Similarly, one can easily prove that $\nu_{(A\cap B)^{\xi}}(xy) \leq \max\{\nu_{(A\cap B)^{\xi}}(x), \nu_{(A\cap B)^{\xi}}(y)\}$.
Also,

$$\begin{aligned} \mu_{(A\cap B)^{\xi}}(x^{-1}) &= \psi\{\mu_{(A\cap B)}, \xi\} \\ &= \psi[\min\{\mu_A(x^{-1}), \xi\}, \min\{\mu_B(x^{-1}), \xi\}] \\ \text{implies that} & \\ \min\{\mu_{A^{\xi}}(x^{-1}), \mu_{B^{\xi}}(x^{-1})\} &= \min\{\mu_{A^{\xi}}(x), \mu_{B^{\xi}}(x)\} \\ &= \mu_{A^{\xi}\cap B^{\xi}}(x). \end{aligned}$$

Similarly, one can easily prove that $\nu_{(A\cap B)^{\xi}}(x^{-1}) = \nu_{(A\cap B)^{\xi}}(x)$.
This concludes the proof. □

Definition 3.9. For any $\xi - IFSG$ A^{ξ} of G and an element x of G, the ξ-intuitionistic fuzzy right coset of A^{ξ} in G may be defined as follows: $A^{\xi}x(g) = (\mu_{A^{\xi}x}(g), \nu_{A^{\xi}x}(g))$, where, $\mu_{A^{\xi}x}(g) = \psi\{\mu_A(gx^{-1}), \xi\}$ and $\nu_{A^{\xi}x}(g) = \hat{\psi}\{\nu_A(gx^{-1}), 1-\xi\}$, for all $g \in G$.
Similarly, one can define the ξ-intuitionistic fuzzy left coset of A^{ξ} in the same way.

Definition 3.10. An $\xi - IFSG$ A^{ξ} of G is called ξ-intuitionistic fuzzy normal subgroup ($\xi - IFNSG$) of G if $A^{\xi}x = xA^{\xi}$, for all $x \in G$.

The following result shows that every $IFNSG$ of G is also $\xi - IFNSG$ of G.

Proposition 3.11. If A is $IFNSG$ of G is also $\xi - IFNSG$ of G.

Proof. For any $IFNSG$ A of a group G, we have
$\mu_A(x^{-1}g) = \mu_A(gx^{-1})$ and $\nu_A(x^{-1}g) = \nu_A(gx^{-1})$' which follows that

$$\begin{aligned}\mu_{A^\xi}(x^{-1}g) &= \psi\{\mu_A(x^{-1}g), \xi\} \\ &= \psi\{\mu_A(gx^{-1}), \xi\} \\ &= \mu_{A^\xi}(gx^{-1}).\end{aligned}$$

Similarly, one can easily prove that $\nu_{A^\xi}(x^{-1}g) = \nu_{A^\xi}(gx^{-1})$.
Consequently, $xA^\xi = A^\xi x$. □

The converse of above proposition need not to be true, which may be viewed in the following example.

Example 3.12. Let $G = \{e, \alpha, \alpha^2, \alpha^3, \alpha^4, \beta, \alpha\beta, \alpha^2\beta, \alpha^3\beta, \alpha^4\beta\}$ be the dihedral group with ten elements.
Define $IFNSG$ A of G by:

$$\mu_A(x) = \begin{cases} 0.8 & if\ x \in <\beta> \\ 0.7 & otherwise \end{cases}$$

And

$$\nu_A(x) = \begin{cases} 0.13 & if\ x \in <\beta> \\ 0.15 & otherwise \end{cases}$$

$A = \{< e, 0.8, 0.13 >, < \alpha, 0.7, 0.15 >, < \alpha^2, 0.7, 0.15 >, < \alpha^3, 0.7, 0.15 >, < \alpha^4, 0.7, 0.15 >, < \beta, 0.8, 0.13 >, < \alpha\beta, 0.7, 0.15 >, < \alpha^2\beta, 0.7, 0.15 >, < \alpha^3\beta, 0.7, 0.15 >, < \alpha^4\beta, 0.7, 0.15 >\}$.
Clearly, A is not $IFNSG$. For $\mu_A(\alpha\beta) = 0.7 \neq 0.8 = \mu_A(\beta\alpha)$.
Now, we take $\xi = 0.6$, we get
$A^{0.6} = \{< e, 0.5, 0.3 >, < \alpha, 0.5, 0.3 >, < \alpha^2, 0.5, 0.3 >, < \alpha^3, 0.5, 0.3 >, < \alpha^4, 0.5, 0.3 >, < \beta, 0.5, 0.3 >, < \alpha\beta, 0.5, 0.3 >, < \alpha^2\beta, 0.5, 0.3 >, < \alpha^3\beta, 0.5, 0.3 >, < \alpha^4\beta, 0.5, 0.3 >\}$.
Thus A is $\xi - IFNSG$.

The following result illustrates another approach to obtain the normality of $\xi - IFNSG$.

Proposition 3.13. Every $\xi - IFNSG$ A^ξ of G admits the following properties:
$\mu_{A^\xi}(xy) = \mu_{A^\xi}(yx)$ and $\nu_{A^\xi}(xy) = \nu_{A^\xi}(yx)$, holds for all $x, y \in G$.

Proof. For any $\xi - IFNSG$ A^{ξ} of G we have $xA^{\xi} = A^{\xi}x$, holds for all $x \in G$.

$$\text{Implies that} \quad xA^{\xi}(y^{-1}) = A^{\xi}x(y^{-1}), \text{holds for all } y^{-1} \in G.$$
$$\text{which follows that} \quad \psi\{\mu_A(x^{-1}y^{-1}), \xi\} = \psi\{\mu_A(y^{-1}x^{-1}), \xi\}$$
$$\mu_{A^{\xi}}(x^{-1}y^{-1}) = \mu_{A^{\xi}}(y^{-1}x^{-1})$$
$$\Longrightarrow \quad \mu_{A^{\xi}}(yx)^{-1} = \mu_{A^{\xi}}(xy)^{-1}$$
$$\Longrightarrow \quad \mu_{A^{\xi}}(yx) = \mu_{A^{\xi}}(xy).$$

Similarly, one can easily prove that $\nu_{A^{\xi}}(yx) = \nu_{A^{\xi}}(xy)$, holds for all $y \in G$. This concludes the proof. □

In the following result, we prove the condition for an $\xi - IFSG$ to be $\xi - IFNSG$ of G.

Proposition 3.14. Consider A^{ξ} is an $\xi - IFSG$ of G in which $\xi < \min\{r, 1-s\}$, where $r = \min\{\mu_A(x)|\forall x \in G\}$ and $s = \max\{\nu_A(x)|\forall x \in G\}$ then A^{ξ} is $\xi - IFNSG$ of G.

Proof. In view of given condition, we have $r > \xi$ and $s < 1-\xi$, which follows that
$\min\{\mu_A(x)|\forall x \in G\} > \xi$ and $\max\{\nu_A(x)|\forall x \in G\} < 1 - \xi$,
which implies that $\mu_A(x) > \xi$ and $\nu_A(x) < 1 - \xi$, for all $x \in G$.
$\mu_{A^{\xi}x}(g) = \psi\{\mu_A(gx^{-1}), \xi\} = \gamma_1$ and $\nu_{A^{\xi}x}(g) = \hat{\psi}\{\nu_A(gx^{-1}), 1 - \xi\} = \lambda_1$, for all $g \in G$. Similarly, $\mu_{xA^{\xi}}(g) = \psi\{\mu_A(x^{-1}g), \xi\} = \gamma_1$ and $\nu_{xA^{\xi}}(g) = \hat{\psi}\{\nu_A(x^{-1}g), 1 - \xi\} = \lambda_1$. This concludes the proof. □

4 Algebraic Properties of ξ-Intuitionistic Fuzzy Homomorphism of ξ-Intuitionistic Fuzzy Subgroups

In this section, we define the study of the notion of ξ-intuitionistic fuzzy homomorphism and establish a number of important fundamental theorems of this concept.

Definition 4.1. Let A^{ξ} and B^{ξ} be any two $\xi - IFSG's$ of groups G and G' respectively, ρ be a group homomorphism from G to G', then ρ is called ξ-intuitionistic fuzzy homomorphism from A^{ξ} to B^{ξ} if it satisfies the following condition: $\rho(A^{\xi}) = B^{\xi}$.

The following result indicates that ξ-intuitionistic fuzzy homomorphic image of $\xi - IFSG$ is always $\xi - IFSG$.

Theorem 4.2. Let A^ξ be $\xi - IFSG$ of a group G and ρ be a surjective homomorphism from G to G', then $\rho(A^\xi)$ is $\xi - IFSG$ of a group G'.

Proof. In view of the given condition, for any two elements y_1 and y_2 in G', there exist x_1 and x_2 in G such that $\rho(x_1) = y_1$ and $\rho(x_2) = y_2$.
Consider, $\rho(A^\xi)(y_1y_2) = (\mu_{\rho(A^\xi)}(y_1y_2), \nu_{\rho(A^\xi)}(y_1y_2))$,
which follows that

$$\begin{aligned}\mu_{\rho(A^\xi)}(y_1y_2) &= \mu_{(\rho(A))^\xi}(y_1y_2)\\ &= \psi\{\mu_{\rho(A)}(\rho(x_1)\rho(x_2)), \xi\}\\ &= \psi\{\mu_{\rho(A)}(\rho(x_1x_2)), \xi\}\\ &= \mu_{A^\xi}(x_1x_2)\\ &\geq \min\{\mu_{A^\xi}(x_1), \mu_{A^\xi}(x_2)\} \text{ for all } x_1, x_2 \in G\end{aligned}$$

such that$\rho(x_1) = y_1$ and $\rho(x_2) = y_2$,

$$\begin{aligned}&= \min[\vee\{\mu_{A^\xi}(x_1) : \rho(x_1) = y_1\}, \vee\{\mu_{A^\xi}(x_2) : \rho(x_2) = y_2\}]\\ &= \min\{\mu_{A^\xi}(\rho^{-1}y_1), \mu_{A^\xi}(\rho^{-1}y_2)\}\\ &= \min\{\mu_{\rho(A^\xi)}(y_1), \mu_{\rho(A^\xi)}(y_2)\}.\end{aligned}$$

Similarly, one can easily prove that $\nu_{\rho(A^\xi)}(y_1y_2) \leq \max\{\nu_{\rho(A^\xi)}(y_1), \nu_{\rho(A^\xi)}(y_2)\}$.
Further,

$$\begin{aligned}\mu_{\rho(A^\xi)}(y^{-1}) &= \vee\{\mu_{A^\xi}(x^{-1})|\rho(x^{-1}) = y^{-1}\}\\ &= \vee\{\mu_{A^\xi}(x)|\rho(x) = y\}\\ &= \mu_{\rho(A^\xi)}(y).\end{aligned}$$

Similarly, one can easily prove that $\nu_{\rho(A^\xi)}(y^{-1}) = \nu_{\rho(A^\xi)}(y)$. This concludes the proof. □

The following result illustrates that every ξ-intuitionistic fuzzy homomorphic image of $\xi - IFNSG$ is always $\xi - IFNSG$.

Theorem 4.3. Let A^ξ be $\xi - IFNSG$ of a group G and ρ be a bijective homomorphism from groups G to G', then $\rho(A^\xi)$ is $\xi - IFNSG$ of a group G'.

Proof. In view of the given condition, for any two elements y_1 and y_2 in G', there exist x_1 and x_2 in G such that $\rho(x_1) = y_1$ and $\rho(x_2) = y_2$.

Consider, $(\rho(A))^{\xi}(y_1y_2) = (\mu_{(\rho(A))^{\xi}}(y_1y_2), \nu_{(\rho(A))^{\xi}}(y_1y_2))$,
which follows that

$$\begin{aligned}\mu_{(\rho(A))^{\xi}}(y_1y_2) &= \psi\{\mu_{\rho(A)}(\rho(x_1)\rho(x_2)), \xi\}\\ &= \psi\{\mu_{\rho(A)}(\rho(x_1x_2)), \xi\}\\ &= \psi\{\mu_A(x_1x_2), \xi\}\end{aligned}$$

which implies that

$$\begin{aligned}\mu_{A^{\xi}}(x_1x_2) &= \mu_{A^{\xi}}(x_2x_1)\\ &= \psi\{\mu_A(x_2x_1), \xi\}\\ &= \psi\{\mu_{\rho(A)}(\rho(x_2x_1)), \xi\}\end{aligned}$$

which implies that

$$\begin{aligned}\psi\{\mu_{\rho(A)}(\rho(x_2)\rho(x_1)), \xi\} &= \psi\{\mu_{\rho(A)}(y_2y_1), \xi\}\\ &= \mu_{(\rho(A))^{\xi}}(y_2y_1).\end{aligned}$$

Similarly, one can easily prove that $\nu_{(\rho(A))^{\xi}}(y_1y_2) = \nu_{(\rho(A))^{\xi}}(y_2y_1)$.
This concludes the proof. □

The following result specifies that every ξ-intuitionistic fuzzy homomorphic inverse image of $\xi - IFSG$ is always $\xi - IFSG$.

Theorem 4.4. Let B^{ξ} be $\xi - IFSG$ of a group G' and ρ be a group homomorphism from groups G to G', then $\rho^{-1}(B^{\xi})$ is $\xi - IFSG$ of a group G.

Proof. Since, B^{ξ} be $\xi - IFSG$ of a group G' and there exist elements x_1 and x_2 in G, we have
$\rho^{-1}(B^{\xi})(x_1x_2) = (\mu_{\rho^{-1}(B^{\xi})}(x_1x_2), \nu_{\rho^{-1}(B^{\xi})}(x_1x_2))$. Where,

$$\begin{aligned}\mu_{\rho^{-1}(B^{\xi})}(x_1x_2) &= \mu_{B^{\xi}}(\rho(x_1x_2))\\ &= \mu_{B^{\xi}}(\rho(x_1)\rho(x_2))\\ &\geq \min\{\mu_{B^{\xi}}(\rho(x_1)), \mu_{B^{\xi}}(\rho(x_2))\}\end{aligned}$$

which implies that $\min\{\mu_{\rho^{-1}(B^{\xi})}(x_1), \mu_{\rho^{-1}(B^{\xi})}(x_2)\}$.
Similarly, one can easily prove that $\nu_{\rho^{-1}(B^{\xi})}(x_1x_2) \leq \max\{\nu_{\rho^{-1}(B^{\xi})}(x_1), \nu_{\rho^{-1}(B^{\xi})}(x_2)\}$.
Moreover,

$$\begin{aligned}\mu_{\rho^{-1}(B^{\xi})}(y^{-1}) &= \mu_{(B^{\xi})}\rho(y^{-1})\\ &= \mu_{(B^{\xi})}\rho(y)^{-1}\\ &= \mu_{(B^{\xi})}\rho(y)\\ &= \mu_{\rho^{-1}(B^{\xi})}(y).\end{aligned}$$

Similarly, $\nu_{\rho^{-1}(B^{\xi})}(y^{-1}) = \nu_{\rho^{-1}(B^{\xi})}(y)$.
Consequently, $\rho^{-1}(B^{\xi})$ is $\xi - IFSG$ of G. □

The following result explains that every ξ-intuitionistic fuzzy homomorphic inverse image of $\xi - IFNSG$ is always $\xi - IFNSG$.

Theorem 4.5. Let B^{ξ} be $\xi - IFNSG$ of a group G' and ρ be a group homomorphism from groups G to G', then $\rho^{-1}(B^{\xi})$ is $\xi - IFNSG$ of a group G.

Proof. Since, B^{ξ} be $\xi - IFNSG$ of a group G' and there exist elements x_1 and x_2 in G, we have $\rho^{-1}(B^{\xi})(x_1x_2) = (\mu_{\rho^{-1}(B^{\xi})}(x_1x_2), \nu_{\rho^{-1}(B^{\xi})}(x_1x_2))$. Where,

$$\begin{aligned}\mu_{\rho^{-1}(B^{\xi})}(x_1x_2) &= \mu_{B^{\xi}}(\rho(x_1x_2))\\ &= \mu_{B^{\xi}}(\rho(x_1)\rho(x_2))\\ &= \mu_{B^{\xi}}(\rho(x_2)\rho(x_1))\end{aligned}$$

which implies that $\mu_{B^{\xi}}(\rho(x_2x_1)) = \mu_{\rho^{-1}(B^{\xi})}(x_2x_1)$.
Similarly, we can show that $\nu_{\rho^{-1}(B^{\xi})}(x_1x_2) = \nu_{\rho^{-1}(B^{\xi})}(x_2x_1)$.
Thus, $\rho^{-1}(B^{\xi})$ is $\xi - IFNSG$ of G. □

5 Conclusion

The notion of ξ-intuitionistic fuzzy set is a valuable speculation of established fuzzy set which estimates the vulnerability of a fuzzy situation in much proficient way. In this paper, we have characterized ξ-intuitionistic fuzzy subgroup and an ξ-intuitionistic fuzzy coset of a given group and have utilized them to present the concept of an ξ-intuitionistic fuzzy normal subgroup and have proved a few imperative properties. We have likewise examined the impact of the image and inverse image of an ξ-intuitionistic fuzzy normal subgroup under ξ-intuitionistic fuzzy homomorphism.

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