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# On 2 and 3-Rainbow Domination Number of Circulant Graph $G(n;\pm\{1,2,3\})$

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**Abstract**

A $k$-rainbow dominating function of a graph $G$ is a function $f$ that assigns to each vertex a set of colors chosen from the power set $\{1,2,\ldots,k\}$. A function $f: V(G) \to \rho(\{1,2,\ldots,k\})$ such that for every $v \in V(G)$, $f(v)=\phi$, implies $\bigcup_{u\in N(v)} f(u) = \{1,2,\ldots,k\}$. The minimum weight of $k$-rainbow dominating function is called $k$-rainbow domination number of $G$. In this paper, we investigate 2 and 3-rainbow domination number of Circulant Graph $G(n;\pm\{1,2,3\})$.

## 1 Introduction

The concept of domination was originated by S. T. Hedetniemi and P. J. Slater [7]. Domination in graph is a model for many location problem in the operation research. The applications of domination in graphs lies in social networks, land surveying, communication networks, radio stations, interconnection networks.

$k$-Rainbow domination was introduced by Bresar *et a*l. [1] and it is a variant of classical domination. The $k$-rainbow domination problem is a location problem in operation research. Bresar *et a*l. [2] proved that 2-rainbow domination is NP-complete even when restricted to chordal graphs.

**Key words and phrases:** 2-Dominating number, 3-rainbow domination number, Circulant graph.
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Let $f : V(G) \rightarrow \rho(\{1, 2, \dots, k\})$ be a function that assigns to each vertex of $G$, a set of colors chosen from the power set of $\{1, 2, \dots, k\}$. For each vertex $v \in V$, $f(v) = \phi$ implies

$$\bigcup_{u\in N(v)} f(u) = \{1, 2, \dots, k\}$$

Then the function $f$ is called a $k$-rainbow dominating function of $G$. The weight $w(f)$ of the function $f$ is defined as $w(f) = \Sigma_{v\in V(G)}|f(v)|$. The minimum weight of a $k$-rainbow dominating function is called the $k$-rainbow domination number of $G$ and it is denoted as $\gamma_{rk}(G)$.

Circulant graph denoted as $G(n; \pm\{1, 2 \dots j\})$,$1 \leq j \leq \lfloor n/2 \rfloor$, $n \geq 3$, is a graph with vertex set $V = \{0, 1, 2, \dots, n-1\}$ and edge set $E = \{(i, j) : |j - i| \equiv s(mod\ n), s \in \{1, 2, \dots, j\}$.

The properties of circulant graph have been observed by Bermond et al.[3]. Circulant graphs have been used for decades in the design of computer, telecommunication networks and also in VLSI design.

## 2 Literature Survey

Bresar et al. [2] have found 2 rainbow domination in graphs. A study on 3-rainbow domination number of some special classes of graphs have been investigated by K. Ameenal Bibi et al. [4]. 3-rainbow domination in hexagonal networks and honeycomb network have been determined by P.Sivagami et al.[6]. Cynthia et al. [5] have investigated the inverse domination number of circulant graphs.

## 3 2-Rainbow Domination Number of Circulant Graph

**Theorem 3.1.** *The 2-rainbow domination number of Circulant graph $G(n; \pm\{1, 2, 3\})$ is*

$$\gamma_{r2}(G) = \begin{cases} 2\lceil n/7 \rceil, & n \equiv 0, 2, 3, 4, 5, 6 \pmod 7 \\ 2\lceil n/7 \rceil - 2, & n \equiv 1 \pmod 7 \end{cases}$$

*Proof.* Let $G$ be an undirected circulant graph. Let $\{v_1, v_2, \dots, v_n\}$ be the set of vertices of $G$. We derive 2-rainbow domination number of circulant graph $G(n; \pm\{1, 2, 3\})$ using the following algorithm .

**Algorithm**
**Input**: Circulant graph $G(n;\pm\{1,2,3\})$
**Step1**: Label the vertices with the pattern 1, 0, 0, 0, 2, 0, 0 starting with the vertex $v_1$ successively, where 1 and 2 represent $\{1\}$ and $\{2\}$ and 0 represent $\phi$.
**Step2**: When $n \equiv 0, 4, 5, 6$ $(mod\ 7)$ follow the coloring pattern consecutively as in step 1.
When $n \equiv 1$ $(mod\ 7)$ follow the coloring pattern consecutively as in step 1 until $v_{n-1}$ and label the vertex $v_n$ as 0.
When $n \equiv 2, 3$ $(mod\ 7)$ label the vertices $v_{n-3}, v_{n-2}, v_{n-1}, v_n$ as 1,0,0 and 2.

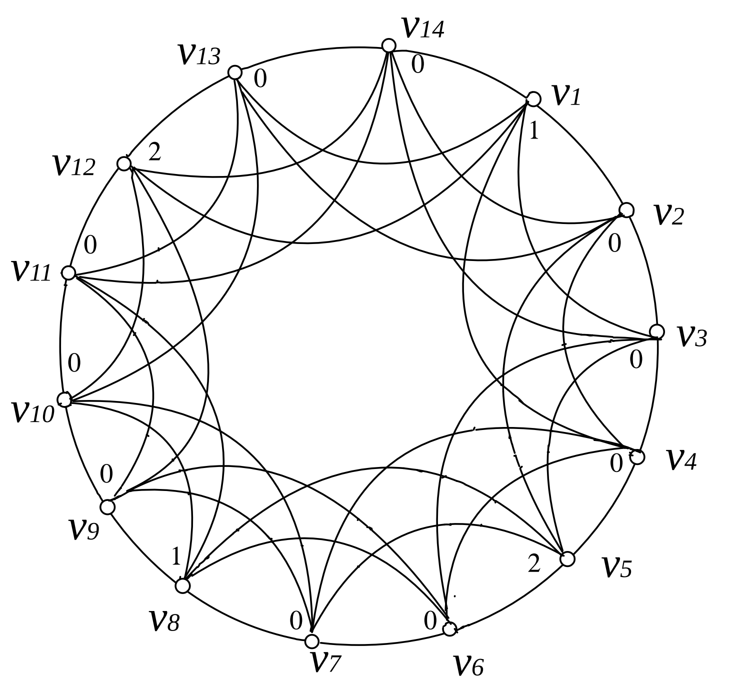


Figure 1: 2-rainbow domination number of Circulant graph $G(14;\pm\{1,2,3\})$

Output: 2-rainbow domination number of circulant graph.

$$\gamma_{r2}(G) = \begin{cases} 2\lceil n/7\rceil, \ n \equiv 0,\ 2,\ 3,\ 4,\ 5,\ 6\ (mod\ 7) \\ 2\lceil n/7\rceil - 2,\ n \equiv 1\ (mod\ 7) \end{cases}$$

Proof of correctness: Let $\phi$ in $\rho(\{1,2\})$ denoted as 0 and singleton sets $\{x\}$ in $\rho(\{1,2\})$ as $x$. Then vertices that are assigned empty set has the color set {1,2} in its neighbourhood. Consider the following cases of $n$ namely $n \equiv 0, 1, 2, 3, 4, 5, 6 \ (mod\ 7)$.

**Case 1** : $n \equiv 0 (mod\ 7)$, $n \geq 7$
Define a function $f : V(G) \to \rho(\{1, 2\})$ by

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \ (mod\ 7) \\ 2, & i \equiv 5 \ (mod\ 7) \\ 0 & otherwise \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f) = 2\lceil n/7 \rceil$. Thus $\gamma_{r2}(G) = 2\lceil n/7 \rceil$.

**Case 2** : $n \equiv 1 (mod\ 7)$, $n \geq 8$
Define a function $f : V(G) \to \rho(\{1, 2\})$ by

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \ (mod\ 7) \\ 2, & i \equiv 5 \ (mod\ 7) \\ 0 & otherwise \\ & v_n = 0 \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= $2\lceil n/7 \rceil - 2$. Thus $\gamma_{r2}(G) = 2\lceil n/7 \rceil - 2$.

**Case 3** : $n \equiv 2 (mod\ 7)$, $n \geq 9$
Define a function $f : V(G) \to \rho(\{1, 2\})$ by

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \ (mod\ 7) \\ 2, & i \equiv 5 \ (mod\ 7) \\ 0 & otherwise \\ & v_{n-3} = 1 \\ & v_{n-2} = 0 \\ & v_{n-1} = 0 \\ & v_n = 2 \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= $2\lceil n/7 \rceil$ . Thus $\gamma_{r2}(G) = 2\lceil n/7 \rceil$.

**Case 4** : $n \equiv 3 (mod\ 7)$, $n \geq 10$

Define a function $f : V(G) \to \rho(\{1,2\})$ by

$$f(v_i) = \begin{cases} 1, \quad i \equiv 1 \ (mod\ 7) \\ 2, \ i \equiv 5 \ (mod\ 7) \\ 0 \quad otherwise \\ v_{n-3} = 1 \\ v_{n-2} = 0 \\ v_{n-1} = 0 \\ v_n = 2 \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= $2\lceil n/7 \rceil$. Thus $\gamma_{r2}(G) = 2\lceil n/7 \rceil$.

**Case 5** : $n \equiv 4(mod\ 7)$, $n \geq 11$
Define a function $f : V(G) \to \rho(\{1,2\})$ by

$$f(v_i) = \begin{cases} 1, \quad i \equiv 1 \ (mod\ 7) \\ 2, \ i \equiv 5 \ (mod\ 7) \\ 0 \quad otherwise \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= $2\lceil n/7 \rceil$ . Thus $\gamma_{r2}(G) = 2\lceil n/7 \rceil$.

**Case 6** : $n \equiv 5(mod\ 7)$, $n \geq 12$
Define a function $f : V(G) \to \rho(\{1,2\})$ by

$$f(v_i) = \begin{cases} 1, \quad i \equiv 1 \ (mod\ 7) \\ 2, \ i \equiv 5 \ (mod\ 7) \\ 0 \quad otherwise \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= $2\lceil n/7 \rceil$ . Thus $\gamma_{r2}(G) = 2\lceil n/7 \rceil$.

**Case 7** : $n \equiv 6(mod\ 7)$, $n \geq 13$
Define a function $f : V(G) \to \rho(\{1,2\})$ by

$$f(v_i) = \begin{cases} 1, \quad i \equiv 1 \ (mod\ 7) \\ 2, \ i \equiv 5 \ (mod\ 7) \\ 0 \quad otherwise \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= $2\lceil n/7 \rceil$ . Thus $\gamma_{r2}(G) = 2\lceil n/7 \rceil$.

□

# 4 3-Rainbow Domination Number of Circulant Graph

**Theorem 4.1.** *The 3- rainbow domination number of Circulant graph is*

$$\gamma_{r3}(G) = \begin{cases} 3\lceil n/7\rceil, & n \equiv 0,\ 3,\ 4,\ 5,\ 6\ (mod\ 7) \\ 3\lceil n/7\rceil - 1, & n \equiv 2\ (mod\ 7) \\ 3\lceil n/7\rceil - 2, & n \equiv 1\ (mod\ 7) \end{cases}$$

*Proof.* Let $G$ be an undirected Circulant graph. Let $\{v_1, v_2, \ldots, v_n\}$ be the set of vertices of $G$. We derive 3-rainbow domination number of circulant graph using the following algorithm .

**Algorithm**

**Input**: Circulant graph $G(n; \pm\{1,2,3\})$

**Step1**: Label the vertices as 1, 0, 0, 2, 3, 0, 0 starting with a vertex $v_1$ successively .

**Step2**: When $n \equiv 0, 4, 5, 6\ (mod\ 7)$ follow the coloring pattern consecutively as in step 1.

When $n \equiv 1\ (mod\ 7)$ follow the coloring pattern consecutively as in step 1 until $v_{n-1}$ and label the vertex $v_n$ as 2.

When $n \equiv 2\ (mod\ 7)$ : Label the vertices $v_{n-2}, v_{n-1}, v_n$ as 1, 0 and 2.

When $n \equiv 3\ (mod\ 7)$ : Label the vertices $v_{n-3}, v_{n-2}, v_{n-1}, v_n$ as 1, 0, 2 and 3.

**Output**: 3-rainbow domination number of circulant graph

$$\gamma_{r3}(G) = \begin{cases} 3\lceil n/7\rceil, & n \equiv 0,\ 3,\ 4,\ 5,\ 6\ (mod\ 7) \\ 3\lceil n/7\rceil - 1, & n \equiv 2\ (mod\ 7) \\ 3\lceil n/7\rceil - 2, & n \equiv 1\ (mod\ 7) \end{cases}$$

Proof of correctness : Let $\phi$ in $\rho(\{1,2,3\})$ denoted as 0 and singleton sets $\{x\}$ in $\rho(\{1,2,3\})$ as $x$. Then vertices that are assigned empty set has the color set $\{1,\ 2,\ 3\}$ in its neighborhood. Consider the following cases of $n$ namely $n \equiv 0,1,2,3,4,5,6\ (mod\ 7)$.

**Case 1** : $n \equiv 0 (mod\ 7)$, $n \geq 7$

Define a function $f : V(G) \to \rho(\{1,2,3\})$ by

$$f(v_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 7) \\ 2, & i \equiv 4\ (mod\ 7) \\ 3, & i \equiv 5\ (mod\ 7) \\ 0 & otherwise \end{cases}$$

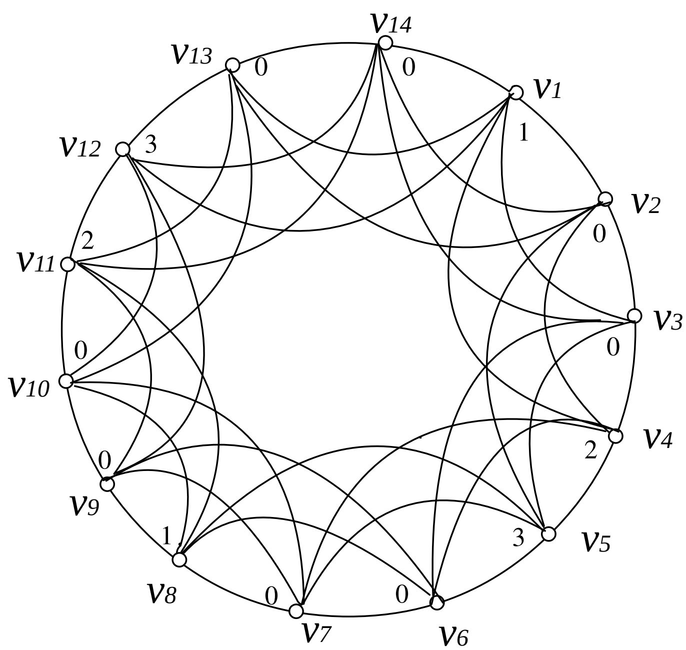


Figure 2: 3-rainbow domination number of Circulant graph $G(14;\pm\{1,2,3\})$

$w(f)$=$\sum_i |v_i|$ for $i = 1,2,3,\ldots n$. $w(f)$= $3\lceil n/7\rceil$. Thus $\gamma_{r3}(G) = 3\lceil n/7\rceil$.

**Case 2** : $n \equiv 1(mod\ 7)$, $n \geq 8$
Define a function $f : V(G) \rightarrow \rho(\{1,2,3\})$ by

$$f(v_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 7) \\ 2, & i \equiv 4\ (mod\ 7) \\ 3, & i \equiv 5\ (mod\ 7) \\ 0 & otherwise \\ & v_n = 2 \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1,2,3,\ldots n$. $w(f)$= $3\lceil n/7\rceil$-2. Thus $\gamma_{r3}(G) = 3\lceil n/7\rceil - 2$.

**Case 3** : $n \equiv 2(mod\ 7)$, $n \geq 9$

Define a function $f : V(G) \to \rho(\{1,2,3\})$ by

$$f(v_i) = \begin{cases} 1 \;\; i \equiv 1 \; (mod\; 7) \\ 2 \; i \equiv 4 \; (mod\; 7) \\ 3 \; i \equiv 5 \; (mod\; 7) \\ 0 \quad otherwise \\ v_{n-2} = 1 \\ v_{n-1} = 0 \\ v_n = 2 \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= 3$\lceil n/7 \rceil$-1 . Thus $\gamma_{r3}(G) = 3\lceil n/7 \rceil -$ 1.

**Case 4** : $n \equiv 3(mod\; 7)$, $n \geq 10$
Define a function $f : V(G) \to \rho(\{1,2,3\})$ by

$$f(v_i) = \begin{cases} 1, \;\; i \equiv 1 \; (mod\; 7) \\ 2, \; i \equiv 4 \; (mod\; 7) \\ 3, \; i \equiv 5 \; (mod\; 7) \\ 0 \quad otherwise \\ v_{n-3} = 1 \\ v_{n-2} = 0 \\ v_{n-1} = 2 \\ v_n = 3 \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n,$. $w(f)$= 3$\lceil n/7 \rceil$. Thus $\gamma_{r3}(G) = 3\lceil n/7 \rceil$.

**Case 5** : $n \equiv 4(mod\; 7)$, $n \geq 11$
Define a function $f : V(G) \to \rho(\{1,2,3\})$ by

$$f(v_i) = \begin{cases} 1, \;\; i \equiv 1 \; (mod\; 7) \\ 2, \; i \equiv 4 \; (mod\; 7) \\ 3, \; i \equiv 5 \; (mod\; 7) \\ 0 \quad otherwise \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= 3$\lceil n/7 \rceil$ . Thus $\gamma_{r3}(G) = 3\lceil n/7 \rceil$.

**Case 6** : $n \equiv 5(mod\; 7)$, $n \geq 12$
Define a function $f : V(G) \to \rho(\{1,2,3\})$ by

$$f(v_i) = \begin{cases} 1, \;\; i \equiv 1 \; (mod\; 7) \\ 2, \; i \equiv 4 \; (mod\; 7) \\ 3, \; i \equiv 5 \; (mod\; 7) \\ 0 \quad otherwise \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= $3\lceil n/7 \rceil$ . Thus $\gamma_{r3}(G) = 3\lceil n/7 \rceil$.

**Case 7** : $n \equiv 6(mod\ 7)$, $n \geq 13$
Define a function $f : V(G) \to \rho(\{1, 2, 3\})$ by

$$f(v_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 7) \\ 2, & i \equiv 4\ (mod\ 7) \\ 3, & i \equiv 5\ (mod\ 7) \\ 0 & otherwise \end{cases}$$

$w(f)$=$\sum_i |v_i|$ for $i = 1, 2, 3, \ldots n$. $w(f)$= $3\lceil n/7 \rceil$ . Thus $\gamma_{r3}(G) = 3\lceil n/7 \rceil$ .
□

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