International Journal of Mathematics and
Computer Science, **15**(2020), no. 3, 827–838

Derivation of Nondimensional Synthetic Unit Hydrograph of ITS-2 Using the 2-Parameter Gamma Distribution

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(Received February 17, 2020, Accepted April 8, 2020)

Abstract

ITS-2 is one of the latest synthetic unit hydrograph-based flood estimation models in Indonesia. Developed at the Institut Teknologi Sepuluh Nopember (ITS) Surabaya in 2017, the basic parameters of the model were formulated using fractal and morphometric characteristics of eight observed river basin in Central Sulawesi, Indonesia. The fundamental concern with developing this model is how to derive

Key words and phrases: Statistical distribution, Performance, Hydrology model, Hydrograph.
AMS (MOS) Subject Classifications: 38B15, 62E10, 86A05.
ISSN 1814-0432, 2020, http://ijmcs.future-in-tech.net

a curve equation which illustrates the shape of the hydrograph. This paper aims to formulate this equation using the 2-parameter Gamma distribution, one of the statistical distribution model that is widely used for hydrological modeling especially hydrographs. The derivation of the equation was carried out by simplifying the parameters of the Gamma function which corresponds to the shape of the hydrograph such as the rising limb, peak and, recession limb. This research has succeeded in formulating an equation of dimensionless synthetic unit hydrograph curve expressed by the ratio of time (T) and peak time (T_p) of the hydrograph. A coefficient of C_3 states that the hydrograph shape parameters can be optimized to improve model performance, when a series of observation discharge is available for calibration. The performance of the synthetic unit hydrograph equation was assessed using Mean Absolute Percentage Error (MAPE) and the Coefficient of correlation (r). The analysis results based on a series of verification indicated that the performance of the hydrograph equation was very high with MAPE below 20% and a r above 0.8.

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1 Introduction

Hydrological modeling, especially the synthetic hydrograph-based flood prediction model plays an important role in environmental management [1–7]. This type of this model is generally developed based on mathematical models with various approaches such as analytic modeling, numerical modeling and statistical modeling [8]. Analytical modeling can be used for modeling hydrographs with very limited variables while numerical modeling can be applied involving many simplifications and assumptions. The limitation of variables and simplification of concepts causes the performance of the hydrograph model to be not optimal. The last approach based on statistics is considered to be able to accommodate the weaknesses of the two methods so that most hydrograph models are developed based on this principle.

The development of synthetic unit hydrograph models in Indonesia has been carried out by several researchers. Harto (1985) initiated the development of synthetic unit hydrograph models (GAMA I) using a statistical approach at Gadjah Mada University [9]. The main equation of the hydrograph curve consists of two formulas: a linear equation on the rising side and an exponential equation on the down side. In general, this model shows good performance, but because the parameters contain quite a number of variables, It causes less efficiency in terms of calculation time. Using the same principle, Lasidi et al. (2003) developed the ABG Model (now known as the ITS-1 Model) by proposing a curve equation adopted from the Delay-Stokage Method [10]. The performance of this model is relatively similar to GAMA I especially for application in Java Island, but the ABG Equation coefficient must be reviewed for the efficiency of its coefficient optimization. Considering these weaknesses, Limatara (2009) proposed a hydrograph model with a concept similar to GAMA I and ITS-1 [11]. The number of variables in the basic parameter equation of the hydrograph is simplified and formulated by two main equations of hydrographs: non linear equations on the rising side and exponential equations on the recession side.

Natakusumah (2011) formulated two hydrograph equations using exponential equations named ITB-1 and ITB-2 [12]. The equation used in both models is much simpler than the previous three models so that for now this model is relatively widely used by practitioners for analysis and planning. However, this model also has a weakness especially ITB-1, where at $t = 0$ the ordinate hydrograph cannot be calculated (undefined) [13]. To overcome the weaknesses in the synthetic unit hydrograph equation, Tunas et al. proposed the ITS-2 Model at the Institut Teknologi Sepuluh Nopember by adopting the modeling principles used in the previous equation [14]. The interesting thing about this model is that the parameters use a combination of variables derived from fractal and morphometric characteristic of catchment and a curve equation derived from 2-Parameter Gamma Distribution [15]. The development of this model is expected to improve the accuracy of predictions and time efficiency calculations, especially for watersheds in Central Sulawesi.

2 Basic Parameters Equations of ITS-2 Model

This research is based on the three basic equations of ITS-2 Model, namely the peak discharge equation (Q_p), the peak time equation (T_p) and the basic time equation (T_b) formulated as [13–15]:

$$Q_p = \frac{R}{3.6T_p}\frac{A}{A_{SUH}} \tag{1}$$

$$T_p = C_1(0.102L - 0.162D - 0.524R_L + 1.24) \tag{2}$$

$$T_b = C_2(0.136A - 43.0S + 11.5), \tag{3}$$

where Q_p, T_p and T_b denote the peak of discharge (m^3/s), the peak of time (hour) and the base of time (hour) successively. R is unit rainfall (mm), A is watershed area (km^2) and A_{SUH} is area under hydrograph curve. L is main river length (m), D is stream density (km/km^2), and R_L is the ratio of river length. S is the slope of the main river bed, C_1 and C_2 are peak time and base time coefficients.

3 The 2-Parameter Gamma Distribution

Gamma distribution is one of the statistical models [16] for hydrograph curve transformation based on a continuous probability distribution. The general form of the 2-Parameter Gamma Distribution equation (2PGDF), expressed by:

$$f(x) = \frac{1}{b^a \Gamma(a)} (x)^{a-1} e^{-\left(\frac{x}{b}\right)}, \quad x > 0, \quad a > 0, \quad b > 0 \tag{4}$$

$f(x) = 0$ for $x \leq 0$, $\Gamma(a)$ denotes the Gamma Function, a is a shape parameter and b is a scale parameter. This distribution is a family of other statistical distributions such as Beta, Chi-Square, Normal, Pearson and Weibull Distributions. In various applications in the field of hydrology, Gamma Distribution shows better performance compared to other probability distributions, especially 2-Parameter Gamma Distribution (2PGDF) [17]. Therefore many synthetic unit hydrograph methods are developed based on Gamma Distribution such as Nash (1957) and H2U by Duchesne and Cudennec (1998) [13].

The shape parameter and scale parameter affect the graph shape of the 2-Parameter Gamma Distribution, both the Probability Density Function (PDF) graph (Figure 1a) and the Cumulative Distribution Function (CDF) graph (Figure 1b) [18]. The PDF graph is getting blunt if the parameters a and b increase and vice versa. Both of these parameters work simultaneously in determining the shape of the two graphs. The PDF graph was then adopted to form synthetic unit hydrographs, by simplifying the parameters and modifying their equations using analogies and simple mathematical logic operating techniques.

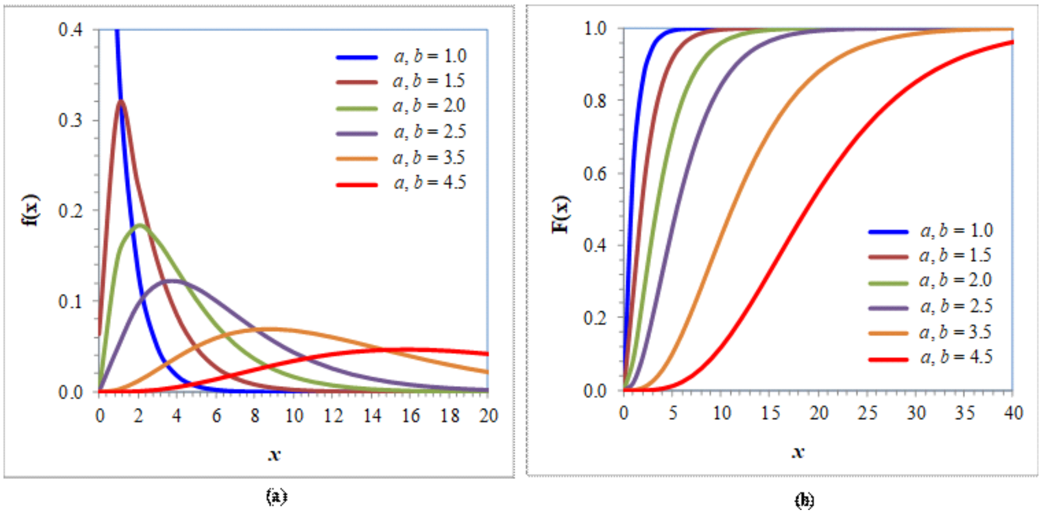

Figure 1: Typical graph of the 2-Parameter Gamma Distribution (a). Probability Density Function–PDF, (b). Cumulative Distribution Function–CDF

4 The Curve Equation of ITS-2 Model

Derivation of 2-Parameter Gamma Distribution for expressing the shape of the hydrograph was initiated by Edson (1951, in Singh, 2000) followed by Nash and Dooge (1959) transform these equations into the equation of the instantaneous unit hydrograph in the form [19–23]:

$$q(t) = \frac{1}{K\Gamma(n)} \left(\frac{t}{K}\right)^{n-1} e^{-\left(\frac{t}{K}\right)} \tag{5}$$

n states the number of linear reservoirs (shape factor) and K denotes the storage coefficient of the catchment (scale factor), q = runoff depth per unit time. $\Gamma(n) = (n-1)!$. The relationship between K and n is expressed by [24]:

$$K = \frac{t_p}{(n-1)} \tag{6}$$

The form of Eq. (5) is very identical to the Gamma Distribution Equation in Eq. (4), where a is expressed by n, x is expressed with t and b is expressed in K. In other words, the shape factor and scale factor are expressed by the number of linear reservoirs and storage coefficient.

Referring to Eq. (5), if the parameters of the Gamma Distribution in Eq. (4) is substituted with three coefficients ω_1, ω_2, and ω_3, as in the following equation:

$$f(x) = \overbrace{\frac{1}{b^a \Gamma(a)}}^{\omega_1} (x)^{\overbrace{a-1}^{\omega_2}} e^{-\overbrace{\left(\frac{1}{b}\right)}^{\omega_3} x}, \tag{7}$$

then the equation becomes:

$$f(x) = \omega_1 (x)^{\omega_2} e^{-\omega_3 (x)} \tag{8}$$

ω_2 and ω_3 are coefficients that control the PDF graph shape simultaneously. Because the roles of the two coefficients are identical, they can be replaced by a power coefficient that represents all the equations.

Simplifying the two power coefficients produces an equation into one coefficient with a simpler form like the following:

$$f(x) = \left\{\omega_4 (x) e^{-x}\right\}^{\omega_5} \tag{9}$$

If the PDF in the equation describes the hydrograph as a function of time, then

$$f(t) = \left\{\omega_4 (t) e^{-t}\right\}^{\omega_5} \tag{10}$$

is a dimensionless equation, where $q(t) = \frac{Q(t)}{Q_p}$ and $t = \frac{T}{T_p}$. Therefore

$$q(t) = \frac{Q(t)}{Q_p} = \left\{\omega_4 \left(\frac{t}{T_p}\right) e^{-\frac{t}{T_p}}\right\}^{\omega_5} \tag{11}$$

$$q(t) = \frac{Q(t)}{Q_p} = \left\{\omega_4 \left(\frac{t}{T_p}\right) exp \left(-\frac{t}{T_p}\right)\right\}^{\omega_5}, \tag{12}$$

where ω_4 is a constant that acts to control so that Q_p corresponds to T_p. If it is wrong to set ω_4, then it is possible that at the peak time of the hydrograph, peak discharge is not reached. In this case the peak discharge can be reached before or after the peak time. This allows peak discharge to be of greater or smaller than the actual value. Therefore the determination of ω_4 must be based on these considerations. To get the compatibility of the two parameters, ω_4 can be determined by setting *q(t)*=1 and *T*=1 according to the following:

$$1 = \{\omega_4 (1)\ exp\ (-1)\}^{\omega_5}. \tag{13}$$

The right term will be equal to 1 if $\omega_4(1)\ exp\ (-1) = 1$ for any value of ω_5. In the form of another equation, it can be written:

$$\overbrace{1^{\frac{1}{\omega_5}}}^{1} = \omega_4(1)\ exp\ (-1). \tag{14}$$

The left term in the above equation indicates that power coefficient $(1/\omega_5)$ does not affect the results of the mathematical operation regardless of the value of ω_5. Therefore

$$1 = \omega_4(1)\ exp\ (-1) \tag{15}$$

$$1 = \frac{\omega_4}{exp\ (1)} \tag{16}$$

$$\omega_4 = exp\ (1). \tag{17}$$

The calculation results show that $\omega_4 = e$. Based on these results, the final unit hydrograph equation can be formulated by replacing ω_5 with C_3, the standard symbol of the coefficient on the ITS-2 Model, as a very simple equation with a coefficient for optimization such as the following:

$$q(t) = \left\{ exp\ (1) \left(\frac{t}{T_p} \right)\ exp \left(-\frac{t}{T_p} \right) \right\}^{C_3} \tag{18}$$

$$q(t) = \left\{ \left(\frac{t}{T_p} \right)\ exp \left(1 - \frac{t}{T_p} \right) \right\}^{C_3}, \tag{19}$$

where $q(t)$ is unit discharge (dimensionless) which represent the equation of unit hydrograph curve, C_3 is the coefficient of hydrograph shape. Eq. (19) is known as the curve equation of the ITS-2 Model

5 Performance Evaluation

To find out the performance of the model, the application was carried out on a medium sized watershed (Wuno Catchment) with an area of 173.19 km^2. This watershed is situated in Central Sulawesi Province and is part of the Palu Watershed. This watershed is located at an altitude of 80 m to 1700 meters above sea level (Figure 2a) with a river network system on the order of 6 (Figure 2b). For calibration and evaluation, rainfall and discharge data were obtained from rainfall and discharge gauges installed at the study

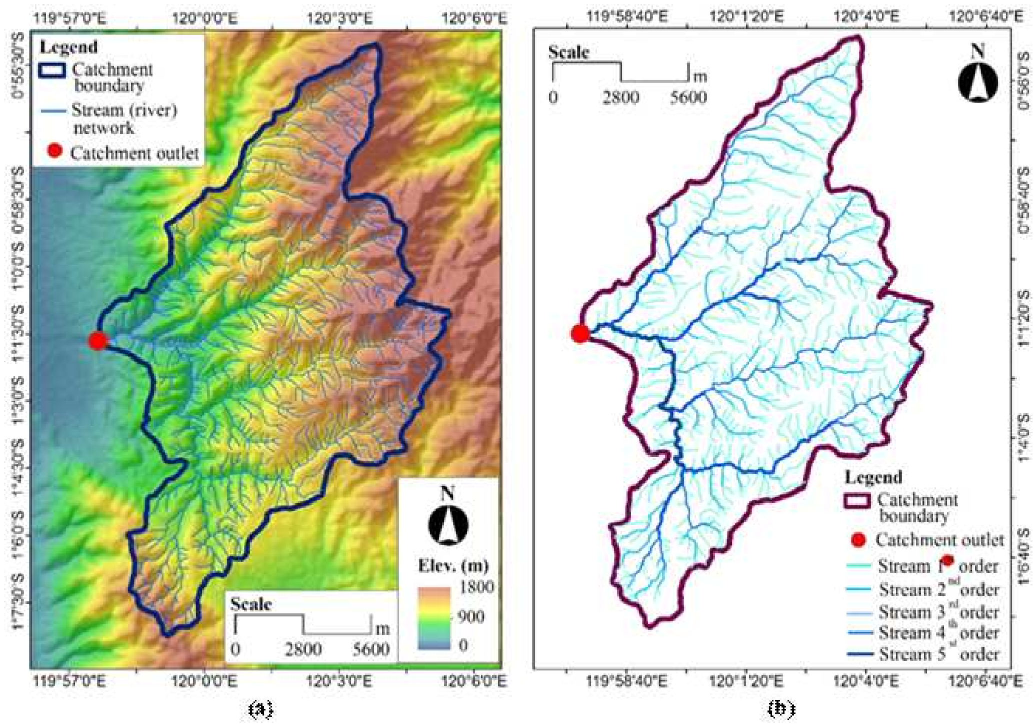

Figure 2: Watershed of Wuno (the sub watershed of Palu), (a). Topographic map, (b). Stream network

site under the management of the Poso Palu Watershed Management Board (BPDAS).

Evaluation of the ITS-2 model using these data pairs indicates that the model's performance is very good as shown in Figure 3, especially after optimization of the coefficients of the model parameters. The performance of the model is measured by two indicators, namely the indicator of model error with the Mean Absolute Percentage Error (MAPE) [25–26] and the conformity indicator with the correlation coefficient (r) [27]. The decrease in MAPE from 62.4% to 11.28% and the increase in r from 0.93 to 0.96 after optimization shows that the coefficients of the parameters of the ITS-2 model are very sensitive especially C_1 and C_3 with optimal values reached at 0.94 and 0.45.

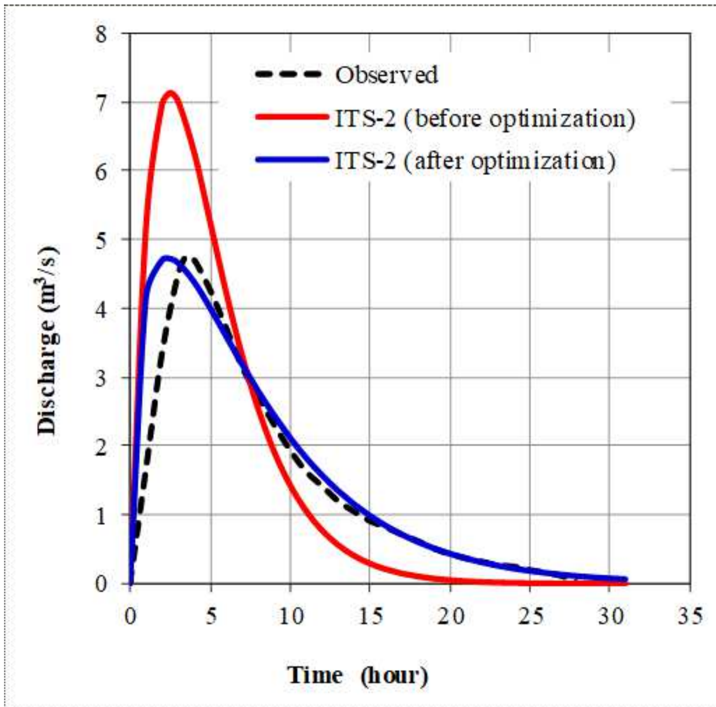

Figure 3: Unit hydrograph of Wuno Watershed derived from observed data and ITS-2 Model before and after parameter optimization

6 Conclusion

This research has succeeded in formulating a dimensionless synthetic unit hydrograph curve equation of ITS-2 Model. The dimensionless formula is expressed by the ratio of time (T) and peak time (T_p) of hydrograph. This very simple equation was derived from the 2-Parameter Gamma Distribution, one of the statistical models for hydrographs transformation based on a continuous probability distribution. A coefficient of C_3 states that the hydrograph shape parameter can be optimized to improve the performance of the model, when a series of observation flow is available for calibration.

The performance of the synthetic unit hydrograph equation was assessed using Mean Absolute Percentage Error (MAPE) and the Coefficient of Correlation (r). The analysis results based on a series of verification indicated that the performance of the hydrograph equation was very high with MAPE below 20% and r above 0.8. This is an indication that the formula of the synthetic unit hydrograph model shows very good performance.

Acknowledgment

This research was partially supported by BPPDN Scholarship from Ministry of Research, Technology and Higher Education, Republic of Indonesia. Therefore, the authors would like to express his highest appreciation for the support of the fund.

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