International Journal of Mathematics and
Computer Science, **15**(2020), no. 4, 1155–1159

Geometric Mean Cordial Labeling of certain Graphs

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(Received July 8, 2020, Accepted September 6, 2020)

Abstract

Let $G = (V, E)$ be a graph and f be a mapping from $V(G)$ into $\{0, 1, 2\}$. For each edge uv assign the label $\lceil\sqrt{f(u)f(v)}\rceil$, f is called a geometric mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x, x \in \{0, 1, 2\}$ respectively. A graph with a geometric mean cordial labeling is called geometric mean cordial graph [6]. In this paper, we prove that the Comb graph $[P_n \odot K_1]$, Quadrilateral snake graph Q_n and Crown graph $[C_n \odot K_1]$ are geometric mean cordial graph.

1 Introduction

A graph labeling is an assignment of integers to the vertices or edges, or both subject to certain conditions [8]. A detailed study on applications of graph labeling was discussed by Bloom and Golomb [4]. According to Beineke and Hedge, graph labeling serves as a frontier between number theory and structure of graphs [3]. A Geometric mean labeling of a graph G with p

Key words and phrases: Geometric mean cordial labeling, Geometric mean cordial graph, Comb graph, Crown graph and Quadrilateral snake graph.
AMS Subject Classifications: 05C76, 05C78.
ISSN 1814-0432, 2020, http://ijmcs.future-in-tech.net

vertices and q edges is an injective function $f : V(G) \to \{1, 2, \dots, q + 1\}$ such that the induced edge labeling $f^* : E(G) \to \{1, 2, \dots, q\}$ defined as $f^*(uv) = \lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$ is bijective [10]. Let $G = (V, E)$ be a graph. For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u)f(v)|$. A binary vertex labeling of a graph G is called a Cordial Labeling, if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling [5]. Let $G = (V, E)$ be a graph and f be a mapping from $V(G) \to \{0, 1, 2\}$. For each edge uv assign the label $\lceil\sqrt{f(u)f(v)}\rceil$, f is called a geometric mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x, x = \{0, 1, 2\}$ respectively. A graph with a geometric mean cordial labeling is called geometric mean cordial graph [6].

2 Comb Graph

Definition 2.1. [9] *The Comb graph $[P_n \odot K_1]$ is the graph obtained from a path P_n by attaching a pendent edge to each vertex of the path P_n.*

Theorem 2.2. *The Comb graph $[P_n \odot K_1]$ is a geometric mean cordial graph.*

Proof. Let G be a Comb Graph $[P_n \odot K_1]$ with the vertex set $V(G) = \{v_1, v_2, v_3, , v_n, u_1, u_2, u_3, \dots, u_n\}$, i.e. $V(G) = \{v_i, u_i/1 \leq i \leq n\}$, where v_i represents the vertices of the path and u_i represents the pendent vertices corresponding to each v_i respectively and the edge set be $E(G) = \{v_1v_2, v_2v_3, v_3v_4, \dots, v_{n-1}v_n\} \cup \{v_1u_1, v_2u_2, v_3u_3, \dots, v_{n-1}u_{n-1}, v_nu_n\}$,
i.e. $E(G) = \{v_iv_{i+1}/1 \leq i \leq n - 1\} \cup \{v_iu_i/1 \leq i \leq n\}$.

Define $f : V(G) \to \{0, 1, 2\}$ as follows:
Case (i): $n \equiv 0(mod3)$. Let $n = 3t, t \geq 1$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq t \\ 1, t + 1 \leq i \leq 2t \\ 2, 2t + 1 \leq i \leq 3t \end{cases}, \ f(u_i) = \begin{cases} 0, 1 \leq i \leq t \\ 1, t + 1 \leq i \leq 2t \\ 2, 2t + 1 \leq i \leq 3t \end{cases}$$

Then $v_f(0) = v_f(1) = v_f(2) = 2t$ and $e_f(0) = e_f(2) = 2t, e_f(1) = 2t - 1$.

Case (ii): $n \equiv 1(mod3)$. Let $n = 3t + 1, t \geq 1$

$$f(v_i)=\begin{cases}0, 1\leq i\leq t\\ 1, t+1\leq i\leq 2t+1\\ 2, 2t+2\leq i\leq 3t+1\end{cases},\ f(u_i)=\begin{cases}0, 1\leq i\leq t\\ 1, t+1\leq i\leq 2t\\ 2, 2t+1\leq i\leq 3t+1\end{cases}$$

Then $v_f(0)=2t, v_f(1)=v_f(2)=2t{+}1$ and $e_f(0)=e_f(1)=2t, e_f(2)=2t{+}1$.

Case (iii): $n\equiv 2(mod 3)$. Let $n=3t+2, t\geq 1$

$$f(v_i)=\begin{cases}0, 1\leq i\leq t+1\\ 1, t+1\leq i\leq 2t+2\\ 2, 2t+3\leq i\leq 3t+2\end{cases},\ f(u_i)=\begin{cases}0, 1\leq i\leq t+1\\ 1, t+2\leq i\leq 2t+1\\ 2, 2t+2\leq i\leq 3t+2\end{cases}$$

Then $v_f(0)=v_f(2)=2t{+}1, v_f(1)=2t{+}2$ and $e_f(0)=e_f(1)=e_f(2)=2t{+}1$.

From all of the above three cases, we see that $|v_f(i)-v_f(j)|\leq 1$ and $|e_f(i)-e_f(j)|\leq 1$ for all $i,j\in\{0,1,2\}$. Hence, the Comb graph $[P_n\odot K_1]$ is a geometric mean cordial graph. □

3 Crown Graph

Definition 3.1. [7] *The Crown graph $[C_n\odot K_1]$ is the graph obtained by attaching a pendent edge to each vertex of the cycle C_n.*

Theorem 3.2. *The Crown graph $[C_n\odot K_1]$ is a geometric mean cordial graph if $n\equiv 1,2(mod 3)$*

Proof. Let G be a Crown graph $[C_n\odot K_1]$ with the vertex set $V(G)=\{v_1,v_2,v_3,\ldots,v_n,u_1,u_2,u_3,\ldots,u_n\}$; i.e., $V(G)=\{v_i,u_i/1\leq i\leq n\}$, where v_i represents the vertices of the cycle and u_i represents the pendent vertices corresponding to each v_i respectively and the edge set

$$E(G)=\{v_1v_2,v_2v_3,v_3v_4,\ldots,v_{n-1}v_n,v_nv_1\}\cup\{u_1v_1,u_2v_2,u_3v_3,\ldots,u_{n-1}v_{n-1},u_nv_n\};$$

i.e., $E(G)=\{v_iv_{i+1}/1\leq i\leq n-1\}\cup\{v_nv_1\}\cup\{u_iv_i/1\leq i\leq n\}$. Let l denote the number of vertices and k the number of edgesof $[C_n\odot K_1]$. Here $k=l$.

Define $f:V(G)\to\{0,1,2\}$ as follows:

Case (i): $n\equiv 1(mod 3)$. Let $n=3t+1, t\geq 1$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq t \\ 1, t+1 \leq i \leq 3t+1 \end{cases}, f(u_i) = \begin{cases} 0, 1 \leq i \leq t \\ 2, t+1 \leq i \leq 3t+1 \end{cases}$$

Then $l \equiv 2(mod 3)$; i.e., $l = 3s + 2$. So $v_f(0) = s, v_f(1) = v_f(2) = s + 1$ and $e_f(0) = e_f(2) = s + 1, e_f(1) = s$.

Case (ii): $n \equiv 2(mod 3)$. Let $n = 3t + 2, t \geq 1$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq t \\ 1, t+1 \leq i \leq 3t+2 \end{cases}, f(u_i) = \begin{cases} 0, 1 \leq i \leq t+1 \\ 2, t+2 \leq i \leq 3t+2 \end{cases}$$

Then $l \equiv 1(\mod 3)$; i.e., $l = 3s+1$, so $v_f(0) = v_f(2) = s, v_f(1) = s+1$ and $e_f(0) = s+1, e_f(1) = e_f(2) = s$.

From the above two cases, we see that $|v_f(i)-v_f(j)| \leq 1$ and $|e_f(i)-e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$. Hence, te Crown graph $[C_n \odot K_1]$ is a geometric mean cordial graph. □

4 Quadrilateral Snake Graph

Definition 4.1. [1] *The Quadrilateral snake graph Q_n is obtained from the path P_n by replacing every edge of a path by a cycle C_4.*

Theorem 4.2. *The Quadrilateral snake graph Q_n is a geometric mean cordial graph.*

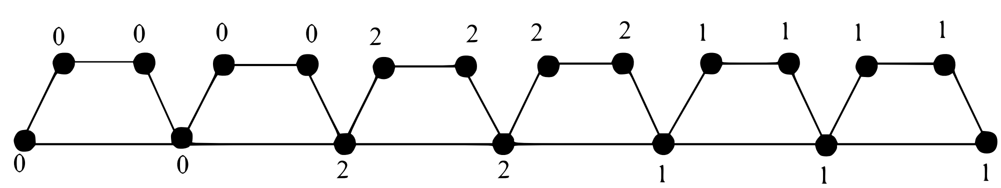

Figure 1: Quadrilateral Snake Graph Q_7

Here $v_f(0) = v_f(2) = 6$, $v_f(1) = 7$, and $e_f(0) = e_f(2) = e_f(1) = 8$. Therefore $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$.

5 Conclusion

In this paper we have proved that the Comb graph $[P_n \odot K_1]$, the Crown graph $[C_n \odot K_1]$ and the Quadrilateral snake graph Q_n are Geometric mean cordial graph. To investigate analogous results for different families of graphs that admit geometric mean cordial labeling is an open area of research.

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