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Mean Square Cordial Labeling of a Shell-butterfly Graph

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Abstract

In this paper, it is analyzed that shell-butterfly graphs are mean square cordial graphs.

1 Introduction

Graph labeling has been viewed as one of the fastest growing research areas of graph theory in recent decades. Gallian [1] is referred for thorough survey of various graph labeling methods. Most of the labeling techniques trace their roots to Rosa in 1967 [2]. The graphs considered are simple and finite [3]. The layout pattern of computer interconnections within a network is called network topology. Network topology of path and star are unfortified for attacks and failures. To overcome those difficulties, a new configuration called shell topology is formed which is the connection of both path and star. Shell graph (shell topology) is not affected by attacks and failures and so it is one of the most reliable networks. Shell network topology is used in Local Area Network (LAN) which is capable of transmitting data much faster with low error rate.

Cahit [4] initiated cordial labeling and finds its use in DNA code word design problems and in noisy communication problems. Cordial labeling

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plays a vital role in computer science as it deals with 0's and 1's, which is the integral part of the same. Various labeling techniques are introduced by the researchers within the theme of cordial labeling. Mean square cordial is one of them and it was first introduced by Murugan [7] who investigated the same labeling for some special graphs [5,6,7]. Dhanalakshmi et al. [8,9] analyzed the mean square cordial labeling of some special graphs and they studied the same labeling for some families of star and snake graphs [10,11]. J. Jeba et al. [12] discussed shell graphs in their research contributions. Mean square cordial labeling of some shell related graphs are taken for discussion as it has its own application in computer networks. In particular, this paper analyzed the mean square cordial labeling of a Shell-butterfly graph.

2 Definitions

Definition 2.1. *A Mean Square Cordial Labeling (MSCL) of a Graph $G(V,E)$ with p vertices and q edges is a surjection from V onto $\{0,1\}$ such that each edge uv is assigned the label ($\lceil (f(u)^2 + f(v)^2)/2 \rceil$)w $\lceil x \rceil$, where $(ceil(x))$ is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled 0 and the number of edges labeled with 1 differ by at most 1.*

Definition 2.2. *A multiple shell is defined to be a collection of edge disjoint shells that have common apex vertex. Hence a double shell consists of two edge disjoint shells with a common apex vertex. A Shell-butterfly graph is a double shell in which each shell has any order with exactly two pendant edges at the apex.*

Definition 2.3. *A shell-butterfly graph in which each shell has the same order is called uniform shell-butterfly graph.*

3 Main Result

Theorem 3.1. *A Shell-butterfly graph with shell orders $m(m > 2)$ and $n(n > 2)$ admits mean square cordial labeling.*

Proof.
Consider the shell-butterfly graph as G. $V(G) = \{u_0, v_i\text{: } 1 \le i \le m, w_i : 1 \le i \le n, v_0 \text{ and } w_0\}$ and $E(G) = \{[(u_0v_i) : 1 \le i \le m] \bigcup [(u_0w_i) : 1 \le i \le n] \bigcup [(v_iv_{i+1}) : 1 \le i \le m-1] \bigcup [(w_iw_{i+1}) : 1 \le i \le n-1] \bigcup [u_0v_0] \bigcup [u_0w_0]\}$, where u_0 is the apex vertex, v_0 and w_0 are the vertices on the pendant edge and v_i, w_i are vertices of the cycles of the two shells respectively except the apex vertex. $|V| = m+n+3$ and $|E| = 2m+2n$.

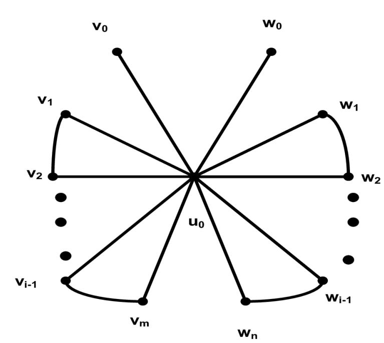

Figure 1: Shell-butterfly graph of orders m and n.

Define $f : V(G) \to \{0,1\}$
To prove that G admits mean square cordial labeling, we consider three cases:
(i) $m = n$
(ii) $m \neq n$, where $m + n \equiv 0(\mod 2)$
(iii) $m \neq n$, where $m + n \equiv 1(\mod 2)$.

Case 1: $m = n$:
$f(u_0) = 0$
$f(v_0) = 0$
$f(w_0) = 1$
$f(v_i) = 0, 1 \leq i \leq n$
$f(w_i) = 0, 1 \leq i \leq n$.

Thus the labeling of edges are $f(u_0v_0) = 0$
$f(u_0w_0) = 1$
$f(u_0v_i) = 0, 1 \leq i \leq m$
$f(u_0w_i) = 1, 1 \leq i \leq n$
$f(v_iv_{i+1}) = 0, 1 \leq i \leq m - 1$.

Case 2: $m \neq n$, where $m + n \equiv 0(\mod 2)$:
Subcase 2.1: $m > n$:
$f(u_0) = 0$
$f(v_0) = 0$
$f(w_0) = 1$
$f(v_i) = 0, 1 \leq i \leq \frac{m+n}{2}$
$1, \frac{m+n}{2} + 1 \leq i \leq m$
$f(w_i) = 1, 1 \leq i \leq n$.
Thus the labeling of edges are
$f(u_0w_0) = 0$
$f(u_0v_i) = 0, 1 \leq i \leq m$
$1, \frac{m+n}{2} + 1 \leq i \leq m$
$f(u_0w_i) = 1, 1 \leq i \leq n$
$f(v_iv_{i+1}) = 0, 1 \leq i \leq \frac{m+n}{2} - 1\ 1, \frac{m+n}{2} \leq i \leq m - 1$
$f(w_iw_{i+1}) = 1, 1 \leq i \leq n - 1$.
Subcase 2.2: $m < n$:
$f(u_0) = 0$
$f(v_0) = 1$
$f(w_0) = 0$
$f(v_i) = 1, 1 \leq i \leq m$
$f(w_i) = 1, 1 \leq i \leq \frac{n-m}{2}$
$0, \frac{n-m}{2} + 1 \leq i \leq n$.

Thus the labeling of edges are

$f(u_0v_0) = 1$
$f(u_0w_0) = 1$
$f(u_0v_i) = 1, 1 \leq i \leq m$
$f(u_0w_i) = 1, 1 \leq i \leq \frac{n-m}{2}$
$0, \frac{n-m}{2} + 1 \leq i \leq n$
$f(v_iv_{i+1}) = 1, 1 \leq i \leq m - 1$
$f(w_iw_{i+1}) = 1, 1 \leq i \leq \frac{n-m}{2} - 1$
$0, \frac{n-m}{2} \leq i \leq n - 1.$

Case 3: $m \neq n$, where $m + n \equiv 1(\mod 2)$:
Subcase 3.1: $m > n$**:**
$f(u_0) = 0$
$f(v_0) = f(w_0) = 1$
$f(v_i) = 0, 1 \leq i \leq \frac{m+n+1}{2}$
$1, \frac{m+n+3}{2} \leq i \leq m$
$f(w_i) = 1, 1 \leq i \leq n.$
Thus the labeling of edges are
$f(u_0v_0) = 0$
$f(u_0w_0) = 1$
$f(u_0w_0) = 1, 1 \leq i \leq \frac{m+n+1}{2}$
$1, \frac{m+n+3}{2} \leq i \leq m$
$f(u_0w_i) = 1, 1 \leq i \leq n$
$f(v_iv_{i+1}) = 0, 1 \leq i \leq \frac{m+n-1}{2}$
$1, \frac{m+n+1}{2} \leq i \leq m - 1$
$f(w_iw_{i+1}) = 1, 1 \leq i \leq n - 1.$
Subcase 3.2: $m > n$:
$f(u_0) = 0$
$f(v_0) = f(w_0) = 1$ $f(v_i) = 1, 1 \leq i \leq m$
$f(w_i) = 1, 1 \leq i \leq \frac{n-m-1}{2}$
$0, \frac{n-m+1}{2} \leq i \leq n.$
Thus the labeling of edges are
$f(u_0v_0) = 1$
$f(u_0w_0) = 1$
$f(u_0v_i) = 1, 1 \leq i \leq m$
$f(u_0w_i) = 1, 1 \leq i \leq \frac{n-m-1}{2}$
$0, \frac{n-m+1}{2} \leq i \leq n$
$f(v_iv_{i+1}) = 1, 1 \leq i \leq m$
$f(w_iw_{i+1}) = 0, 1 \leq i \leq \frac{n-m-3}{2}$
$1, \frac{n-m-1}{2} \leq i \leq n - 1.$

Hence, all shell-butterfly graphs of orders $m(m > 2)$ and $n(n > 2)$ are mean square cordial graphs.

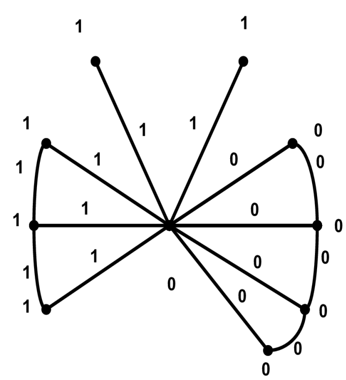

Figure 2: MSCL of shell-butterfly graph of orders 3 and 4.

Remark 3.2. *From case (i) of theorem 3.1, we observe that uniform butterfly graph are mean square cordial graphs.*

4 Conclusion

In this paper, mean square cordiality of shell-butterfly graphs were studied. The author suggests that the same labeling behavior can be studied for standard graphs and also for graph operations. It will be quite interesting for researchers to analyze it in their future work.

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