International Journal of Mathematics and
Computer Science, **15**(2020), no. 4, 1309–1319

Contra Continuity on Double Fuzzy Topological Space

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(Received August 7, 2020, Accepted September 15, 2020)

Abstract

In this paper, we introduce a new class of contra continuity namely double fuzzy contra continuity (for short df contra c), double fuzzy contra α^m- continuity (for short df contra $\alpha^m - c$), and double fuzzy contra α^m-generalized continuity (for short df contra α^m-gc). The relationship among them is studied and some basic properties are proved.

1 Introduction

After Zadeh [1] in 1965 developed the concept of fuzzy sets, Chang [2] in 1968 introduced a fuzzy topological concept through the use of fuzzy sets. Later Coker and Dimirci [3] in 1996 generalized fuzzy topological space by introducing the term intuitionistic. In 2005, Garcia and Rodabaugh [4] further

Key words and phrases: Double fuzzy topology, double fuzzy contra $\alpha^m - c$ontinuous function, double fuzzy α^m-open function, double fuzzy contra α^m-generalized continuous.
AMS (MOS) Subject Classifications: 54A40.
ISSN 1814-0432, 2020, http://ijmcs.future-in-tech.net

generalized fuzzy intuitionistic topological space to double fuzzy topological space by replacing the term intuitionistic with the term double. Mohammed et al. [9] in 2017 presented and studied the concept $(r1, s1)-\alpha^m$-fuzzy closed sets in double fuzzy topological spaces. In the same year, they generalized and studied some types of functions across $(r1, s1) - \alpha^m$-fuzzy closed sets ([10], [11]) by introducing the notions of $\alpha^m - c$ontinuous and α^m-generalized continuous in double fuzzy topological space.
In this paper, we present a concept of contra-closed set in double fuzzy topological space and provide a study of different types of functions by utilizing this concept. Illustrative examples are provided.

2 Preliminaries

In this section, we give some basic definitions that are useful to our research.
Definition 2.1 [4]: A double fuzzy topology (τ_X, τ_X*) on a non-empty set X is a pair of functions $\tau_X, \tau_X* : I^X \to$I, which satisfies the following properties:

(i) $\tau_X(\delta_1) \leq 1 - \tau_X * (\delta_1)$ for each $\delta_1 \in I^X$.

(ii) $\tau_X(\delta_1 \cap \delta_2) \geq \tau_X(\delta_1) \cap \tau_X(\delta_2)$ and
$\tau_X * (\delta_1 \cap \delta_2) \leq \tau_X * (\delta_1) \cup \tau_X * (\delta_2)$ for each $\delta_1, \delta_2 \in I^X$.

(ii) $\tau_X(\cup_1^i \delta_i) \geq \cap_1^i \tau_X(\delta_i)$ and $\tau_X * (\cup_1^i \delta_i) \leq \cup_1^i \tau_X * (\delta_i)$ for each $\delta_i \in I^X$, $i \in \mathbb{N}$.

The triplex (X, τ_X, τ_X*) is called a double fuzzy topological spaces (dfts, for short), $\tau_X(\delta_1)$ and $\tau_X * (\delta_1)$ may be interpreted as a gradation of openness and gradation of non-openness for δ_1.

Definition 2.2 [4,8]: Let X be a dfts, then, for each $r_1 \in I_{r1}, s_1 \in I_{s1}$ and $\delta \in I^X$, we define the double fuzzy closure and interior operator C_{τ_x,τ_x*} : $I^X \times I_{r_1} \times I_{s_1} \to I^X$ as follows:
$C_{\tau_x,\tau_x*}(\delta, r_1, s_1) = \cap\{\lambda \in I^X : \delta \leq \lambda, \tau_X(1-\lambda) \geq r_1, \tau_X * (1-\lambda) \leq s_1\}$.

$I_{\tau_x,\tau_x*}(\delta, r_1, s_1) = \cup\{\lambda \in I^X : \lambda \leq \delta, \tau_X(\lambda) \geq r_1, \tau_X * (\lambda) \leq s_1\}$.

Definition 2.3: Let X be a dfts $\delta, \lambda \in I^X, r_1 \in I_{r1}$ and $s_1 \in I_{s1}$. A

fuzzy set δ is said to be:
(1) An (r_1, s_1)-fuzzy open set [8] ((r_1, s_1)-fo, for short) if $\tau_X(\delta) \geq r_1$ and $\tau_X*(\delta) \leq s_1$. A fuzzy set δ is called an (r_1, s_1)-fuzzy closed set if $\tau_X(1-\delta) \geq r_1$ and $\tau_X*(1-\delta) \leq s_1$.

(2) An (r_1, s_1)-fuzzy α-open set ([5], [6]) ((r_1, s_1)-fα-open, for short), if $\lambda \leq I_{\tau_x,\tau_x*}(C_{\tau_x,\tau_x*}(I_{\tau_x,\tau_x*}(\lambda, r_1, s_1), r_1, s_1), r_1, s_1)$ and an (r_1, s_1)-fuzzy α-closed set ((r_1, s_1)-fα-closed, for short), if $C_{\tau_x,\tau_x*}(I_{\tau_x,\tau_x*}(C_{\tau_x,\tau_x*}(\lambda, r_1, s_1), r_1, s_1), r_1, s_1) \leq \lambda$.

(3) An (r_1, s_1)-generalized fuzzy-closed set [7] ((r_0, s_1)-gf-closed, for short) if $C_{\tau_x,\tau_x*}(\delta, r_1, s_1) \leq \lambda$, whenever $\delta \leq \lambda, \tau_X(\lambda) \geq r_1, \tau_X*(\lambda) \leq s_1$. δ is called (r_1, s_1)-generalized fuzzy open ((r_1, s_1)-gf-open, for short) if and only if $1-\delta$ is (r_1, s_1)-gf-closed set.

(4) An (r_1, s_1)-fuzzy α^m-closed sets ($(r_1, s_1)-f\alpha^m$-closed, for short) ([9],[11]) if and only if $I_{\tau_x,\tau_x*}(C_{\tau_x,\tau_x*}(\delta, r_1, s_1)r_1, s_1) \leq \lambda$, whenever $\delta \leq \lambda$ and λ is $(r_1, s_1) - \alpha$-open. δ is called $(r_1, s_1) - f\alpha^m$-open if and only if $1-\delta$ is $(r_1, s_1) - \alpha^m$-closed.

(5) An $(r_1, s_1) - \alpha^m$-generalized fuzzy-closed set ($(r_1, s_1) - \alpha^m$-gf-closed, for short) ([9], [11]). If $\alpha^m C_{\tau_x,\tau_x*}(\delta, r_1, s_1) \leq \lambda$ such that $\delta \leq \lambda$ and λ is $(r_1, s_1) - f\alpha^m$-open set. δ is called $(r_1, s_1) - \alpha^m$-generalized fuzzy-open set ($(r_1, s_1) - \alpha^m$-gf-open, for short) if and only if $1-\delta$ is an $(r_1, s_1) - \alpha^m$ -gf closed set.

Definition 2.4 [9,11]: If X is a double fuzzy topological space, for each $\delta, \lambda \in I^X$, $r_1 \in I_{r_1}$ and $s_1 \in I_{s_1}$, then the $\alpha^m - c$losure and α^m-Interior operator of δ is defined as:
$\alpha^m C_{\tau_x,\tau_x*}(\delta, r_1, s_1) = \cap\{\lambda \in I^X : \delta \leq \lambda, \lambda \ \ is \ \ (r_1, s_1) - f\alpha^m - closed\}$.

$\alpha^m I_{\tau_x,\tau_x*}(\delta, r_1, s_1) = \cup\{\lambda \in I^X : \delta \geq \lambda, \lambda \ \ is \ \ (r_1, s_1) - f\alpha^m - open\}$.

Definition 2.5 : Let a function $f : X \to Y$, whenever $r_1 \in I_{r_1}$ and $s_1 \in I_{s_1}$. Then f is said to be:
(1) A double fuzzy continuous (see [8]) if and only if $\tau_X(f^{-1}(\delta)) \geq \tau_Y(\delta)$ and $\tau_X*(f^{-1}(\delta)) \leq \tau_Y*(\delta)$ for each $\delta \in I^Y$.

(2) A double fuzzy-α^m-continuous (df-$\alpha^m - c$, for short) ([10], [11]) if $f^{-1}(\delta)$

is an $(r_1, s_1) - f\alpha^m$-open such that $\tau_Y(\delta) \geq r_1$ and $\tau_Y * (\delta) \leq s_1$.

(3) A double fuzzy-α^m-open (df-α^m-open, for short) (see [10], [11]) if $f(\delta)$ is an$(r_1, s_1) - f\alpha^m$-open in Y for each $\tau_X(\delta) \geq r_1$ and $\tau_X * (\delta) \leq s_1$.

(4) A double fuzzy-α^m-generalized continuous (df-α^m-gc, for short)(see[10],[11]) if for each $\tau_Y(1-\delta) \geq r_1$ and $\tau_Y * (1-\delta) \leq s_1[\tau_Y(\delta) \geq r_1, \tau_Y * (\delta) \leq s_1]$, $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-gf-closed set $[(r_1, s_1) - \alpha^m$-gf-open set], whenever $\delta \in I^Y$.

3 Contra continuous function

In this section, we define the concept of double fuzzy contra-continuous function and provide some examples.
Definition 3.1: A function $f : X \to Y$ is called a double fuzzy contra-continuous if $f^{-1}(\delta)$ is $(r_1, s_1) - f$ closed set in X for each (r_1, s_1)-f open δ set in Y.
Example 3.2: Let $X = \{p, q\}, Y = \{m, n\}$ with the fuzzy sets ω_1, ω_2 on X. Then $(\tau_X(\omega), \tau_X * (\omega))$ is define as:

$$\tau_X(\omega) = \begin{cases} 1, & if\ \omega \in \{0,1\} \\ \frac{1}{2}, & if\ \omega(x) = \omega_1 \\ \frac{1}{4}, & if\ \omega(x) = \omega_2 \\ 0, & otherwise \end{cases}$$

$$\tau_X * (\omega) = \begin{cases} 0, & if\ \omega \in \{0,1\} \\ \frac{1}{2}, & if\ \omega(x) = \omega_1 \\ \frac{3}{4}, & if\ \omega(x) = \omega_2 \\ 1, & otherwise \end{cases}$$

such that
$\omega_1(p) = 0.4$, $\omega_1(q) = 0.4$, and $\omega_2(p) = 0.6$ $\omega_2(q) = 0.6$.
Also, take fuzzy set ρ and defined $(\tau_Y(\rho), \tau_Y * (\rho))$ on Y by:

$$\tau_Y(\rho) = \begin{cases} 1, & if\ \rho \in \{0,1\} \\ \frac{1}{2}, & if\ \rho(Y) = \rho_1 \\ 0, & otherwise \end{cases}$$

$$\tau_Y*(\rho)=\begin{cases}0, & if\ \rho\in\{0,1\}\\ \frac{1}{2}, & if\ \rho(Y)=\rho_1\\ 1, & otherwise\end{cases}$$

such that $\rho(m)=0.6$ and $\rho(n)=0.6$.
Let $(\tau_X(\omega),\tau_X*(\omega))$ and $(\tau_Y(\rho),\tau_Y*(\rho))$ be two double fuzzy topological spaces and a function $f:X\to Y$ is define by:

$f(p)=m$ and $f(q)=n$.
Then $\tau_Y(\rho)\geq\frac{1}{2}$, $\tau_Y*(\rho)\leq\frac{1}{2}$, $f^{-1}(\rho)=\omega_2$ and $\tau_X(1-f^{-1}(\rho))\geq\frac{1}{2}$, $\tau_X*(1-f^{-1}(\rho))\leq\frac{1}{2}$.

Now the concept of a double fuzzy contra α^m-continuous function is define by:
Definition 3.3: A function $f:X\to Y$ is called a double fuzzy contra α^m-continuous (df contra α^m-c, for short) function if $f^{-1}(\delta)$ is an $(r_1,s_1)-\alpha^m-f$ closed set in X for each (r_1,s_1)-f open δ set in Y.

Example 3.4: Take the sets X,Y and fuzzy sets ω_1, ω_2, ρ. Define double fuzzy topological spaces $(\tau_X(\omega),\tau_X*(\omega))$ and $(\tau_Y(\rho),\tau_Y*(\rho))$ as in Example 3.2.
Now,$f^{-1}(\rho)=\omega_2\leq 1$
$I_{\tau_x,\tau_x*}(C_{\tau_x,\tau_x*}(f^{-1}(\rho)=\omega_2,\frac{1}{2},\frac{1}{2})\frac{1}{2},\frac{1}{2},)=I_{\tau_x,\tau_x*}(\omega_1,\frac{1}{2},\frac{1}{2})=\omega_2\leq 1$.

So $(f^{-1}(\rho)$ is an $(\frac{1}{2},\frac{1}{2})-f\alpha^m$-closed set. Thus f is df contra α^m-c function.

Theorem 3.5: Let $f:X\to Y$ be a function. Then f is df contra α^m-c function if and only if for each (r_1,s_1)-f closed set δ in Y, $f^{-1}(\delta)$ is an $(r_1,s_1)-\alpha^m$-f open set in X.
Proof: Let $\tau_Y(1-\delta)\geq r_1$, $\tau_Y*(1-\delta)\leq s_1$, then $\tau_Y(\delta)\geq r_1$, $\tau_Y*(\delta)\leq s_1$. Since f is df contra α^m-c function, so $f^{-1}(\delta)$ is an $(r_1,s_1)-\alpha^m$-f closed set in X. Hence, $1-f^{-1}(\delta)=f^{-1}(1-\delta)$ is an $(r_1,s_1)-\alpha^m$-f open set in$X$.

Conversely, let $\tau_Y(\delta)\geq r_1$, $\tau_Y*(\delta)\leq s_1$. Then $\tau_Y(1-\delta)\geq r_1$, $\tau_Y*(1-\delta)\leq s_1$. $f^{-1}(1-\delta)=1-f^{-1}(\delta)$ is an $(r_1,s_1)-\alpha^m-f$ open set in X. Therefore, $f^{-1}(1-\delta)$ is an $(r_1,s_1)-\alpha^m-f$ open set in$X$. Hence $f^{-1}(\delta)$ is an $(r_1,s_1)-\alpha^m-f$ closed set in$X$. That is f is df contra α^m-c function.

Theorem 3.6: Let $f:X\to Y$ be a function. Then the following state-

ments are equivalent.
(1) f is df-contra $\alpha^m - c$ function.
(2) For each $(r_1, s_1) - f$ closed set δ, $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m - f$ open set.
(3) For each (r_1, s_1)-f open set ω in Y, $f^{-1}(\omega)$ is an $(r_1, s_1) - \alpha^m$-f closed set.

Proof: (1) $\longrightarrow$ (2) by theorem 3.5 (2) $\longrightarrow$ (3)
Let $\tau_Y(\delta) \geq r_1$, $\tau_Y * (\delta) \leq s_1$. Put $\delta = 1 - \omega$.
Then $\tau_Y(1-\delta) \geq r_1$, $\tau_Y *(1-\delta) \leq s_1$, then by (2), $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-f open set.
$f^{-1}(\delta) = f^{-1}(1 - \omega) = 1 - f^{-1}(\omega)$ is an $(r_1, s_1) - \alpha^m$-f open set. That is $f^{-1}(\omega)$ is an $(r_1, s_1) - \alpha^m - f$ closed set. (3) $\longrightarrow$ (1) Its proved by definition (2.1).

Theorem 3.7: Suppose $f : X \to Y$ is a df-contra $\alpha^m - c$ function and $g : Y \to Z$ is df- continuous then gof is df-contra $\alpha^m - c$ function.
Proof: Let $\tau_Z(\delta) \geq r_1$,$\tau_Z *(\delta) \leq s_1$, since g is df-continuous, so $\tau_Y(g^{-1}(\delta)) \geq r_1$, $\tau_Y * (g^{-1}(\delta)) \leq s_1$.
Since f is a df-contra $\alpha^m - c$ function, then $f^{-1}(g^{-1}(\delta))$ is an $(r_1, s_1) - \alpha^m - f$ closed set.
Thus gof is df-contra $\alpha^m - c$ function.

Theorem 3.8: Let $f : X \to Y$ and $g : Y \to Z$ be a function, if f is df-α^m irr function and g is df-contra $\alpha^m - c$ function, then gof is df-contra $\alpha^m - c$ function. **Proof:** Let $\tau_Z(\delta) \geq r_1$, $\tau_Z *(\delta) \leq s_1$, and since g is df-contra $\alpha^m - c$ function, so $g^{-1}(\delta)$ is is an $(r_1, s_1) - \alpha^m$-f closed set in Y.
Since f is df- α^m irr function, then $f^{-1}(g^{-1}(\delta))$ is an $(r_1, s_1) - \alpha^m$-f closed set in X.
$f^{-1}(g^{-1}(\delta)) = gof^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-f closed set.
Therefore gof is df-contra $\alpha^m - c$ function.

Theorem 3.9: Let $f : X \to Y$ be a surjective df- α^m-open function and $g : Y \to Z$ is a function such that gof is df-contra $\alpha^m - c$ function then g is df- contra $\alpha^m - c$ function.
Proof: Let $\tau_Z(1 - \delta) \geq r_1$, $\tau_Z * (1 - \delta) \leq s_1$, since gof is df-contra $\alpha^m - c$ function.
then $f^{-1}(g^{-1}(\delta))$ is an $(r_1, s_1) - \alpha^m$-f open set in X and since f is surjective df- α^m-open function.
Then $f(f^{-1}(g^{-1}(\delta))) = g^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-f open set in Y, such that $\tau_Z(1 - \delta) \geq r_1$, $\tau_Z * (1 - \delta) \leq s_1$. Therefore g is df- contra $\alpha^m - c$ function.

Definition 3.10: A function $f : X \to Y$ is called double fuzzy contra α^m-generalized continuous (df contra α^m-g c) function if and only if $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X, for each $\tau_Y(\delta) \geq r_1$, $\tau_Y * (\delta) \leq s_1$.

Example 3.11: Let $X = \{p, q\}, Y = \{m, n\}$ with the fuzzy sets ω_1, ω_2. and we define $(\tau_X(\omega), \tau_X * (\omega))$ on X by:

$$\tau_X(\omega) = \begin{cases} 1, & if\ \omega \in \{0,1\} \\ \frac{1}{2}, & if\ \omega(x) = \omega_1 \\ \frac{1}{4}, & if\ \omega(x) = \omega_2 \\ 0, & otherwise \end{cases}$$

$$\tau_X * (\delta) = \begin{cases} 0, & if\ \omega \in \{0,1\} \\ \frac{1}{2}, & if\ \omega(x) = \omega_1 \\ \frac{3}{4}, & if\ \omega(x) = \omega_2 \\ 1, & otherwise \end{cases}$$

Also take fuzzy sets ρ_1 , ρ_2 and we define $(\tau_Y(\rho), \tau_Y * (\rho))$ on Y by:

$$\tau_Y(\rho) = \begin{cases} 1, & if\ \rho \in \{0,1\} \\ \frac{1}{2}, & if\ \rho(x) = \rho_1 \\ \frac{1}{4}, & if\ \rho(x) = \rho_2 \\ 0, & otherwise \end{cases}$$

$$\tau_Y * (\psi) = \begin{cases} 0, & if\ \rho \in \{0,1\} \\ \frac{1}{2}, & if\ \rho(x) = \rho_1 \\ \frac{3}{4}, & if\ \rho(x) = \rho_2 \\ 1, & otherwise \end{cases}$$

such that

$\omega_1(p) = 0.3$, $\omega_1(q) = 0.4$
$\omega_2(p) = 0.7$, $\omega_2(q) = 0.6$
$p_1(m) = 0.7$, $p_1(n) = 0.8$,

$p_2(m) = 0.3$, $p_2(n) = 0.2$,

$\rho(m) = 0.6$ and $\rho(n) = 0.6$.
Let $(\tau_X(\omega), \tau_X * (\omega))$ and $(\tau_Y(\rho), \tau_Y * (\rho))$ be two double fuzzy topological spaces and a function $f : X \to Y$ is define by:

$f(p) = m$ and $f(q) = n$.
Then $\tau_Y(\rho) \geq \frac{1}{2}$, $\tau_Y * (\rho) \leq \frac{1}{2}$, $f^{-1}(\rho_1) = (p0.7, q0.8)$, $f^{-1}(\rho_1) \leq \omega_2$, ω_2 is an $(\frac{1}{2}, \frac{1}{2}) - f\alpha^m$- closed set.
So, $\tau_Y(\rho) \geq \frac{1}{2}$, $\tau_Y * (\rho) \leq \frac{1}{2}$, $f^{-1}(\rho_1) = (p0.7, q0.8)$ $f^{-1}(\rho_1) \leq \omega_2$, ω_2 is an $(\frac{1}{2}, \frac{1}{2}) - f\alpha^m$- closed set.
. $\alpha^m C_{\tau_x,\tau_x*}(f^{-1}(\rho_1), \frac{1}{2}, \frac{1}{2}) = \cap\{\omega_2 \in I^X : f^{-1}(\rho_1) \leq \omega_2, I_{\tau_x,\tau_x*}(C_{\tau_x,\tau_x*}(\omega_2, \frac{1}{2}, \frac{1}{2})\frac{1}{2}, \frac{1}{2}) = \omega_2\} = \omega_2 \leq \omega_2$. So, $f^{-1}(\rho_1)$ is an $(\frac{1}{2}, \frac{1}{2}) - \alpha^m$-gf-closed set implies df contra α^m-gc.

Theorem 3.12: Every df-contra continuous is a df-contra α^m-g c function.
Proof: Let $f : X \to Y$ be a df-contra continuous, and let $\tau_Y(\delta) \geq r_1$,$\tau_Y * (\delta) \leq s_1$.
So, $\tau_X(1 - f^{-1}(\delta)) \geq r_1$, $\tau_X * (1 - f^{-1}(\delta)) \leq s_1$.
Since, every (r_1, s_1)-f closed set is an $(r_1, s_1) - \alpha^m$-g f closed set, then $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X, for each $\tau_Y(\delta) \geq r_1$,$\tau_Y * (\delta) \leq s_1$.
Therefore f is df-contra α^m-g c function.

Remark 3.13: The converse of the theorem is not true. The following example proves it.
Example 3.14: See Example 3.12, f is a df-contra α^m-g c function, but f is not df-contra continuous.

Theorem 3.15: Every df-contra $\alpha^m - c$ function is a df-contraα^m-g c function.
Proof: Let $f : X \to Y$ be a df-contra $\alpha^m - c$ function, and let $\tau_Y(\delta) \geq r_1$,$\tau_Y * (\delta) \leq s_1$.
So,$f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-f closed set in X.
Since every $(r_1, s_1) - \alpha^m$-f closed set is an $(r_1, s_1) - \alpha^m$-g f closed set, then $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X, for each $\tau_Y(\delta) \geq r_1$,$\tau_Y * (\delta) \leq s_1$.
Thus f is df-contra α^m-g c function.

Remark 3.16: The converse of the theorem is not true. The following example shows it.

Example 3.17: Take the sets X, Y and dftss $(\tau_X(\omega), \tau_X*(\omega))$ and $(\tau_Y(\rho), \tau_Y*(\rho))$ as in Example 3.12 such that, $\omega_1(p) = 0.4$, $\omega_1(q) = 0.5$, $\omega_2(p) = 0.6$, $\omega_2(q) = 0.7$, and $p_1(m) = 0.4$, $p_1(n) = 0.7$, $p_2(m) = 0.4$, $p_2(n) = 0.7$, $p_3(m) = 0.6$, $p_3(n) = 0.5$. Let $(\tau_X(\omega), \tau_X*(\omega))$ and $(\tau_Y(\rho), \tau_Y*(\rho))$ be two double fuzzy topological spaces and a function $f : (X, \tau_X, \tau_X*) \to (Y, \tau_Y, \tau_Y*)$ be define as:
$f(p) = m$ and $f(q) = n$.

So,$\tau_Y(\rho) \geq \frac{1}{2}$,$\tau_Y*(\rho) \leq \frac{1}{2}$, $f^{-1}(\rho_1) = (p0.4, q0.7) \leq \omega_1$, then
$\alpha^m C_{\tau_x,\tau_x*}(f^{-1}(\rho_1), \frac{1}{2}, \frac{1}{2}) = \cap\{\omega_1 \in I^X : f^{-1}(\rho_1) \leq \omega_1, I_{\tau_x,\tau_x*}(C_{\tau_x,\tau_x*}(\omega_1, \frac{1}{2}, \frac{1}{2}), \frac{1}{2}, \frac{1}{2}) = I_{\tau_x,\tau_x*}((0.4, 0.5), \frac{1}{2}, \frac{1}{2}) = \omega_1\} = \omega_1 \leq \omega_1$.

That is $f^{-1}(\rho_1)$ is an $(\frac{1}{2}, \frac{1}{2}) - \alpha^m$-gf-closed set in X implies that df contra α^m-g c. But, $f^{-1}(\rho_1) = (p0.4, q0.7) \leq \omega_2$,
$I_{\tau_x,\tau_x*}(C_{\tau_x,\tau_x*}(f^{-1}(\rho_1), \frac{1}{2}, \frac{1}{2})\frac{1}{2}, \frac{1}{2}) = I_{\tau_x,\tau_x*}((0.4, 0.5), \frac{1}{2}, \frac{1}{2}) = \omega_1 \not\leq \omega_2$.
$f^{-1}(\rho_1)$ is not $(\frac{1}{2}, \frac{1}{2}) - f\alpha^m$-closed set in X implies that f is not df-contra $\alpha^m - c$ function.

Theorem 3.18: A function $f : X \to Y$ is a df-contra α^m-g c function if and only if ,$f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f open set in X, for each $\tau_Y(1-\delta) \geq r_1$,$\tau_Y*(1-\delta) \leq s_1$.
Proof: Let $\tau_Y(1-\delta) \geq r_1$,$\tau_Y*(1-\delta) \leq s_1$.
Then $\tau_Y(\delta) \geq r_1$,$\tau_Y*(\delta) \leq s_1$.
Since, f is df-contra α^m-g c function, so$f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X.
Hence $1 - f^{-1}(\delta) = f^{-1}(1-\delta)$ is an $(r_1, s_1) - \alpha^m$-g f open set in X.
Conversely $\tau_Y(\delta) \geq r_1$,$\tau_Y*(\delta) \leq s_1$.
Then $\tau_Y(1-\delta) \geq r_1$,$\tau_Y*(1-\delta) \leq s_1$.
Therefore $f^{-1}(1-\delta)$ is an $(r_1, s_1) - \alpha^m$-g f open set in X. $f^{-1}(1-\delta) = 1 - f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f open set in X. So, $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X. Thus f is df-contra α^m-g c function.

Theorem 3.19: Let $f : X \to Y$ be a df-contra α^m-g c function and $g : Y \to Z$ a df- continuous function. Then gof is df-contra α^m-g c function.
Proof: Let $\tau_Z(\delta) \geq r_1$, $\tau_Z*(\delta) \leq s_1$. Since g is df- continuous, $\tau_Y(g^{-1}(\delta)) \geq r_1$, $\tau_Y*(g^{-1}(\delta)) \leq s_1$. And, since f df-contra α^m-g c function.
So, $f^{-1}(g^{-1}(\delta)) = (gof)^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X.

Therefore gof is df-contra α^m-g c function.

Theorem 3.20: Let $f : X \rightarrow Y$ be df-contra α^m-g c function and $g : Y \rightarrow Z$ a df-contra continuous function. Then gof is df-contra α^m-g c function.
Proof: Let $\tau_Z(\delta) \geq r_1$, $\tau_Z * (\delta) \leq s_1$. Since g is df- contra- continuous function then $\tau_Y(1-g^{-1}(\delta)) \geq r_1$, $\tau_Y *(1-g^{-1}(\delta)) \leq s_1$. And, since f is df-contra α^m-g c function. then $f^{-1}(g^{-1}(\delta)) = (gof)^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X.
Therefore gof is df-contra α^m-g c function.

Theorem 3.21: Suppose $f : X \rightarrow Y$ is df-contra α^m-g irr function and $g : Y \rightarrow Z$ is df-contra α^m-g c function. Then gof is df-contra α^m-g c function.
Proof: Let $\tau_Z(\delta) \geq r_1$, $\tau_Z * (\delta) \leq s_1$. Since g is df-contra α^m-g c function then $g^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in Y. And, since f is df-contra α^m-g irr function.
Then $f^{-1}(g^{-1}(\delta)) = (gof)^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X.
Therefore gof is df-contra α^m-g c function.

Theorem 3.22: Let $f : X \rightarrow Y$ be df- continuous function. If f an $(r_1, s_1) - \alpha^m$-g f open set in X is equal to $(r_1, s_1) - \alpha^m$-g f closed set in X, then f is df-contra α^m-g c function.
Proof: Let $\tau_Y(\delta) \geq r_1$, $\tau_Y * (\delta) \leq s_1$, so $\tau_Y(f^{-1}(\delta)) \geq r_1$, $\tau_X * (f^{-1}(\delta)) \leq s_1$.
Since every (r_1, s_1)-f open set is an $(r_1, s_1) - \alpha^m$-g f open set, then $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f open set in X. Thus $f^{-1}(\delta)$ is an $(r_1, s_1) - \alpha^m$-g f closed set in X.
Therefore, f is df-contra α^m-g c function.

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