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Corrigendum: On Some Refinements of Hardy-type Integral inequalities

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1 Introduction

In our recent paper [1], a refinement of Hardy-type integral inequality of the form $\int_\alpha^\beta \left(\frac{F(x)}{x}\right)^p \, dx \leq \left(\frac{p}{p-1}\right)^p C_{(\alpha,\beta)} \int_\alpha^\beta f(t)\, dt$ was stated and proved in Theorem 3.1 and Corollary 3.2. However, it was observed that there was an error in the arrangements of α and β in the value of the constant $C_{(\alpha,\beta)}$ which as a result reversed the sign of the constant. We wish to state correctly the formal statement of Theorem 3.1 in [1]. The proof of the theorem and Corollary 3.2 in [1] remain the same.

Theorem 3.1 Let p and q be constants such that $\frac{1}{p} + \frac{1}{q} = 1$ for $p > 1$. If $0 < \alpha < \beta < \infty$, $0 \leq f < \infty$ and $0 < \int_\alpha^\beta f^p(t)\, dt < \infty$. For $F(x) := \int_\alpha^x f(t)\, dt$ then

$$\int_\alpha^\beta \left(x^{-1} \int_\alpha^x f(t)\, dt\right)^p dx \leq \left(\frac{p}{p-1}\right)^p \frac{[\beta^{\frac{1}{q}} - \alpha^{\frac{1}{q}}]^p}{\beta^{\frac{p}{q}}} \int_\alpha^\beta f^p(t)\, dt \qquad (1.1)$$

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Proof. Applying (2.2) in [1] on the left hand side of (1.1), we have

$$\int_\alpha^\beta \left(x^{-1}\int_\alpha^x f(t)dt\right)^p dx \;\le\; \int_\alpha^\beta \left[x^{-1}\left(\frac{p}{p-1}\right)^{\frac{1}{q}}\left(x^{\frac{1}{q}}-\alpha^{\frac{1}{q}}\right)^{\frac{1}{q}}\left(\int_\alpha^x t^{\frac{1}{q}}f^p(t)\,dt\right)^{\frac{1}{p}}\right]^p dx$$

$$= \left(\frac{p}{p-1}\right)^{\frac{p}{q}}\int_\alpha^\beta x^{-p}\left(x^{\frac{1}{q}}-\alpha^{\frac{1}{q}}\right)^{\frac{p}{q}}\int_\alpha^x t^{\frac{1}{q}}f^p(t)\,dt.dx$$

Further simplification and applying Fubini's theorem we have

$$= \left(\frac{p}{p-1}\right)^{\frac{p}{q}}\int_\alpha^\beta \left(\int_t^\beta x^{-p+\frac{p}{q^2}}\left[1-\frac{\alpha^{\frac{1}{q}}}{\beta^{\frac{1}{q}}}\right]^{\frac{p}{q}} dx\right) t^{\frac{1}{q}}f^p t\,dt$$

This can be simplified to give

$$\int_\alpha^\beta \left(x^{-1}\int_\alpha^x f(t)dt\right)^p dx \;\le\; \left(\frac{p}{p-1}\right)^p \frac{\left[\beta^{\frac{1}{q}}-\alpha^{\frac{1}{q}}\right]^p}{\beta^{\frac{p}{q}}}\int_\alpha^\beta f^p(t)\,dt$$

□

Corollary 3.2
If in particular $\alpha = 0$ and $\beta \to \infty$, then (1.1) becomes

$$\int_0^\infty \left(\frac{1}{x}\int_0^x f(t)\,dt\right)^p dx \;\le\; \left(\frac{p}{p-1}\right)^p \int_0^\infty f^p(x)\,dt$$

which is the classical Hardy integral inequality.

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References

[1] Ajisope, M. O. and Rauf, K.(2019), On Refinements of Hardy-type Integral Inequalities. *International Journal of Mathematics and Computer Science,* 14(2019), no. 4, 879–888