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Two new eighth and twelfth order iterative methods for solving nonlinear equations

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Abstract

In this paper, we present two new iteration methods to find the solutions of nonlinear equations, which were developed from a concept of Obadah [14] and Taylor's Series to estimate the second derivative with convergence order analysis of the two new methods. The new methods have eight order convergence with the efficiency index at $(8)^{\frac{1}{3}} \approx 1.5157$, and twelfth order convergence with the efficiency index at $(12)^{\frac{1}{6}} \approx 1.5131$. Numerical examples of the new methods are compared with other methods by exhibiting the effectiveness of the method presented in this paper.

1 Introduction

To find a solution to the nonlinear equation $f(x) = 0$ is one of the essential mathematical problems and the well-known method is the iteration of Newton's method with the second order convergence and the efficiency index $(2)^{\frac{1}{2}} \approx 1.412$ specified by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0, 1, 2, \ldots \tag{1.1}$$

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After that, the iteration of the third order convergence was presented with the efficiency index $(3)^{\frac{1}{3}} \approx 1.4422$ provided by Halley [3]:

$$x_{n+1} = x_n - \frac{2f(x_n) f'(x_n)}{2(f'(x_n))^2 - f(x_n) f''(x_n)}, \qquad n = 0, 1, 2, \ldots \tag{1.2}$$

Many researchers attempted to improve Halley method by increasing the order of convergence or reducing the estimation of functions in each round of iteration without the second order derivative for more precise and effective results. Among these methods, we are interested in the concept of Obadah [14], referred to as an improvement from Halley method.

$$x = x_0 - \frac{f(x_0)}{f'(x_0)} - \frac{2(f(x_0))^2 f'(x_0) f''(x_0)}{4(f'(x_0))^4 - 4f(x_0)(f'(x_0))^2 f''(x_0) + (f(x_0))^2 (f''(x_0))^2} \tag{1.3}$$

The iteration of Chebyshev's method [14] with the third order convergence was presented with efficiency index $(3)^{\frac{1}{3}} \approx 1.4422$, specified by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{(f(x_n))^2 f'(x_n)}{2(f'(x_n))^3} \qquad n = 0, 1, 2, \ldots \tag{1.4}$$

In this paper, we present two methods with three step iterations each. We estimate the second order derivative by using Taylor's Series to reduce functions in each round of iteration. We implement the concept of Obadah [14], equation (1.3), and then rewrite in the form of Newton's method. For the first method, we increase another step of iteration by Newton's method. The second method has one more step by Chebyshev's method [2]. After that, the order of convergence of these two new methods was analyzed. The effectiveness was tested based on numerical examples with other iteration methods, which had an equal order of convergence to each of the new methods.

2 Proposed Methods

In this section, we construct new iterative methods. Assume that $\alpha \in I$ is a simple zero of $f(x)$; that is, $f(\alpha) = 0$ of nonlinear equation $f(x) = 0$, where $f(x)$ is a real function $f : D \subset \mathbb{R} \to \mathbb{R}$ sufficiently differentiable on an open interval D.

2.1 Newton-Obadah-Newton method (NON)

According to the concept of Obadah [14] and Newton's method, the third order convergence and efficiency index at $(3)^{\frac{1}{3}} \approx 1.4422$ could be written as follows

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)} \tag{2.5}$$

$$x_{n+1} = y_n - \frac{2(f(x_n))^2 f'(x_n) f''(x_n)}{4(f'(x_n))^4 - 4f(x_n)(f'(x_n))^2 f''(x_n) + (f(x_n))^2 (f''(x_n))^2} \tag{2.6}$$

After that, the second derivative of function $f''(x_n)$ is estimated by Taylor Series expansion of function $f(y_n)$ around $x = x_n$:

$$f(y_n) \approx f(x_n) + f'(x_n)(y_n - x_n) + \frac{f''(x_n)(y_n - x_n)^2}{2!}, \tag{2.7}$$

where $y_n = x_n - \dfrac{f(x_n)}{f'(x_n)}$.
We have

$$f(y_n) \approx f(x_n) + f'(x_n)\left(\left(x_n - \frac{f(x_n)}{f'(x_n)}\right) - x_n\right) + \frac{f''(x_n)\left(\left(x_n - \frac{f(x_n)}{f'(x_n)}\right) - x_n\right)^2}{2!}$$

$$f(y_n) \approx \frac{f''(x_n)(f(x_n))^2}{2(f'(x_n))^2}.$$

Thus

$$f''(y_n) \approx \frac{2f''(x_n)(f(x_n))^2}{2(f'(x_n))^2} \tag{2.8}$$

Substituting equation (2.8) into (2.6), we obtain

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)} \tag{2.9}$$

$$x_{n+1} = y_n - \frac{(f(x_n))^2 f(y_n)}{(f(x_n))^2 f'(x_n) - 2f(x_n) f'(x_n) f(y_n) + f'(x_n)(f(y_n))^2} \tag{2.10}$$

Equations (2.9) and (2.10) were increased with a step of iteration by Newton's method and the new iteration method is obtained according to Algorithm 2.1.

Algorithm 2.1 For a given x_0, compute approximates solution x_{n+1} by the iterative schemes

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)} \tag{2.11}$$

$$z_n = y_n - \frac{(f(x_n))^2 f(y_n)}{(f(x_n))^2 f'(x_n) - 2f(x_n) f'(x_n) f(y_n) + f'(x_n)(f(y_n))^2} \tag{2.12}$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)} \tag{2.13}$$

Algorithm 2.1 is a new three step iteration method (NON) with the eighth order convergence. Thus, Algorithm 2.1 efficiency index is $(8)^{\frac{1}{5}} \approx 1.5157$

2.2 Newton-Obadah-Chebyshev method (NOC)

In the second method, equation (2.9) and (2.10) were added with a step of iteration by Chebyshev's method and thus the new iteration method is obtained as Algorithm 2.2.

Algorithm 2.2 For a given x_0, compute approximates solution x_{n+1} by the iterative schemes

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)} \tag{2.14}$$

$$z_n = y_n - \frac{(f(x_n))^2 f(y_n)}{(f(x_n))^2 f'(x_n) - 2f(x_n) f'(x_n) f(y_n) + f'(x_n)(f(y_n))^2} \tag{2.15}$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(z)} - \frac{(f(z_n))^2 f''(z_n)}{2(f'(z_n))^3} \tag{2.16}$$

Algorithm 2.2 is a new three step iteration method (NOC) with the twelfth order convergence. Thus, the Algorithm 2.2 efficiency index is $(12)^{\frac{1}{6}} \approx 1.5131$

3 Convergence Analysis

In this part, we discuss the analysis of convergence in Algorithm 2.1 (NON) and Algorithm 2.2 (NOC), respectively.

Theorem 3.1. *Let α be a simple zero of a sufficiently differentiable function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ from an open interval I. If the initial point x_0 is sufficiently close to α, then the algorithm 2.1 (NON) in equations* (2.11)–(2.13) *has eighth order convergence and satisfying the following error equation*

$$e_{n+1} = \left(c_3^2 c_2^3 - 4c_3 c_2^5 + 4c_2^7\right) e_n^8 + O\left(e_n^9\right) \tag{3.17}$$

where $e_n = x_n - \alpha$ *and* $c_n = \dfrac{f^{(n)}(\alpha)}{n! f'(\alpha)}$, $n = 2, 3, ...$

Proof. Let α be a simple zero of $f(x)$ and $e_n = x_n - \alpha$
By using Taylor's expansion of $f(x)$ and $f'(x_n)$ at $x_n = \alpha$, we obtain

$$f(x_n) = f'(\alpha)\left[e_n + c_2 e_n^2 + c_3 e_n^3 + c_4 e_n^4 + c_5 e_n^5 + c_6 e_n^6 + c_7 e_n^7 + c_8 e_n^8 + \dots\right] \tag{3.18}$$

and

$$f'(x_n) = f'(\alpha)\left[1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + 5c_5 e_n^4 + 6c_6 e_n^5 + 7c_7 e_n^6 + 8c_8 e_n^7 + \dots\right] \tag{3.19}$$

Dividing (3.18) by (3.19), we have

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2 e_n^2 - \left(2c_3 - 2c_2^2\right) e_n^3 - \left(3c_4 - 7c_2 c_3 + 4c_2^3\right) e_n^4 + \dots \tag{3.20}$$

Substituting (3.20) into (2.11) and simplifying, we have

$$y_n = \alpha + c_2 e_n^2 + \left(2c_3 - 2c_2^2\right) e_n^3 + \left(3c_4 - 7c_2 c_3 + 4c_2^3\right) e_n^4 + \dots \tag{3.21}$$

Expanding $f(y_n)$ around $y_n = \alpha$ and from (3.21), we have

$$f(y_n) = f'(\alpha)\left[c_2 e_n^2 + \left(2c_3 - 2c_2^2\right) e_n^3 + \left(5c_2^3 - 7c_2 c_3 + 3c_4\right) e_n^4 + \dots\right] \tag{3.22}$$

Substituting equation (3.18), (3.19), (3.21), and (3.22) into (2.12), we get

$$z_n = \alpha + \left(2c_2^3 - c_2 c_3\right) e_n^4 + \left(14c_3 c_2^2 - 10c_2^4 - 2c_3^2 - 2c_2 c_4\right) e_n^5 + \dots \tag{3.23}$$

Using Taylor's expansion for $f\left(z_n\right)$ and $f'\left(z_n\right)$ at $z_n=\alpha$ and equation (3.23), we have

$$f(z_n)=f'(\alpha)\left[\left(2c_2^3-c_2c_3\right)e_n^4+\left(14c_3c_2^2-10c_2^4-2c_3^2-2c_2c_4\right)e_n^5+\ldots\right] \tag{3.24}$$

and

$$f'(z_n)=f'(\alpha)\left[1+2c_2^2\left(2c_2^2-c_3\right)e_n^4-4c_2\left(5c_2^4-7c_3c_2^2+c_3^2+c_2c_4\right)e_n^5\ldots\right] \tag{3.25}$$

Dividing (3.24) by (3.25), we get

$$\frac{f(z_n)}{f'(z_n)}=\left(2c_2^3-c_2c_3\right)e_n^4+\left(14c_3c_2^2-10c_2^4-2c_3^2-2c_2c_4\right)e_n^5+\ldots \tag{3.26}$$

Substituting equation (3.26) into (2.13), we obtain

$$x_{n+1}=\alpha+\left(c_3^2c_2^3-4c_3c_2^5+4c_2^7\right)e_n^8+O\left(e_n^9\right) \tag{3.27}$$

From equation (3.27) and $e_{n+1}=x_{x+1}-\alpha$, we have

$$e_{n+1}=\left(c_3^2c_2^3-4c_3c_2^5+4c_2^7\right)e_n^8+O\left(e_n^9\right)-\alpha \tag{3.28}$$

which shows that Algorithm 2.1 (NON) has eight order convergence. □

Theorem 3.2. *Let α be a simple zero of a sufficiently differentiable function $f: I\subseteq\mathbb{R}\to\mathbb{R}$ from an open interval I. If the initial point x_0 is sufficiently close to α, then the algorithm 2.2 (NOC) in equations* (2.14)–(2.16) *has twelfth order convergence and satisfying the following error equation*

$$e_{n+1}=\left(c_2^3c_3^4-8c_3^3c_2^5+24c_3^2c_2^7-32c_3c_2^9+16c_2^{11}\right)e_n^{12}+O\left(e_n^{13}\right) \tag{3.29}$$

where $e_n=x_n-\alpha$ *and* $c_n=\dfrac{f^{(n)}(\alpha)}{n!f'(\alpha)},\ n=2,3,...$

Proof. Using Taylor's expansion for $f''\left(z_n\right)$ at $z_n=\alpha$ and equation (3.23) we have

$$f''(z_n)=f'(\alpha)\left[2c_2+6c_3c_2(-c_3+2c_2^2)e_n^4-12c_3(-7c_3c_2^2+5c_2^4+c_3^2+c_2c_4)e_n^5+\ldots\right] \tag{3.30}$$

Substituting equation (3.23), (3.26) and (3.20) into (2.16), we get

$$x_{n+1}=\alpha+\left(c_3^2c_2^3-8c_3^3c_2^5+24c_3^2c_2^7-32c_3c_2^9+16c_2^{11}\right)e_n^{12}+O\left(e_n^{13}\right) \tag{3.31}$$

From equation (3.31) and $e_{n+1}=x_{n+1}-\alpha$, we obtain

$$e_{n+1}=\left(c_3^2c_2^3-8c_3^3c_2^5+24c_3^2c_2^7-32c_3c_2^9+16c_2^{11}\right)e_n^{12}+O\left(e_n^{13}\right) \tag{3.32}$$

Therefore equation (3.32) shows that Algorithm 2.1 (NOC) has twelfth order convergence. □

4 Numerical Experiments

In this section, we compare the number of iterations in obtaining an approximated root of our proposed methods with the other methods that had an equal order of convergence. Algorithm 2.1 (NON): equation (2.11) – (2.13) eighth order convergence compare with Chebyshev-Lagrange method (CLM), [13], Lotfi et al (KT) [9], Cordero, and Torregrosa and Vassileva (CTV) [1]. Meanwhile, Algorithm 2.2 (NOC): equation (2.14)–(2.16) twelfth order convergence compare with the method of Kim and Chun (PM) [5], Liu and Wang (NVN) [8], and Hou and Li (GM) [4] according to the consideration numerical examples as follows

$$
\begin{aligned}
&f_1(x) = 4x^4 - 4x^2\\
&f_2(x) = (x-2)^{23} - 1\\
&f_3(x) = e^x \sin x + \ln(x^2+1)\\
&f_4(x) = (x+2)e^x - 1\\
&f_5(x) = e^{x^2+7x-30} - 1\\
&f_6(x) = x^3 - 2x^2 - 5\\
&f_7(x) = (x-1)e^{-x}\\
&f_8(x) = \cos x - x\\
&f_9(x) = \sin^2 x - x^2 + 1\\
&f_{10}(x) = xe^{x^2} - \sin^2 x + 3\cos x + 5
\end{aligned}
$$

All examples were performed on Matlab with 100 digit decimals. The comparison is under the condition that the program will stop when $|x_n - x_{n-1}| < \varepsilon$ and $|f(x_n)| < \varepsilon$ where $\varepsilon = 10^{-15}$.

Table 1-2 show the number of iterations, the absolute value of the functions $|f(x_n)|$, and the absolute difference $|x_n - x_{n-1}|$ to approximation the roots of the iteration method with the eighth and twelfth order convergence, respectively.

Table 1 Comparison of the number of iterations of the NON method (eight order convergence) with the other iterative methods

method	IT	x_n	$\lvert f(x_n)\rvert$	$\lvert x_n - x_{n-1}\rvert$
$f_1(x), x_0 = 0.75$				
CLM	10	1	0	$2.459319e-28$
KT	5	1	0	$8.884929e-31$
CTV	9	1	0	$2.820598e-17$
NON	5	1	0	$4.973714e-73$
$f_2(x), x_0 = 2.9$				
CLM	*div*	–	–	–
KT	6	3	0	$3.935067e-33$
CTV	27	3	0	$5.681689e-41$
NON	6	3	0	$6.396552e-78$
$f_3(x), x_0 = 2.9$				
CLM	3	3.237562984023921	$5.0e-127$	$8.075018e-17$
KT	3	3.237562984023921	$5.0e-127$	$2.898834e-18$
CTV	3	3.237562984023921	$5.0e-127$	$8.429563e-23$
NON	3	3.237562984023921	$5.0e-127$	$2.044221e-21$
$f_4(x), x_0 = -0.9$				
CLM	3	-0.442854401002389	0	$1.535332e-26$
KT	3	-0.442854401002389	0	$5.771482e-24$
CTV	3	-0.442854401002389	0	$1.925221e-32$
NON	3	-0.442854401002389	$6.1e-128$	$5.3604801e-26$
$f_5(x), x_0 = 3.1$				
CLM	3	3	0	$1.073458e-18$
KT	3	3	$2.0e-126$	$6.368692e-18$
CTV	3	3	$2.0e-126$	$1.925221e-32$
NON	3	3	$1.1e-126$	$6.396085e-22$
$f_6(x), x_0 = 2.0$				
CLM	9	2.690647448028614	0	$2.990186e-75$
KT	4	2.690647448028614	0	$1.987149e-82$
CTV	*div*	–	–	–
NON	4	2.690647448028614	0	$1.413885e-108$
$f_7(x), x_0 = 0.25$				
CLM	3	1	0	$0.191866e-18$
KT	3	1	$4.293249e-123$	$4.820894e-16$
CTV	3	1	0	$8.028669e-19$
NON	3	1	0	$2.440301e-17$

method	IT	x_n	$\|f(x_n)\|$	$\|x_n - x_{n-1}\|$
$f_8(x), x_0 = 1.7$				
CLM	3	0.739085133215161	0	$8.610581e-53$
KT	3	0.739085133215161	0	$4.049015e-46$
CTV	3	0.739085133215161	0	$1.922475e-41$
NON	3	0.739085133215161	0	$6.453828e-49$
$f_9(x), x_0 = -2.5$				
CLM	3	-1.404491648215341	$1.1e-127$	$4.083291e-24$
KT	3	-1.404491648215341	$1.1e-127$	$2.432583e-18$
CTV	3	-1.404491648215341	$1.1e-127$	$1.505241e-23$
NON	3	-1.404491648215341	$1.1e-127$	$2.229438e-20$
$f_{10}(x), x_0 = -1.0$				
CLM	3	-1.207647827130919	$1.1e-126$	$1.447895e-30$
KT	3	-1.207647827130919	$1.1e-126$	$1.343613e-31$
CTV	3	-1.207647827130919	$1.1e-126$	$1.070874e-50$
NON	3	-1.207647827130919	$1.1e-126$	$6.294097e-32$

Table 2 Comparison of the number of iterations of the NOC method (twelfth order convergence) with the other iterative methods

method	IT	x_n	$\|f(x_n)\|$	$\|x_n - x_{n-1}\|$
$f_1(x), x_0 = 0.75$				
PM	*div*	–	–	–
NVN	11	1	0	$7.296259e-20$
GM	56	1	265.350429	$4.941043e-16$
NOC	4	1	0	$1.326559e-27$
$f_2(x), x_0 = 2.9$				
PM	*div*	–	–	–
NVN	*div*	–	–	–
GM	*div*	–	–	–
NOC	5	3	0	$7.508217e-44$
$f_3(x), x_0 = 2.9$				
PM	3	3.237562984023921	$5.1e-127$	$6.073023e-63$
NVN	3	3.237562984023921	$2.3e-126$	$1.783472e-45$
GM	3	3.237562984023921	$5.1e-127$	$1.575857e-35$
NOC	3	3.237562984023921	$5.1e-127$	$5.867619e-45$

method	IT	x_n	$\|f(x_n)\|$	$\|x_n - x_{n-1}\|$
$f_4(x), x_0 = -0.9$				
PM	3	-0.442854401002389	0	$1.072745e-77$
NVN	3	-0.442854401002389	0	$7.083838e-49$
GM	3	-0.442854401002389	0	$1.231666e-45$
NOC	3	-0.442854401002389	0	$3.152133e-56$
$f_5(x), x_0 = 3.1$				
PM	3	3	0	$1.161552e-47$
NVN	3	3	$2.0e-126$	$3.425740e-33$
GM	3	3	0	$1.495313e-16$
NOC	3	3	0	$2.094782e-40$
$f_6(x), x_0 = 2.0$				
PM	3	2.690647448028614	0	$4.619743e-45$
NVN	3	2.690647448028614	0	$4.543342e-28$
GM	*div*	−	−	−
NOC	3	2.690647448028614	0	$5.581313e-29$
$f_7(x), x_0 = 0.25$				
PM	4	1	$1.585560e-125$	$1.177534e-31$
NVN	3	1	0	$6.494690e-32$
GM	3	1	0	$3.358262e-27$
NOC	3	1	0	$2.028917e-36$
$f_8(x), x_0 = 1.7$				
PM	3	0.739085133215161	0	$5.432610e-112$
NVN	*div*	−	−	−
GM	*div*	−	−	−
NOC	3	0.739085133215161	0	$6.806911e-104$
$f_9(x), x_0 = -2.5$				
PM	3	-1.404491648215341	$1.1e-127$	$1.991496e-45$
NVN	3	-1.404491648215341	$2.1e-127$	$4.271406e-54$
GM	3	-1.404491648215341	$1.1e-127$	$2.397532e-48$
NOC	3	-1.404491648215341	$1.1e-127$	$3.217353e-42$
$f_{10}(x), x_0 = -1.0$				
PM	3	-1.207647827130919	$1.21e-126$	$4.159033e-101$
NVN	3	-1.207647827130919	$1.10e-126$	$5.765523e-54$
GM	3	-1.207647827130919	$1.21e-126$	$1.699145e-55$
NOC	3	-1.207647827130919	$1.21e-126$	$3.572615e-70$

5 Conclusion

This paper proposed two iteration methods with three-step each for nonlinear equations. The new methods have the eighth and the twelfth order of convergence, respectively. The numerical examples compare the effectiveness of our methods and other well-known iteration methods having equal convergence order, and the result indicated that the proposed methods had fewer rounds of iteration than or equivalent rounds to other methods. It shows that these new methods can be considered as effective alternatives to other methods.

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