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A novel multiobjective Passing Vehicle Search algorithm for signal timing optimization

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Abstract

Many industrial problems in process optimization are Multi-Objective (MO), where each of the objectives represents different facets of the issue. Thus, having in hand multiple solutions prior to selecting the best solution is a seminal advantage. Metaheuristic techniques such as Genetic Algorithm (GA) and Particle Swarm optimization (PSO) have been used in conjunction with scalarization techniques such as weighted sum approach and the normal-boundary intersection (NBI) method to solve MO problems. In this paper, a novel multi-objective Passing Vehicle Search algorithm using the NBI method is used to optimize the signal timing problem, the proposed method calculates optimal solutions to generate surface pareto fronts, by minimizing total delay, number of stops and maximizing the capacity of the junction. To this end, an attempt is made at the intersection of Bab ZAER in Rabat, Morocco; a system that can adapt efficiently with the environment is developed based on the Novel PVS-NBI method to control the flow through the intersection, a database to manage the sensor data and the traffic road simulation software 'AnyLogic' to apply the theory in practice by importing the real-time data relating to the whole traffic situation. The system shows good results with respect to the current situation and existing static model.

Key words and phrases: Multiobjective optimization, isolated signalized intersections, Pareto front, PVS, NBI, AnyLogic, Simulation.
AMS (MOS) Subject Classifications: 65K10.
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1 Introduction

Real time traffic signal control system is the most recent control type of traffic signals.

Adative control system can adjust the signal timing parameters in real time and then is able to efficiently relieve traffic congestion.

Metaheuristics such as PSO [1], Genetic algorithms [2], Tabu Search [3], and Ant Colony algorithms [4], were used for solving the signal timing optimization problem. Cycle length, split, offset etc., are a variety of signal timing parameters that are subject to optimization, and any appropriate traffic performance index including total delay, number of stops, total travel time and queue length can be considered as the objective function.

In the literature, many works have studied the multi-objective signal timing optimization problem, using the weighted sum method. In [5] a multi-objective optimization model to minimize delay time and queue length and to maximize effective capacity was proposed. The original model was transformed to a single-objective problem through three weight coefficients. In [6] a real-time control model was formulated, with three objectives, throughput maximization, to fully use storage capacity, and to provide equitable service.

In addition, a strategy of control based on a predictive model to optimize cycle length duration was developed in [7], using a multi-objective optimization algorithm based on PSO. In [2], NSGA-II and local search were used for muti-objective signal timing optimization, by minimizing total lost time in the traffic and maximizing throughput. NSGA II was also used in [8] to optimize traffic signal timing at over-saturated intersection. the optimization objectives selected are throughput maximization and average queue ratio minimization.

In this paper, a novel multiobjective metaheuristic based on PVS algorithm [9] is proposed to optimize the real time signal timing problem. Similar to other metaheuristic methods, PVS is a population-based method, the optimal solution is selected by following the mathematical characteristics of vehicles that pass on a two-lane highway.

The study focuses on the traffic signal control problem as a multi-objective optimization problem, where the main purpose is to provide decisions taking in consideration several criteria and constraints. A simulation tool is developed to provide real time statistics, analyze the real time data and visualize the traffic evolution.

First, we prove the effectiveness of the proposed PVS NBI algorithm in dealing with some test functions. Secondly, the proposed algorithm is applied to solve the real time traffic signal timing at isolated intersections. For this, a system that can adapt efficiently with the environment is developed in order to organize traffic in Moroccan cities.

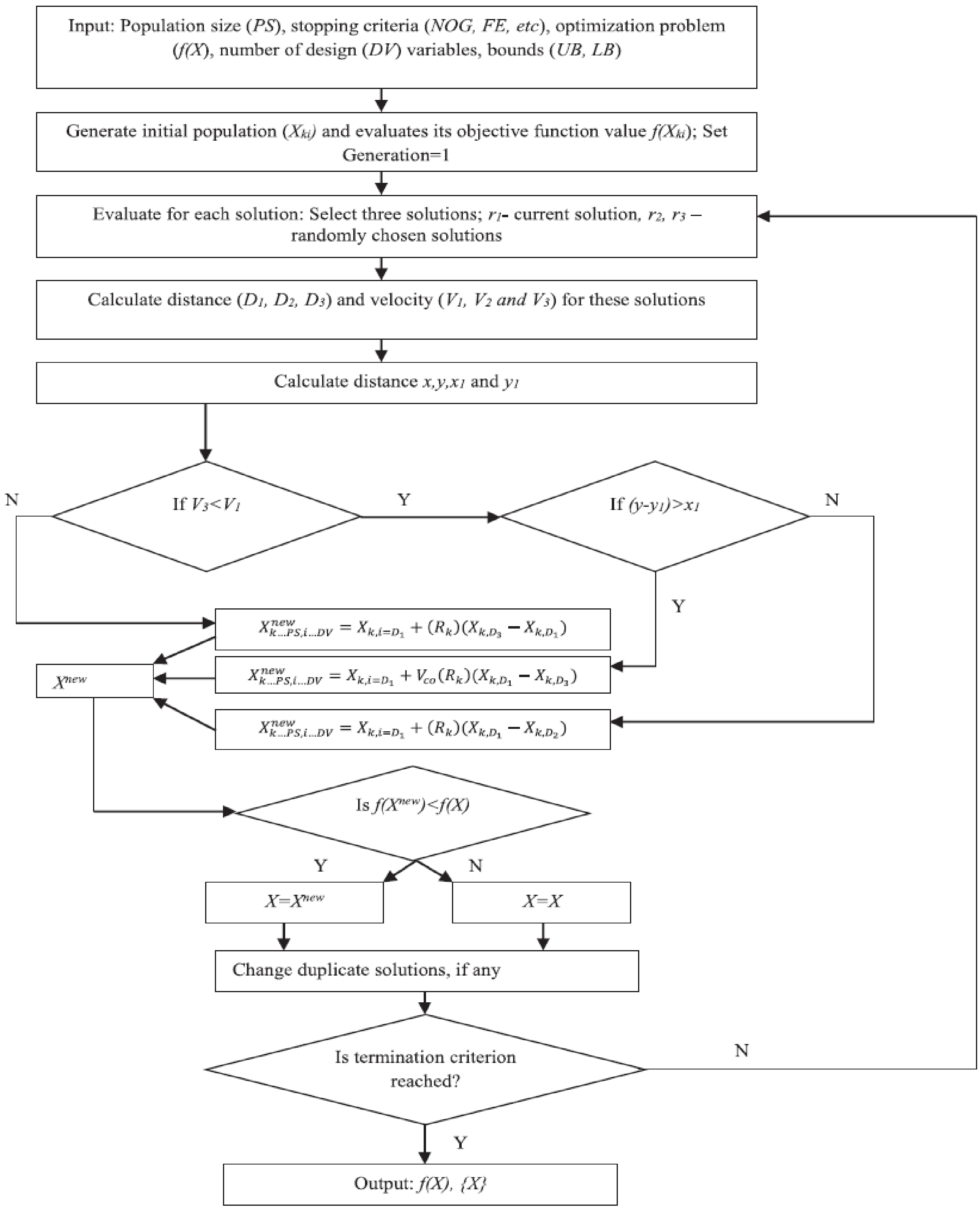
This paper is organized as follows: Section 2 describes the PVS algorithm. In section 3, we present the proposed multiobjective PVS NBI. Section 4 briefly introduces the current traffic management control system in Morocco, the mathematical model used in the case study and the test results.Finally, we conclude our paper in section 5.

2 Passing Vehicle Search algorithm

2.1 Introduction

The "Passing vehicle search" (PVS) algorithm is based on the passing behavior of a vehicle moving on a two-lane highway. Like other meta-heuristics, the objective of the proposed algorithm is to find the global solution or near-optimal solutions for a given function [9].

Let us assume that the set of solutions comprise different vehicles on a two-lane highway. The value of the objective function or fitness value represents the velocity of vehicles; i.e., a solution with a better fitness value is considered to be a vehicle with a higher velocity. The design variables determine the position of the vehicle on a highway. Thus, PVS starts with a set of solutions referred to as a population of vehicles from which three vehicles (solutions) are selected at random. Among these three selected vehicles, the current solution is correlated with BV and the other two can be designated randomly as OV and FV. The distances between the vehicles and their respective velocities are assigned based on the population size and its fitness value. After assigning the distances and velocities, the vehicles are checked to confirm the conditions for passing. Based on the conditions employed, the vehicles change their respective positions on the highway (change the current solution).


Input: Population size (PS), stopping criteria (NOG, FE, etc), optimization problem (f(X), number of design (DV) variables, bounds (UB, LB)
Generate initial population (X_{ki}) and evaluates its objective function value $f(X_{ki})$; Set Generation=1
Evaluate for each solution: Select three solutions; r_1- current solution, r_2, r_3 – randomly chosen solutions
Calculate distance (D_1, D_2, D_3) and velocity (V_1, V_2 and V_3) for these solutions
Calculate distance x,y,x_1 and y_1
If $V_3<V_1$
If $(y-y_1)>x_1$
N
Y
N
Y
$X^{new}_{k...PS,i...DV} = X_{k,i=D_1} + (R_k)(X_{k,D_3} - X_{k,D_1})$
X^{new}
$X^{new}_{k...PS,i...DV} = X_{k,i=D_1} + V_{co}(R_k)(X_{k,D_1} - X_{k,D_3})$
$X^{new}_{k...PS,i...DV} = X_{k,i=D_1} + (R_k)(X_{k,D_1} - X_{k,D_2})$
Is $f(X^{new})<f(X)$
Y
N
$X=X^{new}$
$X=X$
Change duplicate solutions, if any
Is termination criterion reached?
N
Y
Output: $f(X)$, $\{X\}$

3 Multi-Objective PVS algorithm

3.1 Normal Boundary Intersection

Normal-boundary intersection (NBI) was developed by Dennis [10] for solving multi-criteria optimization problems. It is geometrically inspired and has been very successful in practice. NBI is based on the concept of the Convex Hull of Individual Minima (CHIM).

For an n-objective minimization problem, let $F = [f_1, ..., f_n]$, where f_i is the i^{th} function. Optimize F such that $i \in 1, ..., n$. Let $F^* = [f_1^*, ..., f_n^*]$, where f_i^* is the solution to the minimization of f_i and let x_i^* be the corresponding vector that minimizes $f_i(x)$. Let ϕ be the matrix whose i^{th} column is $F(x_i^*) - F^*$. Thus, the Convex Hull of Individual Minima is defined as the set of points that are convex combinations of f_i^*. It can be expressed as:

$$\{\phi\omega / \sum_i \omega_i = 1, 0 \leq \omega_i, \omega \in R^n\} \tag{3.1}$$

The following figure shows an example of the Convex Hull of Individual Minima for two objectives

NBI then proceeds by solving the maximization problem shown in **??** by projecting lines from the CHIM toward $F*$

$$\begin{aligned} \underset{x,t}{\text{maximize}} \quad & t \\ \text{subject to} \quad & \phi\omega + t\hat{n} = F(x) - F^*, \\ & h(x) = 0, \\ & g(x) \leq 0, \end{aligned} \tag{3.2}$$

where n̂=(1,...,1) is the unit normal vector to the CHIM and t represents how far n̂ can project from $\phi\omega$ before crossing the Pareto front.

3.2 Multiobjective test functions

3.2.1 ZDT1 Function

ZDT1: In this ZDT1 function, thirty design variables x_i were chosen ($n = 30$). Each design variable ranged in value from 0 to 1. The Pareto-optimal front appears when $g = 1.0$

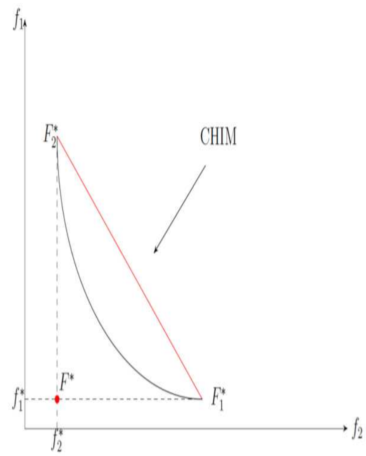


Figure 1: The CHIM

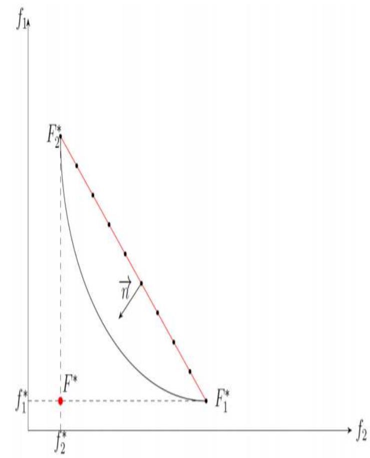


Figure 2: Normal vector to the CHIM

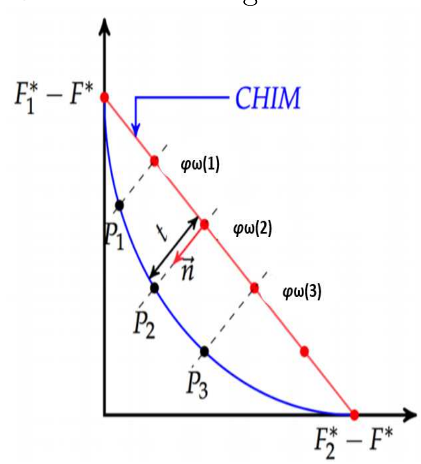


Figure 3: Generation of the pareto front

3.2.2 ZDT2 Function

ZDT2: In this ZDT2 function, thirty design variables x_i were chosen ($n = 30$). Each design variable ranged in value from 0 to 1. The Pareto-optimal front appears when $g = 1.0$

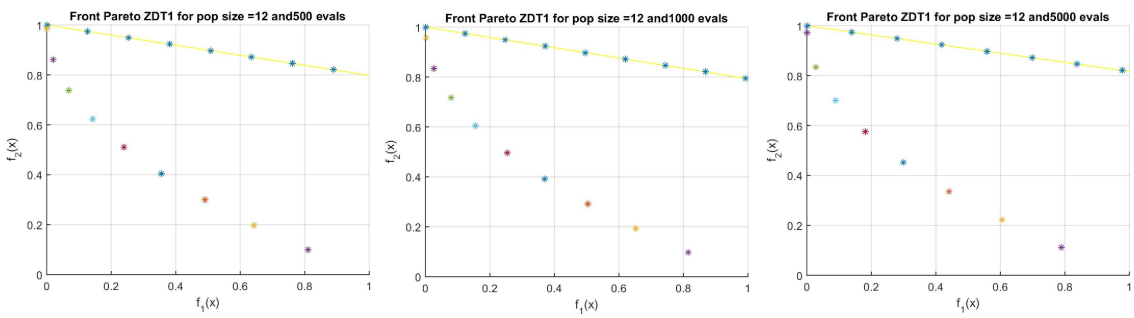


Figure 4: Pareto front of ZDT1 for Population Size 12

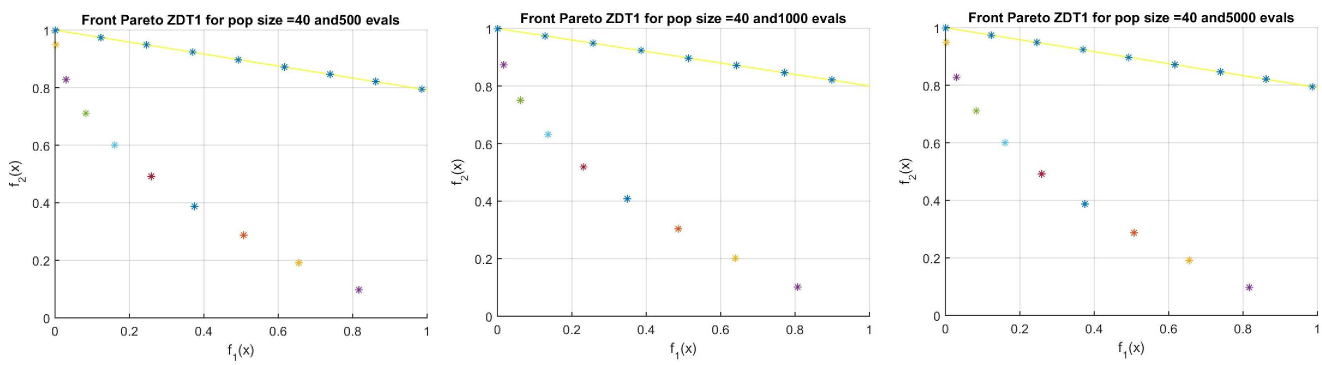


Figure 5: Pareto front of ZDT1 for Population Size 40

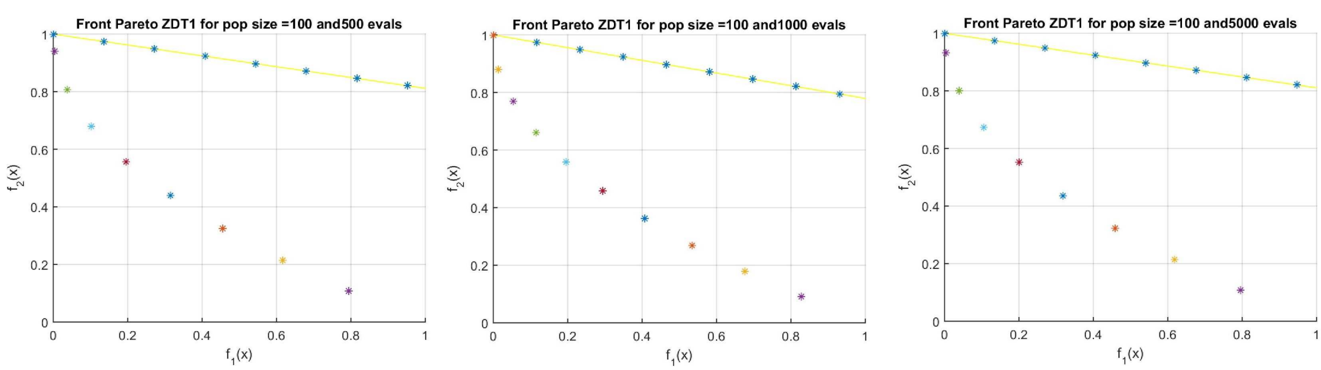


Figure 6: Pareto front of ZDT1 for Population Size 100

3.2.3 ZDT3 Funtion

ZDT3: In this ZDT3 function, thirty design variables x_i were chosen ($n = 30$). Each design variable ranged in value from 0 to 1. The Pareto-optimal

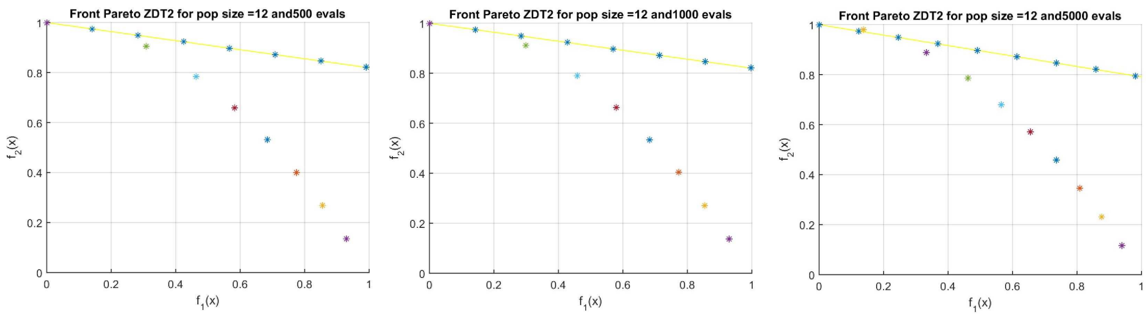


Figure 7: Pareto front of ZDT2 for Population Size 12

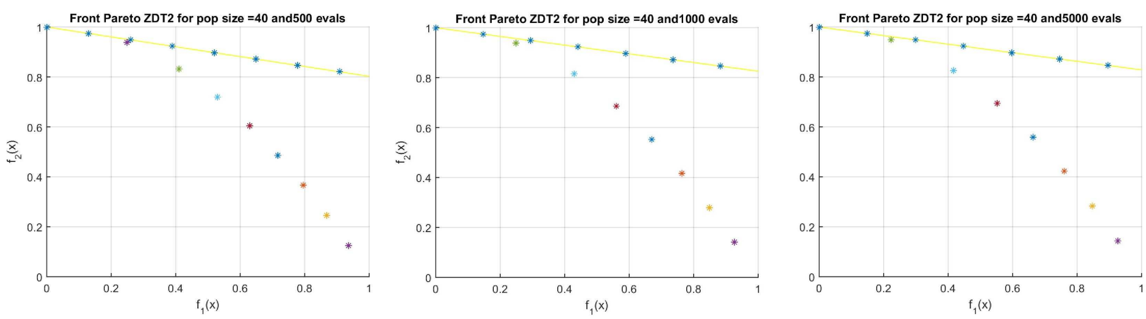


Figure 8: Pareto front of ZDT2 for Population Size 40

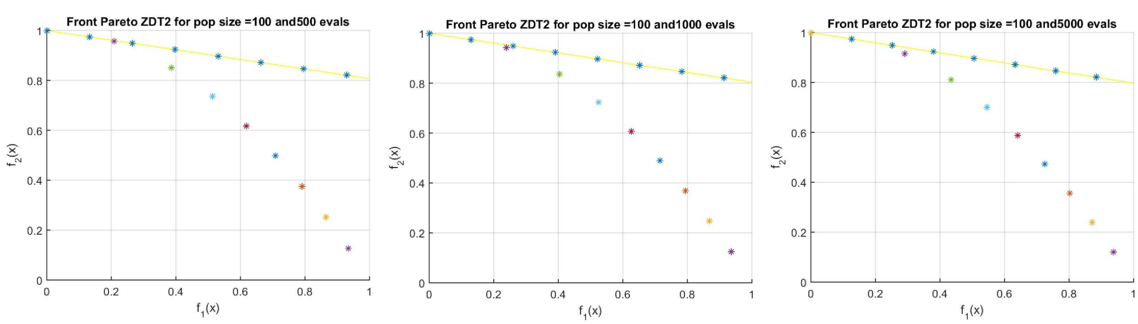


Figure 9: Pareto front of ZDT2 for Population Size 100

front appears when $g = 1.0$

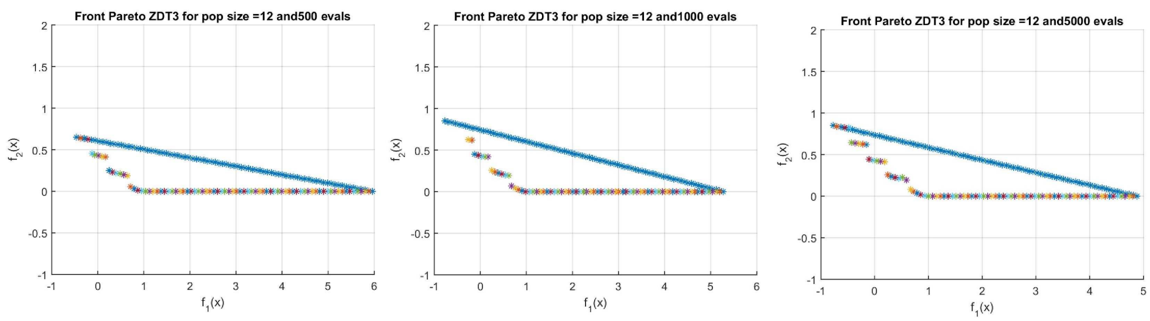


Figure 10: Pareto front of ZDT3 for Population Size 12

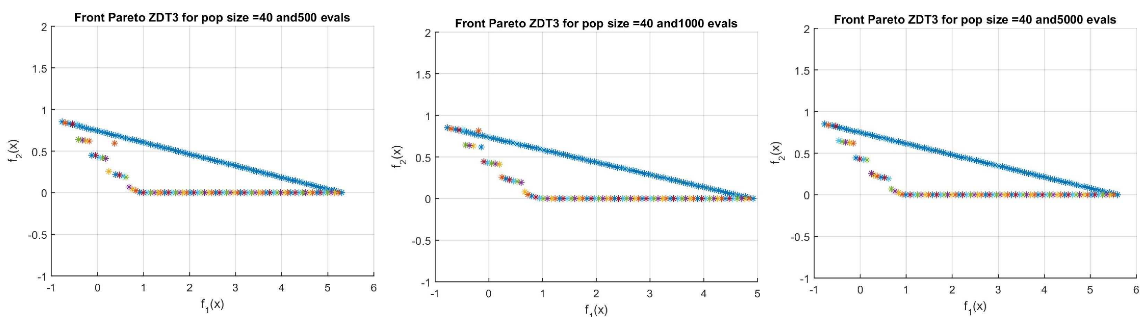


Figure 11: Pareto front of ZDT3 for Population Size 40

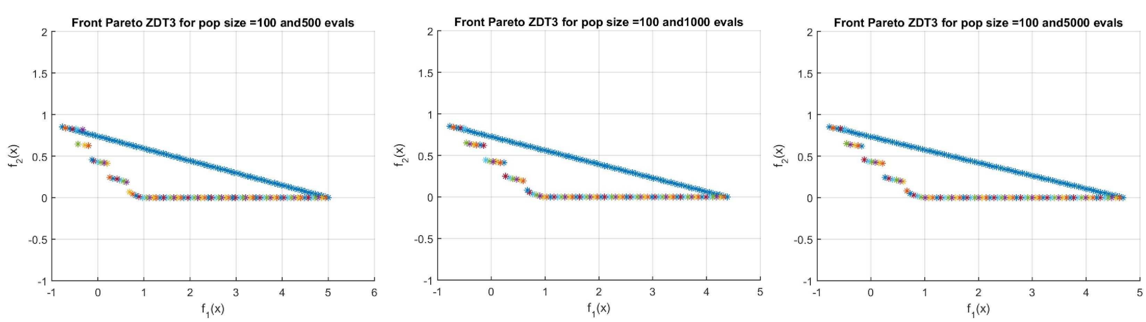


Figure 12: Pareto front of ZDT3 for Population Size 100

4 Application for signal timing optimization

4.1 Mathematical Model

In order to apply the novel multiobjective PVS algorithm, we choose the model developed by Jiao, and Li [11].

To apply the proposed model to our problem, we use the following notations:

T : total cycle length of the intersection signal
t_n : time length of phase $n, n = 1, 2, ..., N$
N : number of phases during a signal cycle (four in this article)
g_n: displayed green time of phase $n, n = 1, 2, ..., N$
g_{en} : effective green time of phase $n, n = 1, 2, ..., N$
R_n : all red time of phase n; that is, red clearance interval of phase $n, n = 1, 2, ..., N$
Y_n : all yellow time of phase n; that is, yellow change interval of phase$n, n = 1, 2, ..., N$
g_n^{min}: minimum displayed green time of phase $n, n = 1, 2, ..., N$
g_n^{max} : maximum displayed green time of phase $n, n = 1, 2, ..., N$
l_n : start-up lost time of phase n, which equals to the sum of start-up lost time of the head car and consequent cars, and other cars are assumed to travel under the saturated time headway without start-up lost time $n = 1, 2, ..., N$
In each signal cycle, according to the basic principles of traffic signal control, the following equations can be formulated :

$$g_n = g_{en} - Y_n + l_n \tag{4.3}$$

$$t_n = g_n + Y_n + R_n \tag{4.4}$$

$$T = \sum_n t_n \tag{4.5}$$

4.2 The multiobjective model

This multi-objective problem consists of optimizing three objective functions while satisfying different equality and inequality constraints. A mathematical formulation is given below:

$$\underset{x}{\text{minimize}} \quad f(x) = f_1(x), f_2(x) \tag{4.6}$$

$$\begin{aligned} \underset{x}{\text{maximize}} \quad & f(x) = f_3(x) \\ \text{subject to} \quad & h(x) = 0, \\ & g(x) \le 0, \\ & x_{LB} \le x \le x_{UB} \end{aligned} \tag{4.7}$$

where x is the design variable vector. Each element x_i is a design variable. $f(x)$ is the objective function. $g(x)$ represents the vector of inequality constraints and $h(x)$ is the vector of the equality constraints; that is, $g_i(x)$ is the i^{th} inequality constraint and $h_j(x)$ is the j^{th} equality constraint. The vectors x_{LB} and x_{UB} are the lower and upper bounds of the design variable vector x, respectively.

4.3 Design Variable

The design variable could be written as:

$$x^T = [t_1, ..., t_n, ..., t_N], \tag{4.8}$$

where t_n represents the time length of phase$n$.

4.4 Objective Constraints

- Inequality Constraints:

Too small green time causes frequent change in signal phases which results in much start-up lost time while too big green time leads to rather long waiting time which brings about intolerable delay and large oil waste. Therefore, we further incorporate the following constraint of the green time:

$$g_n^{min} \leq g_n \leq g_n^{max} \tag{4.9}$$

- Equality constraints:

The vector of equality constraints can be written as follows: $h(x) = [h_1(x), h_2(x), h_3(x)]$

- h_1: green time constraint

The green time equality constraint can be formulated as follows:

$$h_1 = g_n - (g_{en} - Y_n + l_n) \tag{4.10}$$

- h_2 : time length constraint

The time length constraint can be formulated as follows:

$$h_2 = t_n - (g_n + Y_n + R_n) \tag{4.11}$$

- h_3 : Total cycle length

Total cycle length is the sum of the time length of each phase n:

$$h_3 = T - \sum_n t_n \tag{4.12}$$

4.5 Objective functions

- Minimize delay time : f_1

The average delay time can be formulated as follows:

$$d_n = \frac{(\alpha_n^{max})^2}{2q_n^{max}(1-\alpha_n^{max})} + \frac{T(1-g_n/T)^2}{2(1-p_n^{max})}, \tag{4.13}$$

where d_n represents the average delay of arrival vehicles during phase n, α_n^{max} is the maximum volume-to-capacity ratio (VOC) of entering legs during phase n, q_n^{max} is the maximum traffic volume of all turning directions intending to cross the intersection during phase n and p_n^{max} is the maximum volume ratio of all entering legs during phase n.

Therefore, the first objective function can be written as follows:

$$f_1(t_n) = \frac{\sum_n (d_n.q_n)}{\sum_n q_n} \tag{4.14}$$

- Minimize number of stops : f_2

The average number of stops can be written as:

$$s_n = \theta\left(\frac{1-g_n/T}{1-p_n^{max}} + \frac{N_n}{q_n^{max}T}\right), \tag{4.15}$$

where

$$N_n = \frac{e^k}{2(1-\alpha_n^{max})} \tag{4.16}$$

$$k = -1.33\sqrt{\frac{u_n^{max}q_n(1-\alpha_n^{max})}{\alpha_n^{max}}} \tag{4.17}$$

u_n represents the maximum saturation flow of all entering legs during phase n and θ represents maximum saturation flow of all entering legs during phase n.
Therefore, the second objective function can be written as follows:

$$f_2(t_n) = \frac{\sum_n (s_n.q_n)}{\sum_n q_n} \tag{4.18}$$

- Maximize Effective capacity

The effective capacity during phase n can written as follows:

$$cap_n = a^{ij}\frac{(t_n - l_n)}{h^{ij}+1} \tag{4.19}$$

$$t_n = g_n + Y_n + R_n \tag{4.20}$$

$$g_n = \begin{cases} g_n^{max}, g'_n > g_n^{max} \\ g_n^{min}, g'_n < g_n^{min} \\ g'_n, otherwise \end{cases} \tag{4.21}$$

$$g'_n = (q_n^{max} - 1)h^{ij} - Y_n + l_n, \tag{4.22}$$

where a_{ij} is number of lanes from entering leg i to exit leg j and h^{ij} is the saturated time headway of vehicles from entering leg i to exit leg j.

Therefore the third objective function is formulated as follows:

$$f_3(t_n) = \frac{\sum_n cap_n}{N} \tag{4.23}$$

4.6 Case study

To prove performances of the proposed multi-objective PVS NBI and Pareto front, we implemented a survey at the intersection of Bab Zaer in Morocco. Mohammed VI Avenue is calibrated to 3 lanes per direction, the other branches of the crosswords are all calibrated to 2 lanes per direction. At Mohammed VI Avenue, the third lane becomes narrow at the exit On all the branches of the crosswords, there are lanes dedicated to the 2 wheels (track, bicycle, lanes). Pedestrian crossings are located on the Yaacoub Mansour Avenue entrance gate in addition to pedestrian crossings on the various crossroads branches. We collected real-time link traffic volume and turning movement in the morning including both peak and off-peak hours and all the necessary information to build the simulation model. All the above data provided rather rich input and evaluation indices for the case study. We used the following constants in the signal timing plan: yellow time of each phase, $3s$, all red time of each phase, $1s$, minimum displayed green time, $8s$, maximum displayed green time, $50s$. Bby using the input vectors from the data of Bab Zaer intersection, we obtained the following results:

- **Capacity to Delay :** Figure 14

- **Capacity to number of stops:** Figure 15

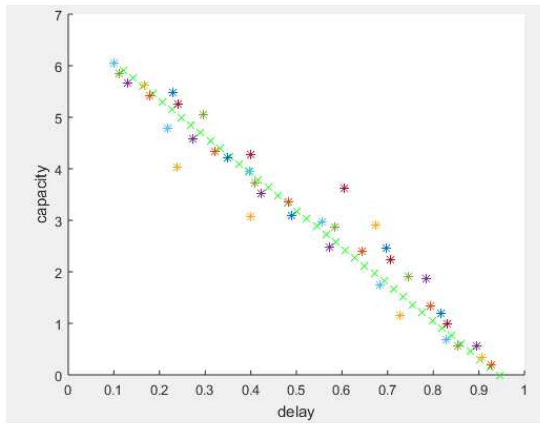


Figure 13: Delay to capacity pareto front

We used "Anylogic" simulation tool, based on the algorithm outputs "cycle phases", to analyze the performance of the multi objective PVS algorithm.

The modeling and simulation results of the system are as follows:

The improved signal timing strategies will lead to the minimization of stops and delays and the maximization of progressive movement through the system.

A static model will allow us to view changes in the objective function Delay time and the Effective capacity to evaluate the strength of the algorithm in terms of computational time and the ability of PVS to find optimum solutions.
Controlling traffic signals at intersections needs a system for managing the duration of different phases of traffic signals in order to decrease the total waiting and total delay and to maximize the flow efficiently.

- Results of Simulation

Running the simulator for an hour improved the capacity of the traffic compared to a test simulation Operational performance at signalized intersections could be significantly enhanced by optimizing phasing and signal timing plans using intelligent traffic control methods.

As shown above, the effective capacity is very interesting and it exceeds 1022 car capacity in 1 hour.

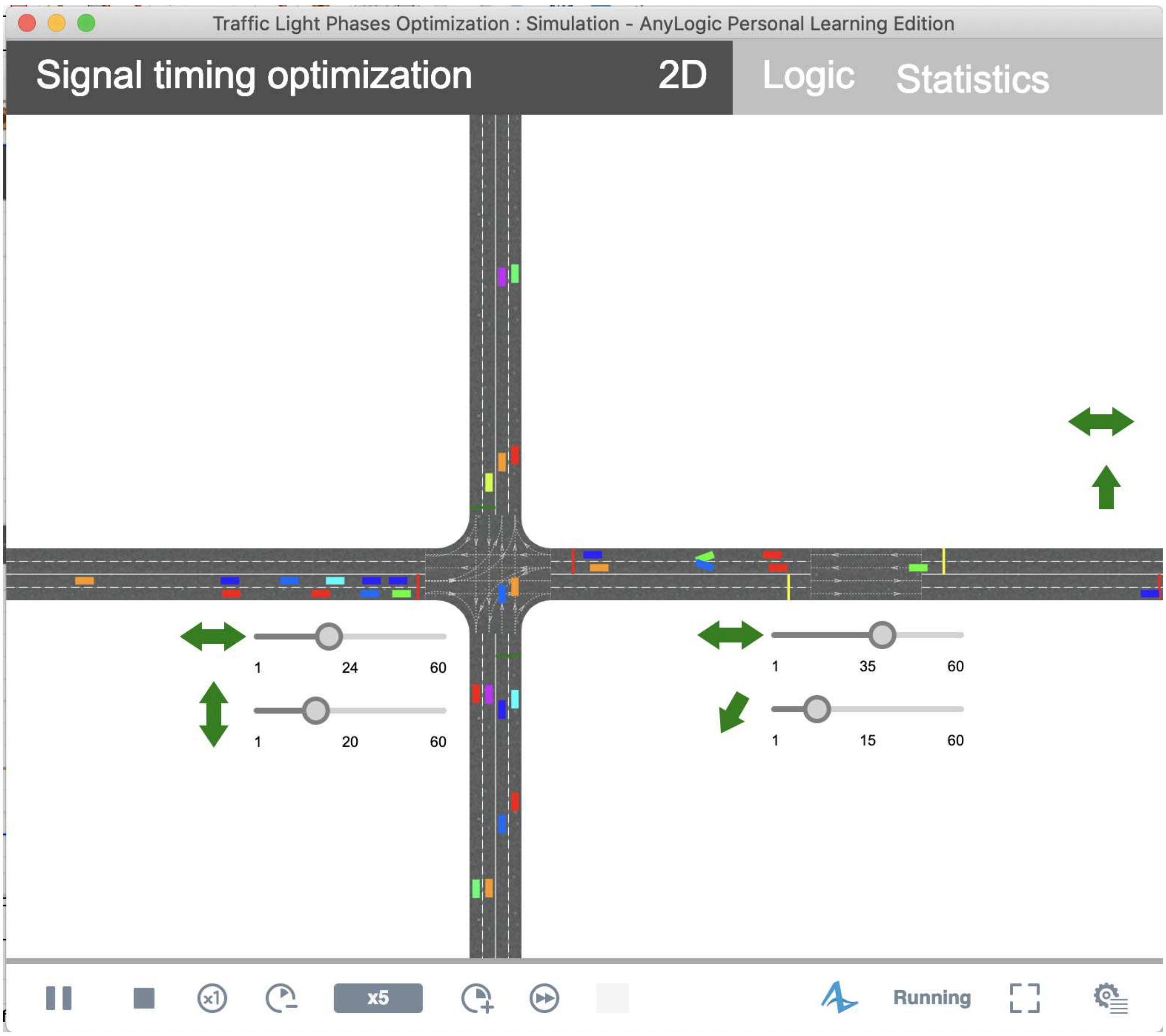


Figure 14: Signal timing optimization

5 Conclusion

The goal of the paper was to present an adaptive multiobjective traffic control system using a novel metaheuristic algorithm based on the passing vehicle search algorithm and the Normal Boundary Intersection Method. The proposed multi-objective optimization algorithm was conducted to provide a trade-off curve among three objectives: delay, number of stops and capacity

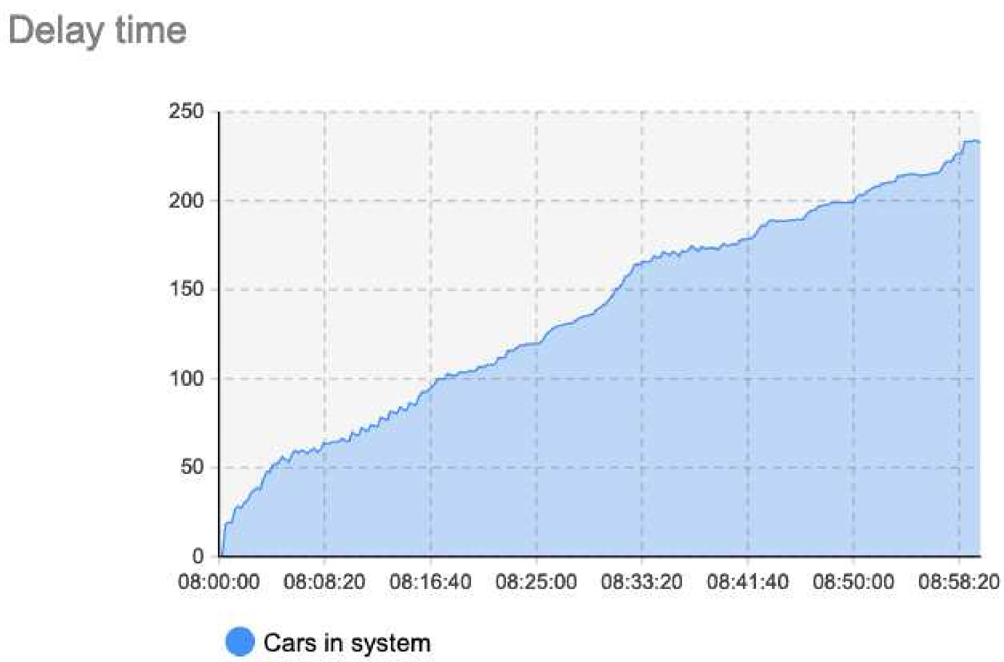


Figure 15: Delay time

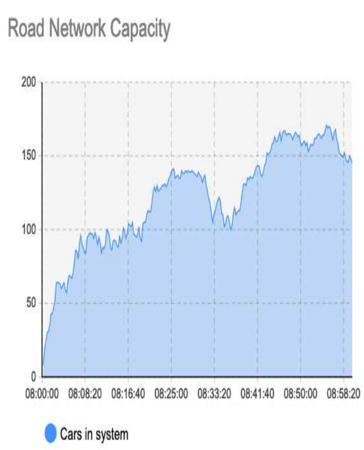


Figure 16: Road Network capacity

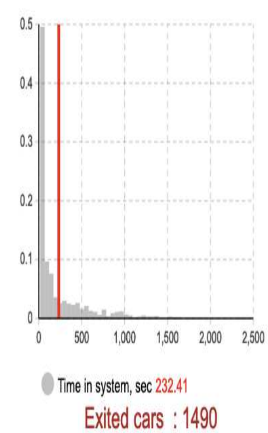


Figure 17: Time in system, existing cars

based on a recent metaheuristic algorithm. PVS is a novel metaheuristic algorithm that is based on a very simple principle; that is, the mechanism of passing vehicles on a two lane road. A critical analysis of the existing liter-

ature on the specific topic revealed that most of these studies have focused on lane-based homogeneous traffic conditions. However, real-time traffic is usually heterogeneous, having non-linear, stochastic, and intricate characteristics.
The case study used an intersection in Rabat, Morocco and the test results obtained were promising in terms of computational time and the ability of the proposed method to find optimum solutions.

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