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A Note on Fuzzy Almost Interior Ideals in Semigroups

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Abstract

The purpose of this paper is to introduce the concepts of fuzzy almost interior ideals and weakly fuzzy almost interior ideals in semigroups with relevance to using the ideas of fuzzy almost ideals and almost interior ideals of semigroups. Moreover, we investigate the properties of fuzzy almost interior ideals and weakly fuzzy almost interior ideals in semigroups. Furthermore, we provide relationships between almost interior ideals and fuzzy almost interior ideals in semigroups.

1 Introduction

Ideal theory plays an important role in studying semigroups and other algebraic structures. In 1976, Lajos gave the concept of interior ideals of semigroups in [2]. Later, in 1981, the notion of almost ideals of semigroups was

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introduced by Grosek and Satko [1] who gave some properties of semigroups which have no proper almost ideals. Recently, almost interior ideals of semigroups have been defined by Kaopusek, Kaewnoi and Chinram [3] as shown in the following definition.

Definition 1.1. [3] A nonempty subset I of a semigroup S is called

1. an *almost interior ideal* of S if $aIb \cap I \neq \emptyset$ for all $a, b \in S$,
2. a *weakly almost interior ideal* of S if $aIa \cap I \neq \emptyset$ for all $a \in S$.

In 1965, Zadeh [8] introduced the concept of fuzzy sets. A fuzzy subset f of a set S is a function defined by $f : S \to [0, 1]$. Next, Kuroki [4] gave the notion of fuzzy ideals of semigroups and studied fuzzy semigroups in [5]. Many interesting results in fuzzy semigroups can be seen in [6]. For any two fuzzy subsets f and g of a set S,

(1) $f \cap g$ is a fuzzy subset of S defined by $(f \cap g)(x) = \min\{f(x), g(x)\}$ for all $x \in S$,

(2) $f \cup g$ is a fuzzy subset of S defined by $(f \cup g)(x) = \max\{f(x), g(x)\}$ for all $x \in S$,

(3) $f \circ g$ is a fuzzy subset of S defined by

$$(f \circ g)(x) = \begin{cases} \sup\limits_{x=ab} \min\{f(a), g(b)\} & \text{if } x \in S^2, \\ 0 & \text{otherwise,} \end{cases}$$

(4) $f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in S$.

For a fuzzy subset f of a set S, the support of f is defined by

$$supp(f) = \{x \in S \mid f(x) \neq 0\}.$$

The characteristic mapping of a subset A of S is a fuzzy subset of S defined by

$$C_A(x) = \begin{cases} 1 & x \in A, \\ 0 & x \notin A. \end{cases}$$

The definition of fuzzy points was given by Pu and Liu [7]. For $x \in S$ and $t \in (0, 1]$, a fuzzy point x_t of a set S is a fuzzy subset of a set S is defined by

$$x_t(y) = \begin{cases} t & y = x, \\ 0 & y \neq x. \end{cases}$$

The purpose of this paper is to introduce and investigate fuzzy almost interior ideals and fuzzy weakly almost interior ideals of semigroups by using the concept of almost interior ideals, weakly almost interior ideals and fuzzy ideals of semigroups. Moreover, we give relationships between almost interior ideals and fuzzy almost interior ideals in semigroups.

2 Fuzzy almost interior ideals of semigroups

We define fuzzy almost interior ideals of semigroups as follows:

Definition 2.1. Let S be a semigroup and f a fuzzy subset of S such that $f \neq 0$. Then f is called

1. a *fuzzy almost interior ideal* of S if $(x_t \circ f \circ y_{t'}) \cap f \neq 0$ for all fuzzy points $x_t, y_{t'}$ of S,

2. a *fuzzy weakly almost interior ideal* of S if $(x_t \circ f \circ x_{t'}) \cap f \neq 0$ for all fuzzy points $x_t, x_{t'}$ of S.

Remark 2.2. *Every fuzzy almost interior ideal of a semigroup is a fuzzy weakly almost interior ideal of that semigroup. However, the converse of this statement is not true.*

Theorem 2.3. *Let f be a fuzzy almost interior ideal of a semigroup S and g be a fuzzy subset of S such that $f \subseteq g$. Then g is a fuzzy almost interior ideal of S.*

Proof. Assume that f is a fuzzy almost interior ideal of a semigroup S and g is a fuzzy subset of S such that $f \subseteq g$. Then for any fuzzy points $x_t, y_{t'}$ of S, we obtain that $(x_t \circ f \circ y_{t'}) \cap f \neq 0$. We have $(x_t \circ f \circ y_{t'}) \cap f \subseteq (x_t \circ g \circ y_{t'}) \cap g$. This implies that $(x_t \circ g \circ y_{t'}) \cap g \neq 0$. Therefore, g is a fuzzy almost interior ideal of S. □

Theorem 2.4. *Let f be a fuzzy weakly almost interior ideal of a semigroup S and g be a fuzzy subset of S such that $f \subseteq g$. Then g is a fuzzy weakly almost interior ideal of S.*

Proof. It is similar to the proof of Theorem 2.3. □

Corollary 2.5. *Let f and g be fuzzy almost interior ideals of a semigroup S. Then $f \cup g$ is also a fuzzy almost interior ideal of S.*

Proof. Since $f \subseteq f \cup g$, by Theorem 2.3, $f \cup g$ is a fuzzy almost interior ideal of S. □

Corollary 2.6. *Let f and g be fuzzy weakly almost interior ideals of a semigroup S. Then $f \cup g$ is also a fuzzy weakly almost interior ideal of S.*

Proof. The proof is similar to that of Corollary 2.5. □

Theorem 2.7. *Let I be a nonempty subset of a semigroup S. Then I is an almost interior ideal of S if and only if C_I is a fuzzy almost interior ideal of S.*

Proof. Assume that I is an almost interior ideal of a semigroup S. Then $xIy \cap I \neq \emptyset$ for all $x, y \in S$. So there exists $z \in S$ such that $z \in xIy$ and $z \in I$. Let $x, y \in S$ and $t, t' \in (0, 1]$. So $(x_t \circ C_I \circ y_{t'})(z) \neq 0$ and $C_I(z) = 1$. Hence, $(x_t \circ C_I \circ y_{t'}) \cap C_I \neq 0$. Therefore, C_I is a fuzzy almost interior ideal of S.

Conversely, suppose that C_I is a fuzzy almost interior ideal of S. Let $x, y \in S$ and $t, t' \in (0, 1]$. Then $(x_t \circ C_I \circ y_{t'}) \cap C_I \neq 0$. Thus there exists $z \in S$ such that $[(x_t \circ C_I \circ y_{t'}) \cap C_I](z) \neq 0$. Hence $z \in xIy \cap I$. So $xIy \cap I \neq \emptyset$. Consequently, I is an almost interior ideal of S. □

Theorem 2.8. *Let I be a nonempty subset of a semigroup S. Then I is a weakly almost interior ideal of S if and only if C_I is a fuzzy weakly almost interior ideal of S.*

Proof. The proof is similar to that of Theorem 2.7. □

Theorem 2.9. *Let f be a fuzzy subset of a semigroup S. Then f is a fuzzy almost interior ideal of S if and only if $supp(f)$ is an almost interior ideal of S.*

Proof. Assume that f is a fuzzy almost interior ideal of a semigroup S. Let $x, y \in S$ and $t, t' \in (0, 1]$. Then $(x_t \circ f \circ y_{t'}) \cap f \neq 0$. So there exists $a \in S$ such that $[(x_t \circ f \circ y_{t'}) \cap f](a) \neq 0$. So $f(a) \neq 0$ and there exists $b \in S$ such that $a = xby$ and $f(b) \neq 0$. Thus $(x_t \circ C_{supp(f)} \circ y_{t'})(a) \neq 0$ and $C_{supp(f)}(a) \neq 0$. Therefore, $(x_t \circ C_{supp(f)} \circ y_{t'}) \cap C_{supp(f)} \neq 0$. Hence, $C_{supp(f)}$ is a fuzzy almost interior ideal of S. By Theorem 2.7, $supp(f)$ is an almost interior ideal of S.

Conversely, suppose that $supp(f)$ is an almost interior ideal of S. By Theorem 2.7, $C_{supp(f)}$ is a fuzzy almost interior ideal of S. Then for any fuzzy point $x_t, y_{t'}$ of S, we have $(x_t \circ C_{supp(f)} \circ y_{t'}) \cap C_{supp(f)} \neq 0$. Thus there exists $a \in S$ such that $[(x_t \circ C_{supp(f)} \circ y_t) \cap C_{supp(f)}](a) \neq 0$. Hence, $(x_t \circ C_{supp(f)} \circ$

$y_{t'})(a) \neq 0$ and $C_{supp(f)}(a) \neq 0$. Then there exists $b \in supp(f)$ such that $a = xby$. Thus $f(a) \neq 0$ and $f(b) \neq 0$. This means that $(x_t \circ f \circ y_{t'}) \cap f \neq 0$. Therefore, f is a fuzzy almost interior ideal of S. □

Theorem 2.10. *Let f be a fuzzy subset of a semigroup S. Then f is a fuzzy weakly almost interior ideal of S if and only if $supp(f)$ is a weakly almost interior ideal of S.*

Proof. The proof is similar to that of Theorem 2.9. □

Next, we examine minimal fuzzy (weakly) almost interior ideals in semigroups and give some relationships between minimal (weakly) almost interior ideals and minimal fuzzy (weakly) almost interior ideals of semigroups.

Definition 2.11. An almost interior ideal I of a semigroup S is called *minimal* if for any almost interior ideal J of S if whenever $J \subseteq I$, then $J = I$.

Definition 2.12. A weakly almost interior ideal I of a semigroup S is called *minimal* if for any weakly almost interior ideal J of S if whenever $J \subseteq I$, then $J = I$.

Definition 2.13. A fuzzy almost interior ideal f of a semigroup S is called *minimal* if for any fuzzy almost interior ideal g of S if whenever $g \subseteq f$, then $supp(g) = supp(f)$.

Definition 2.14. A fuzzy weakly almost interior ideal f of a semigroup S is called *minimal* if for any fuzzy weakly almost interior ideal g of S if whenever $g \subseteq f$, then $supp(g) = supp(f)$.

Theorem 2.15. *Let I be a nonempty subset of a semigroup S. Then I is a minimal almost interior ideal of S if and only if C_I is a minimal fuzzy almost interior ideal of S.*

Proof. Assume that I is a minimal almost interior ideal of a semigroup S. By Theorem 2.7, C_I is a fuzzy almost interior ideal of S. Let g be a fuzzy almost interior ideal of S such that $g \subseteq C_I$. Then $supp(g) \subseteq supp(C_I) = I$. By Theorem 2.9, $supp(g)$ is an almost interior ideal of S. Since I is minimal, $supp(g) = I = supp(C_I)$. Therefore, C_I is minimal.

Conversely, suppose that C_I is a minimal fuzzy almost interior ideal of S. By Theorem 2.7, I is an almost interior ideal of S. To prove I is minimal, let J be an almost interior ideal of S such that $J \subseteq I$. Then C_J is a fuzzy almost interior ideal of S such that $C_J \subseteq C_I$. Hence, $J = supp(C_J) = supp(C_I) = I$. Therefore, I is minimal. □

Theorem 2.16. *Let I be a nonempty subset of a semigroup S. Then I is a minimal weakly almost interior ideal of S if and only if C_I is a minimal fuzzy weakly almost interior ideal of S.*

Proof. The proof is similar to that of Theorem 2.15. □

Corollary 2.17. *Let I be a nonempty subset of a semigroup S. Then I has no proper almost interior ideal of S if and only if for any fuzzy almost interior ideal f of S, $supp(f) = S$.*

Corollary 2.18. *Let I be a nonempty subset of a semigroup S. Then I has no proper weakly almost interior ideal of S if and only if for any fuzzy weakly almost interior ideal f of S, $supp(f) = S$.*

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