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Bipolar fuzzy almost bi-ideal in semigroups

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Abstract

In this paper, we define bipolar fuzzy almost bi-ideal in semigroups. We study the basic properties of bipolar fuzzy almost bi-ideal in semigroups.

1 Introduction

Fuzzy sets, introduced by Zadeh in 1965 [8], are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. In 1994, Zhang [9] extended the concept of a fuzzy set to a bipolar fuzzy set whose membership degree range is $[-1, 0] \cup [0, 1]$. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property and the membership degree $[0, 1]$ of an element

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indicates that the element somehow satisfies that property. These ideas are still central concepts in ring theory and the notion of a one-sided ideal of any algebraic structure is a generalization of the notion of an ideal. In 1980, Grosek and Satko [1] studied the almost ideal theory in semigroups. In 1981, Bogdanvic [2] defined almost bi-ideals in semigroups and studied their properties in semigroups. Later, Chinram [7] defined types of alomst ideals in semigroups such as almost quasi-ideal, almost i-ideal, (m, n)-almost ideal. In 2018, Krishna and Rao [5] defined the bi-interior ideal in semigroups .

In this paper, we define the bipolar fuzzy almost bi-ideal and bipolar fuzzy almost qausi-ideal in semigroups and investigate their basic properties.

2 Preliminaries

In this section, we give some concepts and results which will be helpful in this paper. A *subsemigroup* of a semigroup E is a non-empty set K of E such that $KK \subseteq K$. A *left (right) ideal* of a semigroup E is a non-empty set K of E such that $EK \subseteq K$ $(KE \subseteq K)$. By an *ideal* of a semigroup E, we mean a non-empty set of E which is both a left and a right ideal of E. A subsemigroup K of a semigroup E is called a *bi-ideal* of S if $KEK \subseteq K$. An *almost ideal* K of a semigroup E if $tK \cap K \neq \emptyset$ and $Kp \cap K \neq \emptyset$ for all $t, p \in E$. An *almost bi-ideal* K of a semigroup E if $KrK \cap K \neq \emptyset$ for all $r \in E$ [2].

For any $h_i \in [0, 1]$, $i \in \mathcal{F}$, define

$$\bigvee_{i\in\mathcal{F}} h_i := \sup_{i\in\mathcal{F}}\{h_i\} \quad \text{and} \quad \bigwedge_{i\in\mathcal{F}} h_i := \inf_{i\in\mathcal{F}}\{h_i\}.$$

For any $h, r \in [0, 1]$, we have

$$h \vee r = \max\{h, r\} \quad \text{and} \quad h \wedge r = \min\{h, r\}.$$

A **fuzzy subset** of a non-empty set E is a function $\vartheta : E \to [0, 1]$.

For any fuzzy sets ϑ and ξ of a non-empty set E, we give the following definitions and notations:

(1) for all $h \in E$, $\vartheta \geq \xi \Leftrightarrow \vartheta(h) \geq \xi(h)$,

(2) $\vartheta = \xi \Leftrightarrow \vartheta \geq \xi$ and $\xi \geq \vartheta$,

(3) $(\vartheta\wedge\xi)(h) = \min\{\vartheta(h), \xi(h)\} = \vartheta(h)\wedge\xi(h)$ and $(\vartheta\vee\xi)(h) = \max\{\vartheta(h), \xi(h)\} = \vartheta(h) \vee \xi(h)$ for all $h \in E$,

(4) $\vartheta \subseteq \xi$ if $\vartheta(h) \leq \xi(h)$,

(5) for all $h \in E (\vartheta \cup \xi)(h) = \max\{\vartheta(h), \xi(h)\}$ and $(\vartheta \cap \xi)(h) = \min\{\vartheta(h), \xi(h)\}$,

(6) the *support* of ϑ is $\text{supp}(\vartheta) = \{h \in E \mid \vartheta(h) \neq 0\}$.

Definition 2.1. [6] A **bipolar fuzzy set** (BF set) ϑ on a non-empty set E is an object having the form

$$\vartheta := \{(h, \vartheta^p(h), \vartheta^n(h)) \mid h \in E\},$$

where $\vartheta^p : E \to [0,1]$ and $\vartheta^n : E \to [-1,0]$.

Remark 2.2. For simplicity, we use the symbol $\vartheta = (E; \vartheta^p, \vartheta^n)$ for the BF set $\vartheta = \{(h, \vartheta^p(h), \vartheta^n(h)) \mid h \in E\}$.

The following is an example of a BF set:

Example 2.3. Let $E = \{41, 42, 43...\}$. Define $\vartheta^p : S \to [0,1]$ as

$$\vartheta^p(u) = \begin{cases} 0 & \text{if } h \text{ is old number} \\ 1 & \text{if } h \text{ is even number} \end{cases}$$

and $\vartheta^n : S \to [-1,0]$ as

$$\vartheta^n(u) = \begin{cases} -1 & \text{if } h \text{ is old number} \\ 0 & \text{if } h \text{ is even number.} \end{cases}$$

Then $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF set.

For $h \in E$, define $F_h = \{(h_1, h_2) \in E \times E \mid h = h_1 h_2\}$.

Define products $\vartheta^p \circ \xi^p$ and $\vartheta^n \circ \xi^n$ as follows:
For $h \in E$

$$(\vartheta^p \circ \xi^p)(h) = \begin{cases} \bigvee_{(h_1,h_2) \in F_h} \{\vartheta^p(h_1) \wedge \xi^p(h_2)\} & \text{if } h = h_1 h_2 \\ 0 & \text{otherwise.} \end{cases}$$

and

$$(\vartheta^n \circ \xi^n)(h) = \begin{cases} \bigwedge_{(h_1,h_2) \in F_h} \{\vartheta^n(h_1) \vee \xi^n(h_2)\} & \text{if } h = h_1 h_2 \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.4. [6] Let K be a non-empty set of a semigroup E. **A positive characteristic function and a negative characteristic function** are defined by

$$\lambda_K^p : E \to [0,1], h \mapsto \lambda_K^p(h) := \begin{cases} 1 & h \in K, \\ 0 & h \notin K, \end{cases}$$

and

$$\lambda_K^n : E \to [-1,0], h \mapsto \lambda_K^n(h) := \begin{cases} -1 & h \in K, \\ 0 & h \notin K. \end{cases}$$

respectively.

Remark 2.5. For simplicity, we use the symbol $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ for the BF set $\lambda_K := \{(h, \lambda_K^p(h), \lambda_K^n(h)) \mid h \in K\}$.

For $h \in E$ and $(t,s) \in [0,1] \times [-1,0]$, a BF point $h_{(t,s)} = (E; x_t^p, x_s^n)$ of a set E is a bipolar set of E defined by

$$x_t^p(h) = \begin{cases} t & \text{if } h = x \\ 0 & \text{if } h \neq x \end{cases}$$

and

$$x_s^n(h) = \begin{cases} s & \text{if } h = x \\ 0 & \text{if } h \neq x. \end{cases}$$

Definition 2.6. [4] A BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a **BF subsemigroup** on E if it satisfies the following conditions:
$\vartheta^p(hr) \geq \vartheta^p(h) \wedge \vartheta^p(r)$ and $\vartheta^n(hr) \leq \vartheta^n(h) \vee \vartheta^n(r)$ for all $h, r \in E$.

The following is an example of a BF subsemigroup:

Example 2.7. Let E be a semigroup defined by the following table:

$\cdot$	a	b	c	d	e
a	a	a	a	a	a
a	a	a	a	a	a
c	a	a	c	c	e
d	a	a	c	d	e
e	a	a	c	c	e

Define a BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on E as follows:

S	a	b	c	d	e
ϑ^p	0.9	0.8	0.5	0.3	0.3
ϑ^n	−0.8	−0.8	−0.6	−0.5	−0.3

Then $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF subsemigroup.

Definition 2.8. [4] A BF set $\vartheta = (E; \vartheta_p, \vartheta_n)$ on a semigroup E is called a **BF left (right) ideal** on E if it satisfies the following conditions: $\vartheta^p(hr) \geq \vartheta^p(r)$ $(\vartheta^p(hr) \geq \vartheta^p(h))$ and $\vartheta^n(hr) \leq \vartheta^n(r)$ $(\vartheta^n(hr) \leq \vartheta^n(h))$ for all $h, r \in E$.

Definition 2.9. [4] A BF subsemigroup $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a **BF bi-ideal** on E if $\vartheta^p(hre) \geq \vartheta^p(h) \wedge \vartheta^p(e)$ and $\vartheta^n(hre) \leq \vartheta^n(h) \vee \vartheta^p(e)$ for all $h, r, e \in E$.

Clearly every BF ideal of a semigroup E is a BF bi-ideal of E.

3 Bipolar fuzzy almost bi-ideal

In this section, we define the bipolar fuzzy almost bi-ideal in semigroups and we investigate some of their properties.

Definition 3.1. A BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a *BF almost bi-ideal* of E if $(\vartheta^p \circ x_t^p \circ \vartheta^p) \wedge \vartheta^p \neq \emptyset$ and $(\vartheta^n \circ x_s^n \circ \vartheta^n) \vee \vartheta^n \neq \emptyset$, for any BF point $x_t^p, x_s^n \in E$.

Theorem 3.2. *If ϑ is a BF almost bi-ideal of a semigroup E and ξ is a BF subset of E such that $\vartheta \subseteq \xi$, then ξ is a BF almost bi-ideal of E.*

Proof. Suppose that ϑ is a BF almost bi-ideal of a semigroup E and ξ is a BF subset of E such that $\vartheta \subseteq \xi$. Then, for any BF points $x_t^p, x_s^n \in E$, we have $(\vartheta^p \circ x_t^p \circ \vartheta^p) \wedge \vartheta^p \neq \emptyset$ and $(\vartheta^n \circ x_s^n \circ \vartheta^n) \vee \vartheta^n \neq \emptyset$. Thus $(\vartheta^p \circ x_t^p \circ \vartheta^p) \wedge \vartheta^p \subseteq (\xi^p \circ x_t^p \circ \xi^p) \wedge \xi^p \neq \emptyset$ and $(\vartheta^n \circ x_s^n \circ \vartheta^n) \vee \vartheta^n \subseteq (\xi^n \circ x_s^n \circ \xi^n) \vee \xi^n \neq \emptyset$.
Hence $(\xi^p \circ x_t^p \circ \xi^p) \wedge \xi^p \neq \emptyset$ and $(\xi^n \circ x_s^n \circ \xi^n) \vee \xi^n \neq \emptyset$.
Therefore, ξ is a BF almost bi-ideal of E. □

Theorem 3.3. *Let ϑ and ξ be BF almost bi-ideals of a semigroup E. Then $\vartheta \vee \xi$ is also a BF almost bi-ideal of E.*

Proof. Since $\vartheta \subseteq \vartheta \vee \xi$, $\vartheta \vee \xi$ is also a BF almost bi-ideal of E by Theorem 3.2. □

Theorem 3.4. *Let K be a nonempty subset of a semigroup E. Then K is an almost bi-ideal of E if and only if $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ is a BF almost bi-ideal of E.*

Proof. Suppose that K is an almost bi-ideal of a semigroup E. The $KrK \wedge K \neq \emptyset$ for all $r \in E$; that is, there exists $c \in KrK$ with $c \in K$.
Let $x \in E$, $t \in (0,1]$, and $s \in [-1,0)$. Then $(\lambda^p_K \circ x^p_t \circ \lambda^p_K) = 1$, $\lambda^p_K(c) = 1$ and $(\lambda^n_K \circ x^n_s \circ \lambda^n_K) = -1$, $\lambda^n_K(c) = -1$. Thus $(\lambda^p_K \circ x^p_t \circ \lambda^p_K) \wedge \lambda^p_K \neq 0$, $(\lambda^n_K \circ x^n_s \circ \lambda^n_K) \vee \lambda^n_K \neq 0$. Hence $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ is a BF almost bi-ideal of E.

Conversely, suppose that $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ is a BF almost bi-ideal of E and let $x \in E$, $t \in (0,1]$, and $s \in [-1,0)$. Then $(\lambda^p_K \circ x^p_t \circ \lambda^p_K) \wedge \lambda^p_K \neq 0$ and $(\lambda^n_K \circ x^n_s \circ \lambda^n_K) \vee \lambda^n_K \neq 0$. Thus there exists $c \in E$ such that
$[(\lambda^p_K \circ x^p_t \circ \lambda^p_K) \wedge \lambda^p_K](c) = 1$ and $[(\lambda^n_K \circ x^n_s \circ \lambda^n_K) \vee \lambda^n_K](c) = -1$.
Hence $c \in xKy \wedge K$. So $xKy \wedge K \neq \emptyset$. We conclude that K is an almost bi-ideal of E. □

Theorem 3.5. *Let ϑ be a fuzzy subset of a semigroup E. Then ϑ is a BF almost bi-ideal of E if and only if supp(ϑ) is an almost bi-ideal of E.*

Proof. Assume that ϑ is a BF almost bi-ideal of a semigroup E and let $u \in E$, $t \in (0,1]$, and $s \in [-1,0)$. Then $(\vartheta^p \circ x^p_t \circ \vartheta^p) \neq 0$, $[(\vartheta^n \circ x^n_s \circ \vartheta^n) \wedge \vartheta^n](u) \neq 0$. Thus there exists $z_1, z_2 \in E$ such that $u = z_1 u z_2$, $\vartheta^p(u) \neq 0$, $\vartheta^p(z_1) \neq 0$, $\vartheta^p(z_2) \neq 0$ and $\vartheta^n(u) \neq 0$, $\vartheta^n(z_1) \neq 0$, $\vartheta^n(z_2) \neq 0$. So $u, z_1, z_2 \in \text{supp}(\vartheta)$. This implies that $(\lambda^p_{supp(\vartheta)} \circ x^p_t \circ \lambda^p_{supp(\vartheta)}) \wedge \lambda^p_{supp(\vartheta)} \neq 0$ and $(\lambda^n_{supp(\vartheta)} \circ x^n_s \circ \lambda^n_{supp(\vartheta)}) \vee \lambda^n_{supp(\vartheta)} \neq 0$. Hence $\lambda^p_{supp(\vartheta)}$ is a BF almost bi-ideal of E. By Theorem 3.4, supp(ϑ) is an almost bi-ideal of E.

Conversely, suppose that supp(ϑ) is an almost bi-ideal of E. By Theorem 3.4, $\lambda^p_{supp(\vartheta)}$ is a BF almost bi-ideal of E. Then for any BF points $x^p_t, x^n_s \in E$, we have $(\lambda^p_{supp(\vartheta)} \circ x^p_t \circ \lambda^p_{supp(\vartheta)}) \wedge \lambda^p_{supp(\vartheta)} \neq 0$ and $(\lambda^n_{supp(\vartheta)} \circ x^n_s \circ \lambda^n_{supp(\vartheta)}) \vee \lambda^n_{supp(\vartheta)} \neq 0$. Thus there exists $c \in E$ such that $[(\lambda^p_{supp(\vartheta)} \circ x^p_t \circ \lambda^p_{supp(\vartheta)}) \wedge \lambda^p_{supp(\vartheta)}](c) \neq 0$ and $[(\lambda^n_{supp(\vartheta)} \circ x^n_s \circ \lambda^n_{supp(\vartheta)}) \vee \lambda^n_{supp(\vartheta)}](c) \neq 0$. Hence $(\lambda^p_{supp(\vartheta)} \circ x^p_t \circ \lambda^p_{supp(\vartheta)})(c) = 0$, $\lambda^p_{supp(\vartheta)}(c) \neq 0$ and $(\lambda^n_{supp(\vartheta)} \circ x^n_s \circ \lambda^n_{supp(\vartheta)})(c) = 0$, $\lambda^n_{supp(\vartheta)}(c) \neq 0$. Then there exists $b \in E$ such that $c = xby$ such that $\vartheta^p(c) \neq 0, \vartheta^p(b) \neq 0$ and $\vartheta^n(c) \neq 0, \vartheta^n(b) \neq 0$. So $(\vartheta^p \circ x^p_t \circ \vartheta^p) \wedge \vartheta^p \neq 0$ and $(\vartheta^n \circ x^n_s \circ \vartheta^n) \vee \vartheta^n \neq 0$. Therefore, ϑ is a BF almost bi-ideal of E. □

Next, we investigate minimal BF almost bi-ideals in semigroups and study relationships between minimal almost bi-ideals and minimal BF almost bi-ideals of semigroups.

Definition 3.6. An almost bi-ideal K of a semigroup E is called *minimal* if for any almost bi-ideal M of E with $M \subseteq K$, $M = K$.

Definition 3.7. A BF almost bi-ideal ϑ of a semigroup E is called *minimal* if for any BF almost bi-ideal ξ of E with $\xi \subseteq \vartheta$, sup(ξ) =sup(ϑ).

Theorem 3.8. *Let K be a nonempty subset of a semigroup E. Then K is a minimal almost bi-ideal of E if and only if $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost bi-ideal of E.*

Proof. Assume that K is a minimal almost bi-ideal of a semigroup E.
By Theorem 3.4, $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF almost bi-ideal of E.
Let ξ be a BF almost bi-ideal of E such that $\xi \subseteq \lambda_K$ Then
$\text{supp}(\xi) \subseteq \text{supp}(\lambda_K) = K$. By Theorem 3.5, supp($\xi$) is an almost bi-ideal of E. Since K is minimal we have $\text{supp}(\xi) = K = \text{supp}(\lambda_K)$.
Therefore, λ_K is minimal.

Conversely, suppose that λ_K is a minimal BF almost bi-ideal of E. By Theorem 3.4, K is an almost bi-ideal of E. Let M be an almost bi-ideal of E such that $M \subseteq K$. Then λ_K is a BF almost bi-ideal of E such that $\lambda_M \subseteq \lambda_K$. Hence $M = \text{supp}(\lambda_M) = \text{supp}(\lambda_K) = K$. Therefore, K is minimal. □

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