International Journal of Mathematics and
Computer Science, **17**(2022), no. 1, 143–161

Characterizing Ordered Semihypergroups by Using Generalized Int-soft Hyperideals

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(Received June 8, 2021, Revised July 19, 2021, Accepted July 24, 2021)

Abstract

The central theme of this paper is to study ordered semihypergroups in the context of int-soft left (right) hyperideals. In this paper, we study the generalized int-soft left (right) hyperideals and discuss their related properties. We also characterize regular and intra-regular ordered semihypergroups in terms of generalized int-soft left (right) hyperideals. Finally, we characterize left and right weakly regular ordered semihypergroups through int-soft hyperideals.

1 Introduction

Hyperstructure theory was initialed by Marty [17] when he defined hypergroups. The theory is widely studied from the theoretical point of view as well as for its applications to many subjects of pure and applied properties (see [4, 5, 6]).

Bonansinga and Corsini [3, 5] first investigated semihypergroups (also called hypersemigroup or multisemigroup). Later, this concept was studied by many authors, including Mahmood [16], Hila et al. [9], Davvaz et al.

Key words and phrases: Ordered semihypergroup, int-soft left hyperideal, int-soft right hyperideal, regularities.
AMS (MOS) Subject Classifications: 16Y99, 20N20, 06F99.
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ISSN 1814-0432, 2022, http://ijmcs.future-in-tech.net

[6], Gu and Tang [11], Heidari and Davvaz [10]. The concept of ordered semihypergroups is a generalization of ordered semigroups. Indeed, every ordered semigroup can be considered as an ordered semihypergroup [2].

Molodtsov [18] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties that are free from the difficulties that have troubled the usual theoretical approaches. Anvariyeh et al. [1] initiated the study of several properties of semihypergroups by using soft set theory. Sezgin et al. [19] defined soft intersection interior ideals as a new approach to the classical semigroup theory. Naz and Shabir [15] applied soft set theory to semihypergroups. Khan al et. [14] gave the notion of int-soft interior hyperideals in ordered semihypergroups and proved that the int-soft hyperideals and int-soft interior hyperideals coincide in regular and in intra-regular ordered semihypergroups. Farooq et al. [8] introduced the notion of (M, N)-int-soft bi-hyperideals of ordered semihypergroups and characterized left (M, N) simple and completely regular ordered semihypergroups using (M, N)-int-soft bi-hyperideals.

In this paper, we study the notion of the generalized int-soft left (right) hyperideals and discuss their related properties. Moreover, we give the main theorems that characterize regular and intra-regular ordered semihypergroups through generalized intersection soft left (right) hyperideals. Furthermore, we characterize weakly regular by using int-soft hyperideals.

2 Preliminaries

In this section, we recall the basic terms and definitions of hyperstructure theory and soft set theory.

Definition 2.1 ([6]). *A map* $\circ\colon S \times S \to \mathcal{P}^*(S)$ *is called a* hyperoperation *on* S*, where* S *is a nonempty set and* $\mathcal{P}^*(S)$ *denotes the set of all nonempty subsets of* S*.*

A *hypergroupoid* is a structure $\langle S; \circ\rangle$ comprising of a nonempty set S and a hyperoperation on S.

Let $\langle S; \circ\rangle$ be a hypergroupoid. For nonempty subsets A and B of S, we define

$$A \circ B = \bigcup_{a\in A, b\in B} a \circ b.$$

Definition 2.2 ([6]). *A hypergroupoid* $\langle S; \circ \rangle$ *is a* semihypergroup *if the hyperoperation* $\circ$ *on* S *is associative; that is,*

$$x \circ (y \circ z) = (x \circ y) \circ z,$$

for all $x, y, z \in S$.

Definition 2.3 ([6]). *A structure* $\langle S; \circ, \leq \rangle$ *is called an* ordered semihypergroup *if*

1. $\langle S; \circ \rangle$ *is a semihypergroup,*
2. $\langle S; \leq \rangle$ *is a partially ordered set,*
3. *for any* $a, b, c \in S$, $a \leq b$ *implies* $a \circ c \leq b \circ c$ *and* $c \circ a \leq c \circ b$. *Here,* $A \leq B$ *means that for each* $a \in A$ *there exists* $b \in B$ *such that* $a \leq b$.

For simplicity, we denote an ordered semihypergroup $\langle S; \circ, \leq \rangle$ by its carrier set as a bold letter $\mathbf{S}$.

For $A \subseteq S$, we use the notation

$$(A] := \{a \in S \mid a \leq t \text{ for some } t \in A\}.$$

A nonempty subset A of S is called a *subsemihypergroup* of $\mathbf{S}$ if $\langle A; \circ, \leq \rangle$ is an ordered semihypergroup.

Definition 2.4 ([6]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. A nonempty subset* A *of* S *is called a* left (right) hyperideal *of* $\mathbf{S}$ *if*

1. $S \circ A \subseteq A$ $(A \circ S \subseteq A)$,
2. *for any* $a \in S$ *and* $b \in A$, *if* $a \leq b$, *then* $a \in A$.

A nonempty subset I of S is called a *hyperideal* of $\mathbf{S}$ if it is both a left hyperideal and a right hyperideal of $\mathbf{S}$ (see [6]).

For any $a \in S$, we denote by $\mathrm{L}(a)$ and $\mathrm{R}(a)$ the left hyperideal and the right hyperideal of $\mathbf{S}$ generated by a. One can show that $\mathrm{L}(a) = (a \cup (S \circ a)]$ and $\mathrm{R}(a) = (a \cup (a \circ S)]$.

From now on, U is an initial universe set, E is a set of parameters, $\mathcal{P}(U)$ is the set of all subsets of U and $A, B, C, \ldots$ are a nonempty subsets of E.

Definition 2.5 ([18]). *A* soft set f_A *of* E *over* U *is defined as* $f_A \colon E \to \mathcal{P}(U)$ *such that* $f_A(x) := \emptyset$ *if* $x \notin A$.

Hence, f_A is also called an *approximation function*. A soft set f_A of E over U can be represented by the set of ordered pairs

$$f_A := \{(x, f_A(x)) : x \in E \text{ and } f_A(x) \in \mathcal{P}(U)\}.$$

From the definition, it is clear that a soft set is a parameterized family of subsets of U. Throughout this paper, we denote the set of all soft sets of E over U by $\mathrm{S}_E(U)$.

For a nonempty subset A of E, *the characteristic soft set* is defined to be the soft set χ_A of E over U in which χ_A is given as follows:

$$\chi_A(x) = \begin{cases} U & \text{if } x \in A, \\ \emptyset & \text{if } x \notin A, \end{cases}$$

for all $x \in E$ (see [14]).

In [7], the authors defined binary relations and binary operations on $\mathrm{S}_E(U)$ as follows: Let $f_A, f_B \in \mathrm{S}_E(U)$. Then

1. f_A is called a *soft subset* of f_B, denoted by $f_A \subseteq f_B$, if $f_A(x) \subseteq f_B(x)$, for all $x \in E$,
2. soft sets f_A and f_B are said to be *equal*, denoted by $f_A = f_B$, if $f_A \subseteq f_B$ and $f_B \subseteq f_A$,
3. the *soft union* of f_A and f_B, denoted by $f_A \cup f_B = f_{A\cup B}$, is defined by $(f_A \cup f_B)(x) := f_A(x) \cup f_B(x)$ for all $x \in E$,
4. the *soft intersection* of f_A and f_B, denoted by $f_A \cap f_B = f_{A\cap B}$, is defined by $(f_A \cap f_B)(x) := f_A(x) \cap f_B(x)$ for all $x \in E$.

To study algebraic properties of ordered semihypergroups using soft sets, we need some settings. Let $\mathbf{S}$ be an ordered semihypergroup. Any soft set of S over U will be called a soft set of $\mathbf{S}$ over U. Moreover, the set $\mathrm{S}_S(U)$ will be denoted by $\mathbf{S}(U)$.

Let $\mathbf{S}$ be an ordered semihypergroup and $a \in S$. Then, we define

$$\mathbf{S}_a := \{(x, y) \in S \times S : a \leq x \circ y\}.$$

Let f_A and f_B be elements of $\mathbf{S}(U)$. The soft product of f_A and f_B, denoted by $f_A \otimes f_B$, is defined by

$$(f_A \otimes f_B)(a) = \begin{cases} \bigcup_{(x,y)\in \mathbf{S}_a} (f_A(x) \cap f_B(y)) & \text{if } \mathbf{S}_a \neq \emptyset, \\ \emptyset & \text{if } \mathbf{S}_a = \emptyset, \end{cases}$$

for all $a \in S$.

Now, we always assume that $\emptyset \subseteq \xi \subset \zeta \subseteq U$.

Definition 2.6 ([8]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. A soft set* f_A *of* $\mathbf{S}$ *over* U *is called a* (ξ,ζ)-int-soft semihypergroup *of* $\mathbf{S}$ *over* U *if*

$$\left(\bigcap_{a\in x\circ y} f_A(a)\right)\cup\xi\supseteq(f_A(x)\cap f_A(y))\cap\zeta,$$

for all $x,y\in S$.

Definition 2.7 ([8]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. A soft set* f_A *of* $\mathbf{S}$ *over* U *is called a* (ξ,ζ)-int-soft left hyperideal *of* $\mathbf{S}$ *over* U *if for any* $x,y\in S$,

1. $x\le y$ *implies* $f_A(x)\cup\xi\supseteq f_A(y)\cap\zeta$,
2. $\left(\bigcap_{a\in x\circ y} f_A(a)\right)\cup\xi\supseteq f_A(y)\cap\zeta$.

Definition 2.8 ([8]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. A soft set* f_A *of* $\mathbf{S}$ *over* U *is called a* (ξ,ζ)-int-soft right hyperideal *of* $\mathbf{S}$ *over* U *if for any* $x,y\in S$,

1. $x\le y$ *implies* $f_A(x)\cup\xi\supseteq f_A(y)\cap\zeta$,
2. $\left(\bigcap_{a\in x\circ y} f_A(a)\right)\cup\xi\supseteq f_A(x)\cap\zeta$.

A soft set of $\mathbf{S}$ over U is called a (ξ,ζ)-*int-soft hyperideal* of $\mathbf{S}$ over U if it is both a (ξ,ζ)-int-soft left hyperideal and a (ξ,ζ)-int-soft right hyperideal of $\mathbf{S}$ over U.

3 Main Results

Let $\mathbf{S}$ be an ordered semihypergroup. We define new binary operations on $\mathbf{S}(U)$ and a new soft set from the old one. Then we apply them to characterize some classes of ordered semihypergroups: regular, intra-regular and weakly regular.

We define binary operations $\overset{(\xi,\zeta)}{\bigcap}$ and $\overset{(\xi,\zeta)}{\bigotimes}$ on $\mathbf{S}(U)$ as follows:
For any $f_A, f_B\in\mathbf{S}(U)$, we define

1. $(f_A \overset{(\xi,\zeta)}{\bigcap} f_B)(x) := [(f_A\cap f_B)(x)\cap\zeta]\cup\xi$ for all $x\in S$,

2. $(f_A \overset{(\xi,\zeta)}{\bigotimes} f_B)(x) := [(f_A \otimes f_B)(x) \cap \zeta] \cup \xi$ for all $x \in S$.

Let f_A be a soft set of $\mathbf{S}$ over U. Then we define a new soft set $f_A^{(\xi,\zeta)}$ of $\mathbf{S}$ over U by $f_A^{(\xi,\zeta)}(x) := (f_A(x) \cap \zeta) \cup \xi$ for all $x \in S$.

Remark 3.1. *Let* $\mathbf{S}$ *be an ordered semihypergroup, and* $f_A, f_B \in \mathbf{S}(U)$*. Then, for any* $x \in S$*, we have*

1. $(f_A \overset{(\xi,\zeta)}{\bigcap} f_B)(x) := [(f_A \cap f_B)(x) \cap \zeta] \cup \xi = (f_A^{(\xi,\zeta)} \cap f_B^{(\xi,\zeta)})(x)$,
2. $(f_A \overset{(\xi,\zeta)}{\bigotimes} f_B)(x) := [(f_A \otimes f_B)(x) \cap \zeta] \cup \xi = (f_A^{(\xi,\zeta)} \otimes f_B^{(\xi,\zeta)})(x)$.

We give the condition for which a soft set is a generalized intersection soft semihypergroups of $\mathbf{S}$ over U.

Lemma 3.2. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* f_A *a soft set of* $\mathbf{S}$ *over* U *such that* $x \leq y$ *implies* $f_A(x) \cup \xi \supseteq f_A(y) \cap \zeta$*, for all* $x, y \in S$*. The following statements are equivalent:*

1. f_A *is a* (ξ,ζ)*-int-soft semihypergroup of* $\mathbf{S}$ *over* U*.*
2. $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$.

Proof. (1) ⇒ (2). Let $x \in S$. If $\mathbf{S}_x = \emptyset$, then we obtain

$$\begin{aligned}(f_A \overset{(\xi,\zeta)}{\bigotimes} f_A)(x) &= [(f_A \otimes f_A)(x) \cap \zeta] \cup \xi \\ &= \xi \\ &\subseteq [f_A(x) \cap \zeta] \cup \xi \\ &= f_A^{(\xi,\zeta)}(x).\end{aligned}$$

Suppose that $\mathbf{S}_x \neq \emptyset$. For a given $(u, v) \in \mathbf{S}_x$, there exists $t \in u \circ v$ such that $x \leq t$. So by the assumption of the lemma, we have that $f_A(x) \cup \xi \supseteq f_A(t) \cap \zeta$. This means that $f_A(x) \cup \xi \supseteq \bigcap_{t \in u \circ v}(f_A(t) \cap \zeta)$ for any $(u, v) \in \mathbf{S}_x$. Thus

$$\begin{aligned}(f_A \overset{(\xi,\zeta)}{\bigotimes} f_A)(x) &= [(f_A \otimes f_A)(x) \cap \zeta] \cup \xi \\ &= \left[\left(\bigcup_{(u,v) \in \mathbf{S}_x} (f_A(u) \cap f_A(v))\right) \cap \zeta\right] \cup \xi\end{aligned}$$

$$= \left[\left(\bigcup_{(u,v)\in \mathbf{S}_x} (f_A(u) \cap f_A(v)) \right) \cap \zeta \cap \zeta \right] \cup \xi$$
$$= \left[\left(\bigcup_{(u,v)\in \mathbf{S}_x} (f_A(u) \cap f_A(v) \cap \zeta) \right) \cap \zeta \right] \cup \xi$$
$$\subseteq \left[\left(\bigcup_{(u,v)\in \mathbf{S}_x} \left(\bigcap_{c\in u\circ v} f_A(c) \cup \xi \right) \right) \cap \zeta \right] \cup \xi$$
(by our presumption)
$$\subseteq \left[\left(\bigcup_{(u,v)\in \mathbf{S}_x} (f_A(x) \cup \xi) \right) \cap \zeta \right] \cup \xi$$
$$= \left(\bigcup_{(u,v)\in \mathbf{S}_x} (f_A(x) \cup \xi) \right) \cap \zeta$$
$$= (f_A(x) \cap \zeta) \cup \xi$$
$$= f_A^{(\xi,\zeta)}(x).$$

Hence, $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$.

(2) $\Rightarrow$ (1). Let $x, y \in S$ and $a \in x \circ y$. Then $(x, y) \in \mathbf{S}_a$. Thus, we obtain

$$\begin{aligned} f_A(a) \cup \xi &\supseteq (f_A(a) \cap \zeta) \cup \xi \\ &= f_A^{(\xi,\zeta)}(a) \\ &\supseteq [(f_A \otimes f_A)(a) \cap \zeta] \cup \xi \qquad \text{(by our assumption)} \\ &= \left[\bigcup_{(b,c)\in \mathbf{S}_a} [f_A(b) \cap f_A(c)] \cap \zeta \right] \cup \xi \\ &\supseteq \left[\bigcup_{(b,c)\in \mathbf{S}_a} [f_A(b) \cap f_A(c)] \cap \zeta \right] \\ &\supseteq f_A(x) \cap f_A(y) \cap \zeta. \end{aligned}$$

Thus, $\bigcap_{a\in x\circ y} f_A(a) \cup \xi \supseteq f_A(x) \cap f_A(y) \cap \zeta$. Therefore, f_A is a (ξ, ζ)-int-soft semihypergroup of $\mathbf{S}$ over U. □

A characterization of (ξ, ζ)-int-soft left hyperideal through the characteristic soft set χ_S is shown as follows:

Theorem 3.3. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* f_A *a soft set of* $\mathbf{S}$ *over* U *such that* $x \leq y$ *implies* $f_A(x) \cup \xi \supseteq f_A(y) \cap \zeta$ *for all* $x, y \in S$. *Then the following statements are equivalent:*

1. f_A *is a* (ξ, ζ)*-int-soft left hyperideal of* $\mathbf{S}$ *over* U.

2. $\chi_S \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$.

Proof. (1) ⇒ (2). Let $x \in S$. If $\mathbf{S}_x = \emptyset$, then

$$(\chi_S \overset{(\xi,\zeta)}{\bigotimes} f_A)(x) = \xi \subseteq (f_A(x) \cap \zeta) \cup \xi = f_A^{(\xi,\zeta)}(x).$$

Suppose that $\mathbf{S}_x \neq \emptyset$. For a given $(u, v) \in \mathbf{S}_x$, there exists $t \in u \circ v$ such that $x \leq t$. By the assumption of the lemma, we have that $f_A(x) \cup \xi \supseteq f_A(t) \cap \zeta$. This means that $f_A(x) \cup \xi \supseteq \bigcap_{t \in u \circ v}(f_A(t) \cap \zeta)$ for any $(u, v) \in \mathbf{S}_x$. Thus

$$\begin{aligned}
(\chi_S \overset{(\xi,\zeta)}{\bigotimes} f_A)(x) &= [(\chi_S \otimes f_A)(x) \cap \zeta] \cup \xi \\
&= \left[\bigcup_{(u,v) \in \mathbf{S}_x} \chi_X(u) \cap f_A(v) \cap \zeta \right] \cup \xi \\
&= \left[\bigcup_{(u,v) \in \mathbf{S}_x} f_A(v) \cap U \cap \zeta \right] \cup \xi \\
&\subseteq \bigcup_{(u,v) \in \mathbf{S}_x} \left(\left[\bigcap_{c \in u \circ v} f_A(c) \cup \xi \right] \cap \zeta \right) \cup \xi \text{ (by our presumption)} \\
&= \bigcup_{(u,v) \in \mathbf{S}_x} \left(\left[\bigcap_{c \in u \circ v} f_A(c) \cap \zeta \right] \cup (\zeta \cap \xi) \right) \cup \xi \\
&= \bigcup_{(u,v) \in \mathbf{S}_x} \left[\bigcap_{c \in u \circ v} f_A(c) \cap \zeta \cap \zeta \right] \cup \xi \\
&\subseteq \left[\left(\bigcup_{(u,v) \in \mathbf{S}_x} (f_A(x) \cup \xi) \right) \cap \zeta \right] \cup \xi \\
&= [((f_A(x) \cup \xi)) \cap \zeta] \cup \xi \\
&= [f_A(x) \cap \zeta] \cup \xi
\end{aligned}$$

$$= f_A^{(\xi,\zeta)}(x).$$

This shows that $\chi_S \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$.

(2) $\Rightarrow$ (1). Let $x, y \in S$ and $a \in x \circ y$. Then $(x, y) \in \mathbf{S}_a$. Thus

$$\begin{aligned}
f_A(a) \cup \xi &\supseteq (f_A(a) \cup \xi) \cap \zeta \\
&= (f_A(a) \cap \zeta) \cup (\xi \cap \zeta) \\
&= (f_A(a) \cap \zeta) \cup \xi \\
&\supseteq (\chi_S \overset{(\xi,\zeta)}{\bigotimes} f_A)(a) \\
&= [(\chi_S \otimes f_A)(a) \cap \zeta] \cup \xi \\
&= \left(\bigcup_{a \in u \circ v} \chi_S(u) \cap f_A(v) \cap \zeta \right) \cup \xi \\
&\supseteq (\chi_S(x) \cap f_A(y) \cap \zeta) \cup \xi \\
&= (f_A(y) \cap \zeta) \cup \xi \\
&\supseteq f_A(y) \cap \zeta.
\end{aligned}$$

Thus, $\bigcap_{a \in x \circ y} f_A(a) \cup \xi \supseteq f_A(y) \cap \zeta$. Hence, f_A is a (ξ, ζ)-int-soft left hyperideal of $\mathbf{S}$ over U. □

Similar to Theorem 3.3, we obtain the following results.

Theorem 3.4. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* f_A *a soft set of* $\mathbf{S}$ *over* U *such that* $x \leq y$ *implies* $f_A(x) \cup \xi \supseteq f_A(y) \cap \zeta$, *for all* $x, y \in S$. *Then the following statements are equivalent:*

1. f_A *is a* (ξ, ζ)*-int-soft right hyperideal of* $\mathbf{S}$ *over* U.

2. $f_A \overset{(\xi,\zeta)}{\bigotimes} \chi_S \subseteq f_A^{(\xi,\zeta)}$.

Corollary 3.5. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* f_A *a soft set of* $\mathbf{S}$ *over* U *such that* $x \leq y$ *implies* $f_A(x) \cup \xi \supseteq f_A(y) \cap \zeta$ *for all* $x, y \in S$. *Then the following statements are equivalent:*

1. f_A *is a* (ξ, ζ)*-int-soft hyperideal of* $\mathbf{S}$ *over* U.

2. $\chi_S \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$ *and* $f_A \overset{(\xi,\zeta)}{\bigotimes} \chi_S \subseteq f_A^{(\xi,\zeta)}$.

We now give a property of (ξ,ζ)-int-soft left (right) hyperideals of $\mathbf{S}$ over U.

Theorem 3.6. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* f_A *a* (ξ,ζ)*-int-soft left (right) hyperideal of* $\mathbf{S}$ *over* U*. Then* $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$.

Proof. To obtain our claim, we show that f_A is a (ξ,ζ)-int-soft semihypergroup of $\mathbf{S}$ over U. Let $x,y \in S$. Then

$$\left(\bigcap_{a\in x\circ y} f_A(a)\right) \cup \xi \supseteq f_A(y) \cap \zeta \supseteq (f_A(x) \cap f_A(y)) \cap \zeta.$$

This shows that f_A is a (ξ,ζ)-int-soft semihypergroup of $\mathbf{S}$ over U. By Lemma 3.2, we obtain $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$. □

Corollary 3.7. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* f_A *a* (ξ,ζ)*-int-soft hyperideal of* $\mathbf{S}$ *over* U*. Then* $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$.

We now characterize left (right) hyperideals of $\mathbf{S}$ by the characteristic soft sets.

Lemma 3.8. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* A *be a nonempty set of* S*. Then the following statements are equivalent:*

1. *A is a left (right) hyperideal of* $\mathbf{S}$.
2. *χ_A is a* (ξ,ζ)*-int-soft left (right) hyperideal of* $\mathbf{S}$ *over* U.

Proof. (1) ⇒ (2). Suppose that A is a left hyperideal of $\mathbf{S}$. Let $x,y \in S$. If $y \notin A$, then $\left(\bigcap_{a\in x\circ y} \chi_A(a)\right) \cup \xi \supseteq \emptyset = \chi_A(y) \cap \zeta$. If $y \in A$, then $x \circ y \subseteq S \circ A \subseteq A$. This implies that, for every $a \in x \circ y$, $a \in A$. Thus

$$\left(\bigcap_{a\in x\circ y} \chi_A(a)\right) \cup \xi = U \supseteq \chi_A(y) \cap \zeta.$$

Now, let $x,y \in S$ be such that $x \leq y$. If $y \notin A$, then $\chi_A(x) \cup \xi \supseteq \emptyset = \chi_A(y) \cap \zeta$. If $y \in A$, then, by the left hyperideality of A, we obtain $x \in A$. Then

$$\chi_A(x) \cup \xi = U \supseteq U \cap \zeta = \chi_A(y) \cap \zeta.$$

Therefore, χ_A is a (ξ, ζ)-int-soft left hyperideal of $\mathbf{S}$ over U.

(2) $\Rightarrow$ (1). Let A be a nonempty subset of S and χ_A be a (ξ, ζ)-int-soft left hyperideal of $\mathbf{S}$ over U. Let $x \in S$ and $y \in A$. Then

$$\left(\bigcap_{z \in x \circ y} \chi_A(z) \right) \cup \xi \supseteq \chi_A(y) \cap \zeta = \zeta.$$

This implies that $\emptyset \neq \zeta \setminus \xi \subseteq \bigcap_{z \in x \circ y} \chi_A(z)$. Thus, $\chi_A(z) = U$ and then $z \in A$, for all $z \in x \circ y \subseteq S \circ A$. Hence, $S \circ A \subseteq A$. Now, let $x \in S$ and $y \in A$ be such that $x \leq y$. Since χ_A is a (ξ, ζ)-int-soft left hyperideal of $\mathbf{S}$ over U, we obtain $\chi_A(x) \cup \xi \supseteq \chi_A(y) \cap \zeta = \zeta$. As a result, $\chi_A(x) \supseteq \zeta \setminus \xi$. That is, $\chi_A(x) = U$ and then $x \in A$. This completes the proof.

Similarly, we can show that χ_A is a (ξ, ζ)-int-soft right hyperideal of $\mathbf{S}$ over U if and only if A is a right hyperideal of $\mathbf{S}$. □

Corollary 3.9. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* A *a nonempty set of* S*. Then the following statements are equivalent:*

1. *A is a hyperideal of* $\mathbf{S}$.
2. *χ_A is a (ξ, ζ)-int-soft hyperideal of* $\mathbf{S}$ *over* U.

The following lemma is a useful tool used to characterize particular classes of ordered semihypergroups.

Lemma 3.10 ([13]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements hold:*

1. $\chi_A \cap \chi_B = \chi_{A \cap B}$.
2. $\chi_A \otimes \chi_B = \chi_{(A \circ B]}$.

It is not difficult to show the following result by applying Lemma 3.10.

Lemma 3.11. *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements hold:*

1. $\chi_A \overset{(\xi,\zeta)}{\bigcap} \chi_B = \chi^{(\xi,\zeta)}_{A \cap B}$.
2. $\chi_A \overset{(\xi,\zeta)}{\bigotimes} \chi_B = \chi^{(\xi,\zeta)}_{(A \circ B]}$.

Lemma 3.12. *Let f_A and f_B be a (ξ, ζ)-int-soft right hyperideal and a (ξ, ζ)-int-soft left hyperideal of* $\mathbf{S}$ *over U, respectively. Then,* $f_A \overset{(\xi,\zeta)}{\bigotimes} f_B \subseteq f_A \overset{(\xi,\zeta)}{\bigcap} f_B$.

Proof. Let f_A and f_B be a (ξ,ζ)-int-soft right hyperideal and a (ξ,ζ)-int-soft left hyperideal of $\mathbf{S}$ over U, respectively. Let $z \in S$. If $\mathbf{S}_z = \emptyset$, then

$$\begin{aligned}(f_A \overset{(\xi,\zeta)}{\bigotimes} f_B)(z) &= ((f_A \otimes f_B)(z) \cap \zeta) \cup \xi \\ &= \xi \\ &\subseteq (f_A(z) \cap f_B(z) \cap \zeta) \cup \xi.\end{aligned}$$

Suppose that $\mathbf{S}_z \neq \emptyset$. For a given $(u,v) \in \mathbf{S}_z$, there exists $t \in u \circ v$ such that $z \leq t$. By our assumption, we have $f_A(z) \cup \xi \supseteq f_A(t) \cap \zeta$. It is not difficult to see that $f_A(z) \cup \xi \supseteq (f_A(t) \cup \xi) \cap \zeta$. That is, $f_A(z) \cup \xi \supseteq \left[\bigcap_{t \in u \circ v} (f_A(t) \cup \xi)\right] \cap \zeta$. Similarly, we obtain that $f_B(z) \cup \xi \supseteq \left[\bigcap_{t \in u \circ v} (f_B(t) \cup \xi)\right] \cap \zeta$. Thus

$$(f_A(z) \cap f_B(z)) \cup \xi \supseteq \left[\left(\bigcap_{t \in u \circ v} (f_A(t) \cup \xi)\right) \cap \left(\bigcap_{t \in u \circ v} (f_B(t) \cup \xi)\right)\right] \cap \zeta,$$

for all $(u,v) \in \mathbf{S}_z$. Hence

$$\begin{aligned}(f_A \overset{(\xi,\zeta)}{\bigotimes} f_B)(z) &= ((f_A \otimes f_B)(z) \cap \zeta) \cup \xi \\ &= \bigcup_{(x,y) \in \mathbf{S}_z} (f_A(x) \cap f_B(y) \cap \zeta) \cup \xi \\ &= \bigcup_{(x,y) \in \mathbf{S}_z} (f_A(x) \cap f_B(y) \cap \zeta \cap \zeta) \cup \xi \\ &= \bigcup_{(x,y) \in \mathbf{S}_z} [(f_A(x) \cap \zeta) \cap (f_B(y) \cap \zeta) \cap \zeta] \cup \xi \\ &\subseteq \bigcup_{(x,y) \in \mathbf{S}_z} \left[\left(\left[(\bigcap_{a \in x \circ y} f_A(a) \cup \xi) \cap (\bigcap_{b \in x \circ y} f_B(b) \cup \xi)\right] \cap \zeta\right) \cap \zeta\right] \cup \xi \\ &\subseteq \left[\left(\bigcup_{(x,y) \in \mathbf{S}_z} (f_A(z) \cap f_B(z)) \cup \xi\right) \cap \zeta\right] \cup \xi \\ &= [((f_A(z) \cap f_B(z)) \cup \xi) \cap \zeta] \cup \xi \\ &= [(f_A \cap f_B)(z) \cap \zeta] \cup \xi \\ &= (f_A \overset{(\xi,\zeta)}{\bigcap} f_B)(z).\end{aligned}$$

From these two cases, we obtain $f_A \overset{(\xi,\zeta)}{\bigotimes} f_B \subseteq f_A \overset{(\xi,\zeta)}{\bigcap} f_B$. □

In what follows, we recall some particular classes of ordered semihypergroups. As the main point of the present paper, we provide a characterization of these instance classes through (ξ, ζ)-int-soft left hyperideals and (ξ, ζ)-int-soft right hyperideals.

An ordered semihypergroup $\mathbf{S}$ is

1. *regular* if for each element $a \in S$, there exists an element $x \in S$ such that $a \leq a \circ x \circ a$,

2. *intra-regular* if for each element $a \in S$, there exist elements $x, y \in S$ such that $a \leq x \circ a \circ a \circ y$,

3. *left weakly regular* if for every $a \in S$ there exist $x, y \in S$ such that $a \leq x \circ a \circ y \circ a$,

4. *right weakly regular* if for every $a \in S$ there exist $x, y \in S$ such that $a \leq a \circ x \circ a \circ y$.

We note here that if $\mathbf{S}$ is both a left weakly regular and a right weakly regular ordered semihypergroup, then $\mathbf{S}$ said to be a *weakly regular ordered semihypergroup*. Thus, if $\mathbf{S}$ is commutative and weakly regular, then $\mathbf{S}$ is regular.

Lemma 3.13 ([12, Theorem 3.9]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements are equivalent:*

1. $\mathbf{S}$ *is regular.*

2. $R \cap L = (R \circ L]$ *for every right hyperideal* R *and every left hyperideal* L *of* $\mathbf{S}$.

The following theorem provides a characterization of regular ordered semihypergroups in terms of (ξ, ζ)-int-soft left hyperideals and (ξ, ζ)-int-soft right hyperideals.

Theorem 3.14. *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements are equivalent:*

1. $\mathbf{S}$ *is regular.*

2. $f_A \overset{(\xi,\zeta)}{\bigotimes} f_B = f_A \overset{(\xi,\zeta)}{\bigcap} f_B$ *for every* (ξ, ζ)*-int-soft right hyperideal* f_A *and every* (ξ, ζ)*-int-soft left hyperideal* f_B *of* $\mathbf{S}$ *over* U.

Proof. (1) ⇒ (2). Let $a \in S$. Since $\mathbf{S}$ is regular, there exists $x \in S$ such that $a \leq a \circ x \circ a$. That is, there exists $c \in a \circ x$ such that $a \leq c \circ a$. This implies that $\mathbf{S}_a \neq \emptyset$. Then

$$\begin{aligned}
(f_A \overset{(\xi,\zeta)}{\bigotimes} f_B)(a) &= ((f_A \otimes f_B)(a) \cap \zeta) \cup \xi \\
&= \left[\bigcup_{(u,v)\in \mathbf{S}_a} (f_A(u) \cap f_B(v)) \cap \zeta \right] \cup \xi \\
&= \bigcup_{(u,v)\in \mathbf{S}_a} (f_A(u) \cap f_B(v) \cap \zeta) \cup \xi \\
&\supseteq [f_A(c) \cap f_B(a) \cap \zeta] \cup \xi \\
&= (f_A(c) \cup \xi) \cap [(f_B(a) \cap \zeta) \cup \xi] \\
&\supseteq (\bigcap_{c \in a\circ x} f_A(c) \cup \xi) \cap [(f_B(a) \cap \zeta) \cup \xi] \\
&= (\bigcap_{c \in a\circ x} f_A(c) \cup \xi \cup \xi) \cap [(f_B(a) \cap \zeta) \cup \xi] \\
&\supseteq [(f_A(a) \cap \zeta) \cup \xi] \cap [(f_B(a) \cap \zeta) \cup \xi] \\
&\qquad \text{(since } f_A \text{ is a } (\xi,\zeta)\text{-int-soft right hyperideal)} \\
&= [(f_A(a) \cap f_B(a)) \cap \zeta] \cup \xi \\
&= [(f_A \cap f_B)(a) \cap \zeta] \cup \xi \\
&= (f_A \overset{(\xi,\zeta)}{\bigcap} f_B)(a).
\end{aligned}$$

Thus, $f_A \overset{(\xi,\zeta)}{\bigcap} f_B \subseteq f_A \overset{(\xi,\zeta)}{\bigotimes} f_B$. By Lemma 3.12, we conclude that $f_A \overset{(\xi,\zeta)}{\bigotimes} f_B = f_A \overset{(\xi,\zeta)}{\bigcap} f_B$.

(2) ⇒ (1). Let R and L be a right hyperideal and a left hyperideal of $\mathbf{S}$, respectively. Then, by Lemma 3.8, χ_R and χ_L is a (ξ,ζ)-int-soft right hyperideal and a (ξ,ζ)-int-soft left hyperideal of $\mathbf{S}$ over U, respectively. By hypothesis, we obtain $\chi_R \overset{(\xi,\zeta)}{\bigotimes} \chi_L = \chi_R \overset{(\xi,\zeta)}{\bigcap} \chi_L$. Since $\chi_R \overset{(\xi,\zeta)}{\bigotimes} \chi_L = \chi^{(\xi,\zeta)}_{(R\circ L]}$ and $\chi_R \overset{(\xi,\zeta)}{\bigcap} \chi_L = \chi^{(\xi,\zeta)}_{R\cap L}$, we have $\chi^{(\xi,\zeta)}_{(R\circ L]} = \chi^{(\xi,\zeta)}_{R\cap L}$. Let $a \in (R \circ L]$. Then $\chi^{(\xi,\zeta)}_{(R\circ L]}(a) = \zeta$. This implies that $\chi^{(\xi,\zeta)}_{R\cap L}(a) = \zeta$. Hence, $a \in R \cap L$. That is, $(R \circ L] \subseteq R \cap L$. It is not difficult to see that $R \cap L \subseteq (R \circ L]$. This yields $R \cap L = (R \circ L]$. By Lemma 3.13, $\mathbf{S}$ is regular. □

Lemma 3.15 ([12, Theorem 3.12]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements are equivalent:*

1. $\mathbf{S}$ *is intra-regular.*
2. $R \cap L \subseteq (L \circ R]$ *for every right hyperideal* R *and every left hyperideal* L *of* $\mathbf{S}$.

We present a characterization of intra-regular ordered semihypergroups using (ξ, ζ)-int-soft left hyperideals and (ξ, ζ)-int-soft right hyperideals as follows.

Theorem 3.16. *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements are equivalent:*

1. $\mathbf{S}$ *is intra-regular.*
2. $f_B \overset{(\xi,\zeta)}{\bigcap} f_A \subseteq f_B \overset{(\xi,\zeta)}{\bigotimes} f_A$ *for every* (ξ, ζ)*-int-soft right hyperideal* f_A *of* $\mathbf{S}$ *over* U *and every* (ξ, ζ)*-int-soft left hyperideal* f_B *of* $\mathbf{S}$ *over* U.

Proof. (1) $\Rightarrow$ (2). Let $a \in S$. Since $\mathbf{S}$ is intra-regular, there exist $x, y \in S$ such that $a \leq x \circ a \circ a \circ y$. That is, there exist $c \in x \circ a$ and $d \in a \circ y$ such that $a \leq c \circ d$. This implies that $\mathbf{S}_a \neq \emptyset$. Then

$$\begin{aligned}
(f_B \overset{(\xi,\zeta)}{\bigotimes} f_A)(a) &= ((f_B \otimes f_A)(a) \cap \zeta) \cup \xi \\
&= \left(\bigcup_{(u,v) \in \mathbf{S}_a} (f_B(u) \cap f_A(v) \cap \zeta) \right) \cup \xi \\
&\supseteq \left(\left[\bigcap_{c \in x \circ a} f_B(c) \cap \bigcap_{d \in a \circ y} f_A(d) \right] \cap \zeta \cap \zeta \cap \zeta \right) \cup \xi \\
&= \left[\left(\bigcap_{c \in x \circ a} f_B(c) \cap \zeta \right) \cap \left(\bigcap_{d \in a \circ y} f_A(d) \cap \zeta \right) \cap \zeta \right] \cup \xi \\
&\supseteq [(f_B(a) \cup \xi) \cap (f_A(a) \cup \xi) \cap \zeta] \cup \xi \\
&= [((f_B(a) \cap f_A(a)) \cup \xi) \cap \zeta] \cup \xi \\
&= [(f_B(a) \cap f_A(a) \cap \zeta) \cup (\xi \cap \zeta)] \cup \xi \\
&\supseteq (f_B(a) \cap f_A(a) \cap \zeta) \cup \xi \\
&= ((f_B(a) \cap f_A(a)) \cap \zeta) \cup \xi
\end{aligned}$$

$$= ((f_B \cap f_A)(a) \cap \zeta) \cup \xi$$
$$= (f_B \overset{(\xi,\zeta)}{\bigcap} f_A)(a).$$

Thus, $f_B \overset{(\xi,\zeta)}{\bigcap} f_A \subseteq f_B \overset{(\xi,\zeta)}{\bigotimes} f_A$.

(2) $\Rightarrow$ (1). Let R and L be a right hyperideal and a left hyperideal of $\mathbf{S}$, respectively. Then, by Lemma 3.8, χ_R and χ_L is a (ξ,ζ)-int-soft right hyperideal and a (ξ,ζ)-int-soft left hyperideal of $\mathbf{S}$ over U, respectively. Thus, by our hypothesis, we obtain $\chi_R \overset{(\xi,\zeta)}{\bigcap} \chi_L \subseteq \chi_L \overset{(\xi,\zeta)}{\bigotimes} \chi_R$. Then, we have that $\chi^{(\xi,\zeta)}_{R\cap L} \subseteq \chi^{(\xi,\zeta)}_{(L\circ R]}$. since $\chi_L \overset{(\xi,\zeta)}{\bigotimes} \chi_R = \chi^{(\xi,\zeta)}_{(L\circ R]}$ and $\chi_R \overset{(\xi,\zeta)}{\bigcap} \chi_L = \chi^{(\xi,\zeta)}_{R\cap L}$. Let $a \in R \cap L$. Then, $\chi^{(\xi,\zeta)}_{R\cap L}(a) = \zeta$. This implies that $\chi^{(\xi,\zeta)}_{(L\circ R]}(a) = \zeta$. Hence, $a \in (L \circ R]$. Thus, we have $R \cap L \subseteq (L \circ R]$. By Lemma 3.15, $\mathbf{S}$ is intra-regular. □

Lemma 3.17 ([7]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements are equivalent:*

1. $\mathbf{S}$ *is left weakly regular.*
2. $L = (L \circ L]$ *for every left hyperideal* L *of* $\mathbf{S}$.
3. $\mathrm{L}(a) = (\mathrm{L}(a) \circ \mathrm{L}(a)]$ *for every* $a \in S$.

Lemma 3.18 ([7]). *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements are equivalent:*

1. $\mathbf{S}$ *is right weakly regular.*
2. $R = (R \circ R]$ *for every right hyperideal* R *of* $\mathbf{S}$.
3. $\mathrm{R}(a) = (R(a) \circ \mathrm{R}(a)]$ *for every* $a \in S$.

In the following theorem, we characterize Left weakly regular ordered semihypergroups.

Theorem 3.19. *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements are equivalent:*

1. $\mathbf{S}$ *is left weakly regular.*
2. $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A = f_A^{(\xi,\zeta)}$ *for every int-soft left hyperideal* f_A *of* $\mathbf{S}$ *over* U.

Proof. (1) ⇒ (2). Assume that f_A is a (ξ,ζ)-int-soft left hyperideal of $\mathbf{S}$ over U. By Theorem 3.6, we have $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A \subseteq f_A^{(\xi,\zeta)}$. For the other inclusion, let $a \in S$. Since $\mathbf{S}$ is left weakly regular, there exist $x, y \in S$ such that $a \leq (x \circ a) \circ (y \circ a)$. That is, there exist $u \in (x \circ a)$ and $v \in (y \circ a)$ such that $a \leq u \circ v$. Thus, $\mathbf{S}_a \neq \emptyset$. It follows that

$$\begin{aligned}
(f_A \overset{(\xi,\zeta)}{\bigotimes} f_A)(a) &= ((f_A \otimes f_A)(a) \cap \zeta) \cup \xi \\
&= \bigcup_{(p,q)\in \mathbf{S}_a} (f_A(p) \cap f_A(q) \cap \zeta) \cup \xi \\
&\supseteq (f_A(u) \cap f_A(v) \cap \zeta) \cup \xi \\
&= (f_A(u) \cup \xi) \cap (f_A(v) \cup \xi) \cap \zeta \\
&\supseteq \left[(\bigcap_{u \in x \circ a} f_A(u) \cup \xi) \cap (\bigcap_{v \in y \circ a} f_A(v) \cup \xi) \right] \cap \zeta \\
&= \left[(\bigcap_{u \in x \circ a} f_A(u) \cup \xi \cup \xi) \cap (\bigcap_{v \in y \circ a} f_A(v) \cup \xi \cup \xi) \right] \cap \zeta \\
&\supseteq [(f_A(a) \cap \zeta \cup \xi) \cap (f_A(a) \cap \zeta \cup \xi)] \cap \zeta \\
&= [(f_A(a) \cap \zeta) \cup \xi] \cap [(f_A(a) \cap \zeta) \cup \xi] \\
&= (f_A(a) \cap \zeta) \cup \xi \\
&= f_A^{(\xi,\zeta)}(a).
\end{aligned}$$

Therefore, $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A \supseteq f_A^{(\xi,\zeta)}$. Altogether, we have $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A = f_A^{(\xi,\zeta)}$.

(2) ⇒ (1). Let $a \in S$ and $b \in \mathrm{L}(a)$. Since $\mathrm{L}(a)$ is a left hyperideal of $\mathbf{S}$, we have $\chi_{\mathrm{L}(a)}$ is a (ξ,ζ)-int-soft left hyperideal of $\mathbf{S}$ over U. By our assumption, we obtain

$$(\chi_{\mathrm{L}(a)} \overset{(\xi,\zeta)}{\bigotimes} \chi_{\mathrm{L}(a)})(b) = \chi_{\mathrm{L}(a)}^{(\xi,\zeta)}(b) = (\chi_{\mathrm{L}(a)}(b) \cap \zeta) \cup \xi = \zeta.$$

That is, $(\chi_{\mathrm{L}(a)} \overset{(\xi,\zeta)}{\bigotimes} \chi_{\mathrm{L}(a)})(b) = \zeta$. But, by Lemma 3.11, we have

$$(\chi_{\mathrm{L}(a)} \overset{(\xi,\zeta)}{\bigotimes} \chi_{\mathrm{L}(a)})(b) = \chi_{(\mathrm{L}(a)\circ \mathrm{L}(a)]}^{(\xi,\zeta)}(b).$$

Thus, $\chi_{(\mathrm{L}(a)\circ \mathrm{L}(a)]}^{(\xi,\zeta)}(b) = \zeta$. This implies that $b \in (\mathrm{L}(a) \circ \mathrm{L}(a)]$. Hence, $\mathrm{L}(a) \subseteq (\mathrm{L}(a) \circ \mathrm{L}(a)]$. On the other hand $(\mathrm{L}(a) \circ \mathrm{L}(a)] \subseteq (S \circ \mathrm{L}(a)] \subseteq (\mathrm{L}(a)] = \mathrm{L}(a)$.

Altogether, we have $\mathrm{L}(a) = (\mathrm{L}(a) \circ \mathrm{L}(a)]$. By Lemma 3.17, $\mathbf{S}$ is left weakly regular. □

Similarly, by applying Lemmas 3.11 and 3.18, we obtain the following theorem.

Theorem 3.20. *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then the following statements are equivalent:*

1. $\mathbf{S}$ *is right weakly regular.*

2. $f_A \overset{(\xi,\zeta)}{\bigotimes} f_A = f_A^{(\xi,\zeta)}$, *for every int-soft right hyperideal* f_A *of* $\mathbf{S}$ *over* U.

4 Conclusion

We presented a generalized concept of int-soft left (right) hyperideals in an ordered semihypergroup $\mathbf{S}$, so-called (ξ, ζ)-int-soft left (right) hyperideal of $\mathbf{S}$ over U. We used such generalizations to characterize regular, intra-regular, and left (right) weakly ordered semihypergroups. For future work, we will apply these notions and results to study related notions in other soft algebraic structures.

References

[1] S. M. Anvariyeh, S. Mirvakili, B. Davvaz, On Γ-hyperideals in Γ-semihypergroups, Carpathian J. Math., **16**, no. 1, (2010), 11–23.

[2] L. K. Ardekani, B. Davvaz, Ordered semihypergroup constructions, Boletín de Matemáticas, **25**, no. 2, (2018), 77–99.

[3] P. Bonansinga, P. Corsini, On semihypergroup and hypergroup homomorphisms, Boll. Un. Mat. Ital. B, **6**, no. 1, (1982), 717–727.

[4] P. Corsini, Prolegomena of hypergroup theory, Aviani editore, 1993.

[5] P. Corsini, V. Leoreanu, Applications of hyperstructure theory, **5,** Springer Science & Business Media, 2003.

[6] B. Davvaz, P. Corsini, T. Changphas, Relationship between ordered semihypergroups and ordered semigroups by using pseudoorder, European J. Combin., **44**, (2015), 208–217.

[7] B. Davvaz, A. Khan, M. Farooq, Int-soft structures applied to ordered semihypergroups, Le Matematiche, **73**, no. 2, (2018), 235–259.

[8] M. Farooq, A. Khan, M. Izhar, Characterizations of ordered semihypergroups via (M, N)-int-soft bi-hyperideals, Algebraic Structures and Their Applications, (2021), 23–41.

[9] K. Hila, B. Davvaz, J. Dine, Study on the structure of Γ-semihypergroups, Comm. Algebra, **40**, no. 8, (2012), 2932–2948.

[10] D. Heidari, B. Davvaz, On ordered hyperstructures, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl., **73**, no. 2, (2011), 85–96.

[11] Z. Gu, X. Tang, Ordered regular equivalence relations on ordered semihypergroups, J. Algebra, **450**, (2016), 384–397.

[12] N. Kehayopulu, From ordered semigroups to ordered hypersemigroups, Turkish J. Math., **43** (2019), 21–35.

[13] A. Khan, M. Farooq, B. Davvaz, Int-soft left (right) hyperideals of ordered semihypergroups, (2017), submitted.

[14] A. Khan, M. Farooq, B. Davvaz, Int-soft interior hyperideals of ordered semihypergroups, Int. J. Anal. Appl., **14**, no. 2, (2017), 193–202.

[15] S. Naz, M. Shabir, On soft semihypergroups, J. Intell. Fuzzy Syst., **26**, no. 5, (2014), 2203–2213.

[16] T. Mahmood, Some contributions to semihypergroups, Ph. D Thesis, Department of Mathematics, Quaid-i-Azm University, Islamabad, Pakistan, 2021.

[17] F. Marty, Sur une generalization de la notion de groupe, In 8th congress Math. Scandinaves, (1934), 45–49.

[18] D. Molodtsov, Soft set theory—first results, Comput. Math. Appl., **37**, no. 4–5, (1999), 19–31.

[19] A. Sezgin, N. Çağman, A. O. Atagün, Soft intersection interior ideals, quasi-ideals and generalized bi-ideals: A new approach to semigroup theory II, J. Multi-valued Log. S., **23**, nos. 1–2, (2014), 161–207.