International Journal of Mathematics and
Computer Science, **17**(2022), no. 1, 207–216

Non-uniform bound on normal approximation for call function of locally dependent random variables

Suporn Jongpreechaharn[1], **Kritsana Neammanee**[1,2]

[1]Department of Mathematics and Computer Science
Faculty of Science
Chulalongkorn University
Bangkok, Thailand

[2]Centre of Excellence in Mathematics
Commission on Higher Education
Bangkok, Thailand

email: suporn.jon@student.chula.ac.th, kritsana.n@chula.ac.th

(Received July 2, 2021, Accepted August 8, 2021)

Abstract

In this paper, we concentrate on a mean for call function of random variables which is useful and important, especially in finance. We propose a non-uniform bound from a normal approximation under local dependence by using the Stein's method. Finally, we illustrate an application on a collateralized debt obligation.

1 Introduction and main results

A call function, defined by $(x-k)^+ = \max\{x-k, 0\}$, where x and k are real numbers, is useful and important, especially in finance. In 2009, Karoui and Jiao [8] and Karoui, Jiao and Kurtz [9] approximated a loss on each tranche of a collateralized debt obligation (CDO) in the form of $E(W-k)^+$, where W is a sum of independent random variables and k is a real number.

Key words and phrases: non-uniform bound, Stein's method, local dependence, call function, normal approximation.
AMS (MOS) Subject Classifications: 60F05.
ISSN 1814-0432, 2022, http://ijmcs.future-in-tech.net

They used the Stein's method to obtain error bounds from approximating $E(W-k)^+$ by a mean for call function of Poisson and normal random variables. The refined bounds are proposed by Yonghint and Neammanee [10] for Poisson approximation and Jongpreechaharn and Neammanee [7] for normal approximation under the same assumption. For a normal approximation, Jongpreechaharn and Neammanee [6] extended the assumption to a local dependence motivated by Chen and Shao [4].

Let $X_1, X_2, \ldots, X_n$ be random variables with zero means and finite variances. For $A \subseteq \{1, 2, \ldots, n\}$, let X_A denote $\{X_i : i \in A\}$ and $A^c = \{i \in \{1, 2, \ldots, n\} : i \notin A\}$. We say that $X_1, X_2, \ldots, X_n$ satisfy the local dependence condition if there exists a partition $\{A_i\}_{i=1}^{l_n}$ of $\{1, 2, \ldots, n\}$ such that for each $i \in \{1, 2, \ldots, l_n\}$, X_{A_i} and $X_{A_i^c}$ are independent.

Let Z be a standard normal random variable and let k be a positive real number. Denote $W = \sum_{i=1}^{n} X_i$ and assume that W has a unit variance. Suppose that $X_1, X_2, \ldots, X_n$ satisfy the local dependence condition. Jongpreechaharn and Neammanee [6] approximated $E(W-k)^+$ by $E(Z-k)^+$ and proposed a uniform bound in the following theorem.

Theorem 1.1 ([6]). *For each $i = 1, 2, \ldots, l_n$, let $Y_i = \sum_{j \in A_i} X_j$. Under the local dependence condition, we have*

$$\sup_{k>0} \left|E(W-k)^+ - E(Z-k)^+\right| \leq 24.97 \sum_{i=1}^{l_n} E|Y_i|^3 + 0.80 \left(\sum_{i=1}^{l_n} EY_i^4\right)^{1/2} + \left(l_n EW^4 \sum_{i=1}^{l_n} EY_i^6\right)^{1/2},$$

where $EW^4 \leq \sum_{i=1}^{l_n} EY_i^4 + 3$.

In this work, we use Stein's method introduced in Section 2 to obtain a non-uniform bound as follows.

Theorem 1.2. *For each $i = 1, 2, \ldots, l_n$, let $Y_i = \sum_{j \in A_i} X_j$. Under the local dependence condition and $k \geq 2$, we have*

$$\left|E(W-k)^+ - E(Z-k)^+\right| \leq C_1(k) \sum_{i=1}^{l_n} E|Y_i|^3 + C_2(k) \left(\sum_{i=1}^{l_n} EY_i^4\right)^{1/2} + C_3(k) \left(l_n \sum_{i=1}^{l_n} EY_i^6\right)^{1/2},$$

where $C_1(k) = \dfrac{5.5e^{-k^2/2}}{\sqrt{2\pi}k^2} + \dfrac{5.5}{k} + \dfrac{1}{2k^2}$, $C_2(k) = \dfrac{e^{-k^2/2}}{\sqrt{2\pi}k^2} + \dfrac{1}{k}$,

$$C_3(k) = \frac{3}{k}\sqrt{EW^6} + \frac{15.69}{k}\sqrt{EW^4} + \frac{18.24}{k},\ EW^4 \le \sum_{i=1}^{l_n} EY_i^4 + 3$$

and $$EW^6 \le \sum_{i=1}^{l_n} EY_i^6 + 15\sum_{i=1}^{l_n} EY_i^4 + 10\left(\sum_{i=1}^{l_n} EY_i^3\right)^2 + 15.$$

In addition, applying Theorem 1.2 with independent and identically distributed random variables gives a convergence rate of the non-uniform bound. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with $\Pr(X_1 = 1/\sqrt{n}) = \Pr(X_1 = -1/\sqrt{n}) = 1/2$ and $W = \sum_{i=1}^n X_i$. Thus, by choosing $A_i = \{i\}$ for $i = 1, 2, \ldots, n$ in Theorem 1.2, we obtain the following corollary.

Corollary 1.3. *Adopting the notations in Theorem 1.2, we have for $k \ge 2$ that*

$$\left|E(W-k)^+ - E(Z-k)^+\right| \le \frac{a_k}{\sqrt{n}},$$

where $a_k = \frac{6.5e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{2k^2} + \frac{24.74}{k} + \frac{3}{k}\left(\frac{1}{n^2} + \frac{25}{n} + 15\right)^{1/2} + \frac{15.69}{k}\left(\frac{1}{n} + 3\right)^{1/2}.$

Observe that, a non-uniform bound steadily declines as k gets larger.

2 Stein's method on normal approximation for call function

In 1972, Stein [11] discovered a powerful method for finding a bound from approximation (see also [1] and [3]). The foundation of Stein's method is the differential equation called the Stein equation. Let Z be a standard normal random variable. The Stein equation on normal approximation for a given function h is

$$xf(x) - f'(x) = h(x) - Eh(Z). \tag{2.1}$$

In this work, we let $h(x) = (x-k)^+$ for a fixed positive real number k. Thus, (2.1) becomes

$$xf(x) - f'(x) = (x-k)^+ - E(Z-k)^+. \tag{2.2}$$

From (2.2), we substituting x by any random variable W and taking an expectation on both sides of the equation, we obtain

$$EWf_k(W) - Ef_k'(W) = E(W-k)^+ - E(Z-k)^+, \tag{2.3}$$

where f_k is the Stein solution of (2.2). From (2.3), a targeted bound for $|E(W-k)^+ - E(Z-k)^+|$ can be verified from $|EWf_k(W) - Ef_k'(W)|$ instead. This is a procedure of the Stein's method on normal approximation. In order to verify a bound for $|EWf_k(W) - Ef_k'(W)|$, the properties of the Stein solution f_k and its derivative f_k' are essential. Notice that

$$f_k(x) = \begin{cases} \sqrt{2\pi}e^{x^2/2}E(Z-k)^+\Phi(x), & \text{if } x \le k, \\ 1 - \sqrt{2\pi}e^{x^2/2}[k + E(Z-k)^+]\Phi(-x), & \text{if } x > k \end{cases}$$

and

$$f_k'(x) = \begin{cases} E(Z-k)^+\left(1 + \sqrt{2\pi}x\Phi(x)e^{x^2/2}\right), & \text{if } x \le k, \\ [k + E(Z-k)^+]\left(1 - \sqrt{2\pi}x\Phi(-x)e^{x^2/2}\right), & \text{if } x > k, \end{cases}$$

(see also [6] and [7]), where Φ is the cumulative distribution function of Z. Next, we propose some properties of them in Propositions 2.1 and 2.2 which are used to prove the main theorem.

Proposition 2.1. *For $x \le k$ and $k \ge 1$, we have $|f_k(x)| \le \frac{1}{k^2}$.*

Proof. Let $x \le k$ and $k \ge 1$. Note that $f_k(x) \ge 0$. If $|x| \le k$, then by the fact that

$$E(Z-k)^+ \le \frac{e^{-k^2/2}}{\sqrt{2\pi}k^2} \quad \text{for } k \ge 1, \tag{2.4}$$

(see [7], p.3502), we have $f_k(x) \le \frac{1}{k^2}$. Suppose that $x < -k$. By the fact that

$$\Phi(-a) \le \frac{e^{-a^2/2}}{\sqrt{2\pi}a} \quad \text{for } a > 0, \tag{2.5}$$

(see [11], p.23) and (2.4), we have $f_k(x) \le \frac{E(Z-k)^+}{-x} < \frac{e^{-k^2/2}}{\sqrt{2\pi}k^3}$. Hence $|f_k(x)| \le \frac{1}{k^2}$ for $x \le k$. □

Let $\|g\| = \sup_{x\in\mathbb{R}} |g(x)|$ for every real-valued function g on $\mathbb{R}$.

Proposition 2.2. *For $k \ge 1$, we have $\|f_k'\| \le \frac{e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{k}$.*

Proof. Let $x \le k$. If $x < 0$, by (2.5), we have $0 \le 1 + \sqrt{2\pi}x\Phi(x)e^{x^2/2}$. Hence, $0 \le 1 + \sqrt{2\pi}x\Phi(x)e^{x^2/2} \le 1$. By this inequality and (2.4), we have $0 \le f_k'(x) \le \frac{e^{-k^2/2}}{\sqrt{2\pi}k^2}$ for $x < 0$. Suppose that $x \ge 0$. Then $f_k'(x) \ge 0$. By (2.4), we have $f_k'(x) \le E(Z-k)^+\left(1+\sqrt{2\pi}ke^{k^2/2}\right) \le \frac{e^{-k^2/2}}{\sqrt{2\pi}k^2}+\frac{1}{k}$. Thus, for $x \ge 0$, we have $0 \le f_k'(x) \le \frac{e^{-k^2/2}}{\sqrt{2\pi}k^2}+\frac{1}{k}$. Therefore,

$$0 \le f_k'(x) \le \frac{e^{-k^2/2}}{\sqrt{2\pi}k^2}+\frac{1}{k} \quad \text{for } x \le k.$$

Assume that $x > k$. By (2.5), we obtain $\sqrt{2\pi}x\Phi(-x)e^{x^2/2} \le 1$. Then $f_k'(x) \ge 0$. On the other hand, note that $\Phi(-a) \ge \frac{e^{-a^2/2}}{\sqrt{2\pi}}\left(\frac{1}{a}-\frac{1}{a^3}\right)$ for $a > 0$, (see [7], p.3502). Thus $\sqrt{2\pi}x\Phi(-x)e^{x^2/2} \ge 1-\frac{1}{x^2} > 1-\frac{1}{k^2}$. From this fact and (2.4), we obtain $f_k'(x) \le \frac{k+E(Z-k)^+}{k^2} \le \frac{1}{k}+\frac{e^{-k^2/2}}{\sqrt{2\pi}k^4}$. Hence

$$0 \le f_k'(x) \le \frac{e^{-k^2/2}}{\sqrt{2\pi}k^4}+\frac{1}{k} \quad \text{for } x > k.$$

Combining these two cases, we obtain $\|f_k'\| \le \frac{e^{-k^2/2}}{\sqrt{2\pi}k^2}+\frac{1}{k}$ for $k \ge 1$. □

Next, we use the Stein's method and properties of the Stein solution in Propositions 2.1 and 2.2 to provide a non-uniform bound for $|E(W-k)^+ - E(Z-k)^+|$.

3 Proof of the results

Proof of Theorem 1.2. By the proof of Theorem 1.1, we have

$$EWf_k(W) - Ef_k'(W) = R_1 + R_2 + R_3, \tag{3.6}$$

where
$$|R_1| \le \|f_k'\|\left[\left(\sum_{i=1}^{l_n} EY_i^4\right)^{1/2} + 2\sum_{i=1}^{l_n} E|Y_i|^3\right],$$

$$|R_2| \le 2\|f_k'\|\sum_{i=1}^{l_n} E|Y_i|^3,$$

$$|R_3| \le E\int_{|t|\le 1} |f_k'(W+t) - f_k'(W)|\,\widehat{K}(t)dt$$

and
$$\widehat{K}(t) = \sum_{i=1}^{l_n} Y_i\left[\mathbb{I}\left(-Y_i \le t < 0\right) - \mathbb{I}\left(0 \le t \le -Y_i\right)\right].$$

From Proposition 2.2, we obtain

$$|R_1| \leq \left(\frac{e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{k} \right) \left[\left(\sum_{i=1}^{l_n} EY_i^4 \right)^{1/2} + 2 \sum_{i=1}^{l_n} E|Y_i|^3 \right] \tag{3.7}$$

and
$$|R_2| \leq 2 \left(\frac{e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{k} \right) \sum_{i=1}^{l_n} E|Y_i|^3. \tag{3.8}$$

For R_3, we use the truncation technique to rewrite R_3 by

$$|R_3| \leq R_{3,1} + R_{3,2} + R_{3,3}, \tag{3.9}$$

where
$$R_{3,1} = E \int_{|t|\leq 1} |f_k'(W+t) - f_k'(W)| \, \mathbb{I}(W > k) \widehat{K}(t) dt,$$
$$R_{3,2} = E \int_{|t|\leq 1} |f_k'(W+t) - f_k'(W)| \, \mathbb{I}(W + t > k, W \leq k) \widehat{K}(t) dt$$
and
$$R_{3,3} = E \int_{|t|\leq 1} |f_k'(W+t) - f_k'(W)| \, \mathbb{I}(W + t \leq k, W \leq k) \widehat{K}(t) dt.$$

By Proposition 1 of [6], we have $|f_k'(x+t) - f_k'(x)| \leq 2x^2|t| + 10.46|x||t| +12.16|t|$ for $|t| \leq 1$ and $x \in \mathbb{R}$. This implies that

$$R_{3,1} \leq E \left(2W^2 + 10.46|W| + 12.16 \right) \mathbb{I}(W > k) \int_{|t|\leq 1} |t| \widehat{K}(t) dt$$
and
$$R_{3,2} \leq E \left(2W^2 + 10.46|W| + 12.16 \right) \mathbb{I}(W > k-1) \int_{|t|\leq 1} |t| \widehat{K}(t) dt.$$

By the Hölder's inequality, the Markov's inequality and the fact in [6] that $E \left(\int_{|t|\leq 1} |t| \widehat{K}(t) dt \right)^2 \leq \frac{l_n}{4} \sum_{i=1}^{l_n} EY_i^6$, we obtain

$$\begin{aligned} R_{3,1} \leq & \Big[2 \left(EW^4 \mathbb{I}(W > k) \right)^{1/2} + 10.46 \left(EW^2 \mathbb{I}(W > k) \right)^{1/2} \\ & + 12.16 \left(\Pr(W > k) \right)^{1/2} \Big] \left[E \left(\int_{|t|\leq 1} |t| \widehat{K}(t) dt \right)^2 \right]^{1/2} \\ \leq & \frac{1}{k} \left(\sqrt{EW^6} + 5.23\sqrt{EW^4} + 6.08 \right) \left(l_n \sum_{i=1}^{l_n} EY_i^6 \right)^{1/2}. \end{aligned} \tag{3.10}$$

Using the same argument for bounding $R_{3,1}$, we obtain

$$R_{3,2} \leq \frac{1}{k-1}\left(\sqrt{EW^6} + 5.23\sqrt{EW^4} + 6.08\right)\left(l_n \sum_{i=1}^{l_n} EY_i^6\right)^{1/2}.$$

Since $k \geq 2$, $\frac{1}{k-1} \leq \frac{2}{k}$. From this fact, we have

$$R_{3,2} \leq \frac{2}{k}\left(\sqrt{EW^6} + 5.23\sqrt{EW^4} + 6.08\right)\left(l_n \sum_{i=1}^{l_n} EY_i^6\right)^{1/2}. \tag{3.11}$$

Thus, it remains to consider $R_{3,3}$. By the proof of Proposition 1 of [6], we have $|f_k'(x+t) - f_k'(x)| \leq \|f_k'\|\,|x||t| + |f_k(x+t)||t|$ for $x+t \leq k$ and $x \leq k$. By this inequality and Propositions 2.1 and 2.2, we have

$$|f_k'(x+t) - f_k'(x)| \leq \left(\frac{e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{k}\right)|x||t| + \frac{|t|}{k^2}.$$

This implies that

$$R_{3,3} \leq E\left[\left(\frac{e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{k}\right)|W| + \frac{1}{k^2}\right]\int_{|t|\leq 1} |t|\widehat{K}(t)dt.$$

By the fact that $\int_{|t|\leq 1} |t|\widehat{K}(t)dt \leq \frac{1}{2}\sum_{i=1}^{l_n} |Y_i|(Y_i^2 \wedge 1)$, where $a \wedge b = \min\{a, b\}$ for any real numbers a and b, (see [6]), we obtain

$$R_{3,3} \leq \frac{1}{2}\left(\frac{e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{k}\right)\sum_{i=1}^{n} E|WY_i|(Y_i^2 \wedge 1) + \frac{1}{2k^2}\sum_{i=1}^{n} E|Y_i|^3.$$

By modification of the proof of r_4 in Theorem 2.2 of [4] on p.2013, we obtain $E|WY_i|(Y_i^2 \wedge 1) \leq 3E|Y_i|^3$. This implies that

$$R_{3,3} \leq \left(\frac{3e^{-k^2/2}}{2\sqrt{2\pi}k^2} + \frac{3}{2k} + \frac{1}{2k^2}\right)\sum_{i=1}^{l_n} E|Y_i|^3. \tag{3.12}$$

Combining (3.6)–(3.12), we obtain

$$\begin{aligned}
&\left|E(W-k)^+ - E(Z-k)^+\right| \\
&\leq \left(\frac{5.5e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{5.5}{k} + \frac{1}{2k^2}\right)\sum_{i=1}^{l_n} E|Y_i|^3 + \left(\frac{e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{k}\right)\left(\sum_{i=1}^{l_n} EY_i^4\right)^{1/2} \\
&\quad + \left(\frac{3}{k}\sqrt{EW^6} + \frac{15.69}{k}\sqrt{EW^4} + \frac{18.24}{k}\right)\left(l_n \sum_{i=1}^{l_n} EY_i^6\right)^{1/2}.
\end{aligned} \tag{3.13}$$

Next, we find bounds for EW^4 and EW^6. Notice that

$$EW^4 \le \sum_{i=1}^{l_n} EY_i^4 + 3, \tag{3.14}$$

(see [6]). For EW^6, we first notice that Y_i and Y_j are independent for distinct i and j and $EY_i = 0$. Then $\sum_{i=1}^{l_n} EY_i^2 = EW^2 = 1$, (see [6]). This implies that

$$\begin{aligned} EW^6 &= \sum_{i=1}^{l_n} EY_i^6 + 15 \sum_{i=1}^{l_n} \sum_{\substack{j=1 \\ j\neq i}}^{l_n} EY_i^4 EY_j^2 + 10 \sum_{i=1}^{l_n} \sum_{\substack{j=1 \\ j\neq i}}^{l_n} EY_i^3 EY_j^3 \\ &\quad + 15 \sum_{i=1}^{l_n} \sum_{\substack{j=1 \\ j\neq i}}^{l_n} \sum_{\substack{l=1 \\ l\neq i \\ l\neq j}}^{l_n} EY_i^2 EY_j^2 EY_l^2 \\ &\le \sum_{i=1}^{l_n} EY_i^6 + 15 \sum_{i=1}^{l_n} EY_i^4 + 10 \left(\sum_{i=1}^{l_n} EY_i^3 \right)^2 + 15. \end{aligned} \tag{3.15}$$

Combining (3.13)–(3.15), the proof is complete. □

Proof of Corollary 1.3. For $r \ge 1$, we have $E|Y_1|^r = E\left|X_1\right|^r = \frac{1}{n^{r/2}}$. This implies that $EW^4 \le \frac{1}{n} + 3$ and $EW^6 \le \frac{1}{n^2} + \frac{25}{n} + 15$. From these inequalities and Theorem 1.2, we have Corollary 1.3 as required. □

4 Application on collateralized debt obligation

Consider a collateralized debt obligation (CDO) containing n assets. A total loss of a CDO at time T is defined by $L(T) = \frac{1}{n} \sum_{i=1}^{n} (1 - R_i)\mathbb{I}(\tau_i \le T)$, where $\mathbb{I}(A)$ is an indicator function of A, R_i and τ_i are deterministic recovery rate and stochastic default time for the i^{th} asset, respectively. One is interested in calculation of a loss on each tranche of the CDO given by $(L(T) - AP)^+ - (L(T) - DP)^+$, where AP and DP are attachment and detachment points of the tranche, respectively, (see [2] and [5] for more details). Therefore, we concentrate on $E\left(L(T) - a\right)^+$, where a is a positive real number.

Let $X_i = \frac{(1-R_i)[\mathbb{I}(\tau_i \le T) - E\mathbb{I}(\tau_i \le T)]}{n\sqrt{\operatorname{Var} L(T)}}$ and $W = \sum_{i=1}^{n} X_i$. Then W has zero mean

and unit variance. We classify assets in a CDO by their corresponding workplace. Let l_n be a number of workplaces and let A_i be a group of assets from the same workplace for $i = 1, 2, \dots, l_n$. Then $\{\mathbb{I}(\tau_i \le T)\}_{i=1}^n$ satisfies the local dependence condition.
To simplify the verification, assume that $\{\mathbb{I}(\tau_i \le T)\}_{i=1}^n$ are identically Bernoulli random variables with $\Pr(\mathbb{I}(\tau_i \le T) = 1) = \Pr(\mathbb{I}(\tau_i \le T) = 0) = 1/2$, $E\mathbb{I}(\tau_i \le T)\mathbb{I}(\tau_j \le T) = 1/2$ and $R = R_i$. Suppose that every group A_i contains c assets. Thus $l_n = n/c$.

Theorem 4.1. *Adopting notations defined above, we have*

1. $\sup_{k>0} |E(W-k)^+ - E(Z-k)^+| \le b\sqrt{\frac{c}{n}}$, *where* $b = 25.77 + \left(\frac{c}{n}+3\right)^{1/2}$,
2. *for* $k \ge 2$, *we have* $|E(W-k)^+ - E(Z-k)^+| \le b_k\sqrt{\frac{c}{n}}$,
   where $b_k = \frac{6.5e^{-k^2/2}}{\sqrt{2\pi}k^2} + \frac{1}{2k^2} + \frac{24.74}{k} + \frac{3}{k}\left(\frac{c^2}{n^2} + \frac{25c}{n} + 15\right)^{1/2} + \frac{15.69}{k}\left(\frac{c}{n}+3\right)^{1/2}$.

Proof. Note that $E\mathbb{I}(\tau_i \le T) = 1/2$ and

$$\operatorname{Var} L(T) = \frac{(1-R)^2}{n^2} \sum_{i=1}^{l_n} \sum_{j \in A_i} \sum_{l \in A_i} \operatorname{Cov}\left(\mathbb{I}(\tau_j \le T), \mathbb{I}(\tau_l \le T)\right) = \frac{c(1-R)^2}{4n}.$$

Thus $X_i = \frac{2\mathbb{I}(\tau_i \le T) - 1}{\sqrt{cn}}$ and hence $E|Y_i|^r = E\left|\sum_{j \in A_i} X_j\right|^r = \left(\frac{c}{n}\right)^{r/2}$ for $r \ge 1$. This implies that $EW^4 \le \frac{c}{n} + 3$ and $EW^6 \le \frac{c^2}{n^2} + \frac{25c}{n} + 15$. From these inequalities and Theorems 1.1 and 1.2, we have Theorem 4.1 as required. □

Table 1 shows constants b and b_k, where we bound $\frac{1}{n^2} \le \frac{1}{n} \le 1$ and vary the values of c as shown. Observe that, when the number of correlated random

c	b	b_k				
		$k=5$	$k=10$	$k=50$	$k=100$	$k=1000$
≤ 5	28.5985	21.5508	10.7704	2.1533	1.0766	0.1077
≤ 10	29.3756	27.7453	13.8677	2.7728	1.3864	0.1387
≤ 50	33.0502	64.6288	32.3094	6.4611	3.2305	0.3231

Table 1: Comparison of the constant b for uniform bound and b_k for non-uniform bound.

variables c is increased, we always can find a constant k such that a non-uniform bound is sharper than a uniform bound.

Acknowledgment. The first author appreciates the financial support by Human Resource Development in Science Project (Science Achievement Scholarship of Thailand, SAST).

References

[1] A. D. Barbour, L. H. Y. Chen, An Introduction to Stein's Method, 2005.

[2] X. Burtschell, J. Gregory, J.-P. Laurent, A comparative analysis of CDO pricing models under the factor copula framework, J. Deriv., **16**, (2009), 9–37.

[3] L. H. Y. Chen, L. Goldstein, Q. M. Shao, Normal Approximation by Stein's Method, 2010.

[4] L. H. Y. Chen, Q. M. Shao, Normal approximation under local dependence, Ann. Probab., **32**, no. 3, (2004), 1985–2028.

[5] P. Glasserman, S. Suchintabandid, Correlation expansion for CDO pricing, J. Bank. Finance, **31**, (2007), 1375–1398.

[6] S. Jongpreechaharn, K. Neammanee, Normal approximation for call function of locally dependent random variables, submitted, (2021).

[7] S. Jongpreechaharn, K. Neammanee, Normal approximation for call function via Stein's method, Commun. Stat.-Theor. M., **48**, no. 14, (2019), 3498–3517.

[8] N. El Karoui, Y. Jiao, Stein's method and zero bias transformation for CDO tranche pricing, Financ. Stoch., **13**, (2009), 151–180.

[9] N. El Karoui, Y. Jiao, D. Kurtz, Gaussian and Poisson approximation: application to CDOs tranche pricing, J. Comput. Finance, **2**, (2008), 31–58.

[10] K. Neammanee, N. Yonghint, Poisson approximation for call function via Stein-Chen method, Bulletin Malays. Math. Sci. Soc., **43**, (2020), 1135–1152.

[11] C. Stein, Approximate Computation of Expectations, 1986.