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Modern Roman Domination of Corona of Cycle graph with Some Certain Graphs

Saba Salah Majeed[1], **Ahmed Abed Ali Omran**[2],
Manal Naji Yaqoob[1]

[1]Department of Applied Sciences
University of Technology
Baghdad, Iraq

[2]Department of Mathematics
College of Education for Pure Sciences
University of Babylon
Babylon, Iraq

email: as.18.95@grad.uotechnology.edu.iq,
pure.ahmed.omran@uobabylon.edu.iq,
Manal.N.Alharere@uotechnology.edu.iq

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Abstract

Throughout this paper, we determine the modern Roman domination number by the cycle graph. Moreover, we determine the corona operation to two graphs: the cycle graph and one of the path, complete, cycle, null, complete bipartite, star, or wheel graph.

1 Introduction

One of the most important concepts that were used as an active tool in graph theory is the concept of dominance because, through it, one can control any working system. In 1962, Berge [8] initiated this concept. Afterwards, this notion started to appear in different kinds of forms. In Mathematics, this

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concept appeared in many fields, including, for example, fuzzy graph ([15], [19]-[26]), topological graph [12]. More recently, many new definitions of this concept have been introduced by adding some conditions on the dominating set, out of dominating set, or on the two together as in ([1]-[7], [11], [14], [17]-[18]). A dominating set is a subset of a graph such that every vertex in it either belongs to this set or is adjacent to a vertex in this set. A minimum dominating set is a dominating set that has the least order over all-dominating sets in the graph [16]. Cockayne et al. [9] introduced a new technique to get the Roman dominating function. Omran and Al Hwaeer [13] developed this concept where they dealt with four dimensions instead of three and gave a new structure to the connection of the edges depending on the weight of each vertex as mentioned below. We use the notations in [10].

Definition 1.1. *[13] Let $f : V(G) \to \{0, 1, 2, 3\}$ be a labeling function, where G is a graph such that every vertex with label 0 is adjacent to two vertices, one of which is of label 2 and the other is of label 3 and every vertex with label 1 is adjacent to a vertex with label 2 or 3.Such a s function is called a modern Roman dominating function. The minimum weight of all these functions is called the modern Roman domination number and is denoted by $\gamma_{mR}(G)$.*

Proposition 1.2. *[13] For $n \geq 3, \gamma_{mR}(S_n) = n + 1$, where $S_n \equiv K_{1,n-1}$.*

Proposition 1.3. *The modern roman domination of corona of two paths C_n and C_m graphs is* $\gamma_{mR}(C_n \odot C_m) = \left\{ \begin{array}{c} 3n + 2\left\lceil \frac{m}{3} \right\rceil, \ \ if\ m \equiv 0, 2(mod\ 3) \\ \\ 3n + 2\left\lceil \frac{m}{3} \right\rceil - 1,\ if\ m \equiv 1(mod\ 3) \end{array} \right\}$

2 Main Results

Theorem 2.1. *Let G be a cycle of order n. Then*

$$\gamma_{mR}(C_n) = \left\{ \begin{array}{c} \frac{5n}{4},\ \ if\ n \equiv 0(mod\ 4) \\ \\ 5\left\lfloor \frac{n}{4} \right\rfloor + 2,\ if\ n \equiv 1(mod\ 4) \\ \\ 5\left\lfloor \frac{n}{4} \right\rfloor + 3,\ if\ n \equiv 2(mod\ 4) \\ \\ 5\left\lfloor \frac{n}{4} \right\rfloor + 4,\ if\ n \equiv 3(mod\ 4) \end{array} \right\}$$

Proof. Let the vertex set of the cycle be $\{v_1, v_2, ..., v_n\}$. There are two different ways to label these vertices. One of them is as follows: $v_1 =$

$2, v_2 = 1, v_3 = 1, v_4 = 1, v_5 = 2$ and so on. The other is as follows: $v_1 = 2, v_2 = 0, v_3 = 3, v_4 = 0$ and so on. One can conclude that the second way gives the minimum. So, to get the modern roman domination, the second way must be used. There are four cases that depend on the order of the cycle:.

Case 1. If $n \equiv 0\ (mod\ 4)$, then as mentioned above, let $v_1 = 2, v_2 = 0, v_3 = 3, v_4 = 0$ and so on for every consecutive four vertices. Thus $v_n = 0$ and $v_{n-1} = 3$ which means there is no problem with this label. Therefore, $\gamma_{mR}(C_n) = \frac{5n}{4}$.

Case 2. If $n \equiv 1\ (mod\ 4)$, then again as mentioned above, let $v_1 = 2, v_2 = 0, v_3 = 3, v_4 = 0$ and so on for every consecutive four vertices. It is obvious that $n - 1 \equiv 0\ (mod\ 4)$. Then all first $(n - 1)$ vertices have labels as in case 1. The remaining vertex is v_n which cannot take the 0 or 1 label since the vertex v_{n-1} takes the zero label. Thus the minimum labeled to the vertex v_n is two. Therefore, $\gamma_{mR}(C_n) = 5\left\lfloor \frac{n}{4} \right\rfloor + 2$.

Case 3. If $n \equiv 2\ (mod\ 4)$, then as in case 1, the first $(n - 2)$ vertices have label as in case 1, since $n - 2 \equiv 0\ (mod\ 4)$, Thus, under this labeling, the vertex v_{n-2} takes the zero label and the vertex v_1 take the two label. Therefore, the remaining vertices not taking labels are the two vertices v_{n-1} and v_n. We conclude that the minimum weights to label these vertices are two and one, respectively. Thus $\gamma_{mR}(C_n) = 5\left\lfloor \frac{n}{4} \right\rfloor + 3$.

Case 4. If $n \equiv 3\ (mod\ 4)$, then as in case 1, the first $(n - 3)$ vertices have labels as in case 1, since $n - 3 \equiv 0\ (mod\ 4)$, Thus, under this labeling, the vertex v_{n-2} takes the zero label and the vertex v_1 take the two label. Therefore, the remaining vertices not taking labels are the three vertices v_{n-2}, v_{n-1} and v_n. We conclude that the minimum weights to label these vertices are 2, 1, and 1, respectively. Thus, $\gamma_{mR}(C_n) = 5\left\lfloor \frac{n}{4} \right\rfloor + 4$ Consequently, the result follows. □

Theorem 2.2. *The modern roman domination of corona of two graphs C_n and K_m be to graphs,*

$$\gamma_{mR}(C_n \odot K_m) = \left\{ \begin{array}{ll} \frac{11n}{4} + (m-1)n, & if\ n \equiv 0(mod\ 4) \\ 5\left\lfloor \frac{n}{4} \right\rfloor + 3\left\lfloor \frac{n}{2} \right\rfloor + 3 + (m-1)n, & if\ n \equiv 1(mod\ 4) \\ 8\left\lfloor \frac{n}{4} \right\rfloor + 6 + (m-1)n, & if\ n \equiv 2(mod\ 4) \\ 8\left\lfloor \frac{n}{4} \right\rfloor + 9 + (m-1)n, & if\ n \equiv 3(mod\ 4) \end{array} \right\}$$

where $m = 1, 2,$ otherwise $\gamma_{mR}(C_n \odot K_m) = 5n, m \geq 3$.

Proof. There are three cases depending on m that must be considered:
Case 1. If $m = 1$, then there are four subcases depending on n as follows:
Subcase 1. If $n \equiv 0\ (mod\ 4)$, then, according to Theorem 1, $\gamma_{mR}(C_n) = 5\left\lfloor \frac{n}{4} \right\rfloor$ and by the proof this theorem $(\frac{n}{2})$ vertices of the cycle take the label zero. So the minimum value of the labeled vertices which are adjacent to $(\frac{n}{2})$ vertices of the cycle are two. Also, $(\frac{n}{2})$ vertices of the cycle take labels 2 or 3. So the minimum value of the labeled vertices which are adjacent to $(\frac{n}{2})$ vertices of the cycle is one. Thus $\gamma_{mR}(C_n \odot K_m) = \frac{5n}{4} + 2(\frac{n}{2}) + (\frac{n}{2}) = (\frac{11n}{4})$.
Subcase 2. If $n \equiv 1\ (mod\ 4)$, then, according to theorem 1, $\gamma_{mR}(C_n) = 5\left\lfloor \frac{n}{4} \right\rfloor + 2$ and by the proof this theorem $\left\lfloor \frac{n}{2} \right\rfloor$ vertices of the cycle take labels zero. So the minimum value of the labeled vertices which are adjacent to $\left\lfloor \frac{n}{2} \right\rfloor$ vertices of the cycle is two. Also, $\left\lfloor \frac{n}{2} \right\rfloor$ vertices of the cycle take the label 2 or 3. So the minimum value of labeled vertices which are adjacent to $\left\lfloor \frac{n}{2} \right\rfloor$ vertices of the cycle is one. Thus $\gamma_{mR}(C_n \odot K_m) = 5\left\lfloor \frac{n}{4} \right\rfloor + 2 + 2\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil = 5\left\lfloor \frac{n}{4} \right\rfloor + 2 + 2\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{2} \right\rfloor + 1 = 5\left\lfloor \frac{n}{4} \right\rfloor + 3\left\lfloor \frac{n}{2} \right\rfloor + 3$.
Subcase 3. If $n \equiv 2\ (mod\ 4)$, then. according to theorem 1, $\gamma_{mR}(C_n) = 5\left\lfloor \frac{n}{4} \right\rfloor + 3$ and by the proof this theorem $\left\lfloor \frac{n}{4} \right\rfloor$ vertices of the cycle take the label zero and one vertex takes the label one. So the minimum value of labeled vertices which are adjacent to these vertices of the cycle is two. Also, $\left(\left\lfloor \frac{n}{4} \right\rfloor + 1\right)$ vertices of the cycle take label 2 or 3. So the minimum value of labeled vertices which are adjacent to these vertices of the cycle is one. Thus $\gamma_{mR}(C_n \odot K_m) = 8\left\lfloor \frac{n}{4} \right\rfloor + 6$.
Subcase 4. If $n \equiv 3\ (mod\ 4)$, then, by Theorem 2.1, $\gamma_{mR}(C_n) = 5\left\lfloor \frac{n}{4} \right\rfloor + 4$ and by the proof of this theorem $\left\lfloor \frac{n}{4} \right\rfloor$ vertices of the cycle take the label zero and two vertices take the label one. So the minimum value of labeled vertices which are adjacent to these vertices of the cycle is two. Also, $\left(\left\lfloor \frac{n}{4} \right\rfloor + 1\right)$ vertices of the cycle take the label 2 or 3. So the minimum value of the labeled vertices which are adjacent to these vertices of the cycle is one. Thus $\gamma_{mR}(C_n \odot K_m) = 5\left\lfloor \frac{n}{4} \right\rfloor + 4 + 2\left(\left\lfloor \frac{n}{4} \right\rfloor + 2\right) + \left(\left\lfloor \frac{n}{4} \right\rfloor + 1\right) = 8\left\lfloor \frac{n}{4} \right\rfloor + 9$.
Case 2. If $m = 2$, then the labels of vertices in $C_n \odot K_m$ take in the same manner in case 1; moreover, the other vertex in each K_2 takes the label one. Thus, to get the result, we must be adding the value n to all results obtained in case 1. Therefore, the result follows.
Case 2. If $m = 3$, then we label one vertex 2 and the second vertex 3 and zero for the other vertices for each copy of K_m. Thus $\gamma_{mR}(C_n \odot K_m) = 5n$. Consequently the theorem follows. □

Theorem 2.3. *Let C_n and P_m be graphs. Then $\gamma_{mR}(C_n \odot P_m) = \gamma_{mR}(C_n \odot K_m)$, where $m = 1, 2$, otherwise $\gamma_{mR}(C_n \odot P_m) = n\left(\left\lceil \frac{n}{3} \right\rceil + 3\right)$, $m \geq 3$.*

Proof. Let $V(C_n) = \{v_1, v_2, v_3, ..., v_n\}$ and $V(P_n^i) = \{u_1^i, u_2^i, ..., u_n^i\}$ if $i = 1, 2, ..., n$. If $m = 1, 2$, then it is obvious that $P_m \equiv K_m$. Hence $\gamma_{mR}(C_n \odot P_m) = \gamma_{mR}(C_n \odot K_m)$. Now, if $m \geq 3$, then all vertices in the cycle take the label 3. Moreover, there are three cases depending on the number of vertices of each copy of the path in the graph $C_n \odot P_m$ as follows:

Case 1. If $n \equiv 0$ $(mod\ 3)$, then the vertices of each copy of the path take labels as follows: $f(u_1^i) = 0, f(u_2^i) = 2$ and $f(u_3^i) = 0$ and so on for each three consecutive vertices. Then, the number of vertices that take the label two is $\frac{n}{3}$. Therefore, $\gamma_{mR}(C_n \odot P_m) = n\left(\frac{2n}{3} + 3\right)$.

Case 2. If $n \equiv 1$ $(mod\ 3)$, then the vertices of each copy of the path take labels as follows: $f(u_1^i) = 0, f(u_2^i) = 2, f(u_3^i) = 0, f(u_4^i) = 0, f(u_5^i) = 2, f(u_2^i) = 0, f(u_1^i) = 0, f(u_2^i) = 2, ..., f(u_{n-2}^i) = 2, f(u_{n-1}^i) = 0, f(u_n^i) = 2$ and so on. Then the number of vertices that take label two is $\left\lceil \frac{n}{3} \right\rceil$. Therefore, $\gamma_{mR}(C_n \odot P_m) = n\left(2\left\lceil \frac{n}{3} \right\rceil + 3\right)$.

Case 3. If $n \equiv 2$ $(mod\ 3)$, then the vertices of each copy of the path take labels as follows: $f(u_1^i) = 0, f(u_2^i) = 2, f(u_3^i) = 0, f(u_4^i) = 0, f(u_5^i) = 2, f(u_2^i) = 0, f(u_1^i) = 0, f(u_2^i) = 2, ..., f(u_{n-2}^i) = 0, f(u_{n-1}^i) = 0, f(u_n^i) = 2$ and so on. Again, the number of vertices that take label two is $\left\lceil \frac{n}{3} \right\rceil$. Therefore, $\gamma_{mR}(C_n \odot P_m) = n\left(2\left\lceil \frac{n}{3} \right\rceil + 3\right)$ This completes the proof. □

Theorem 2.4. *The modern roman domination of corona of two paths* C_n *and* $K_{m,n}; m, n \geq 2$; *graphics* $\gamma_{mR}(C_n \odot K_{m,n}) = 7n$.

Proof. Let X and Y be partite sets of each copy of a graph $K_{m,n}$ with $|Y| = m$ and $|X| = n$. The vertices of $\langle C_n \rangle$ take the label 3. Also, label one vertex from each copy of the sets X and Y 2 and the other 0. Under this labeling, each vertex in the set $X(Y)$ that takes the label 0 is adjacent to two vertices one of which is label 2 in the set $Y(X)$. So, this labeling satisfies the condition of the modern roman dominating set. Moreover, under this label, one can conclude that the weight of these labels is minimum. Thus $\gamma_{mR}(C_n \odot K_{m,n}) = 7n$. □

3 Conclusion

From the results above, the modern Roman domination number was determined by the cycle graph. Moreover, the corona of cycle graph with some graphs was determined.

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