International Journal of Mathematics and
Computer Science, **17**(2022), no. 1, 231–242

Stability Analysis and Adaptive Control of Spreading Tuberculosis Disease

Prisayarat Sangapate[1], Paweena Tangjuang[2], Chayakorn Somsila[3]

[1]Department of Mathematics
Faculty of Engineering
Rajamangala University of Technology
Isan Khon kaen Campus
Khon kaen, Thailand

[2]Department of Mathematics and Statistics
Faculty of Science and Technology
Rajamangala University of Technology Tawan-ok
Chonburi, Thailand

[3]Department of Computer Education
Faculty of Education
Rajabhat Maha Sarakham University
Maha Sarakham, Thailand

email: prisayarat.sa@rmuti.ac.th, paweena_ta@rmutto.ac.th,
chayakorn.so@rmu.ac.th

(Received May 20, 2021, Revised June 29, 2021, Accepted August 9, 2021)

Abstract

Tuberculosis disease is a widespread disease that spreads in the air from one person to another. Moreover, this contagious disease can cause damage in the lung, brain, kidney, intestine, bone and skin. Furthermore, this disease can be also latent and reactivated. For these reasons, the control analysis of this disease has a significant importance for helping its epidemic prevention. In this paper, we discuss the stability analysis of a fixed point and present a model for adaptive

Key words and phrases: Stability, Adaptive Control, Tuberculosis Disease
AMS (MOS) Subject Classifications: 93D05, 93D20, 93D21.
Corresponding author: prisayarat.sa@rmuti.ac.th, paweena_ta@rmutto.ac.th
ISSN 1814-0432, 2022, http://ijmcs.future-in-tech.net

control against spreading tuberculosis disease. The stability analysis shows that the two fixed points of the tuberculosis disease are asymptotically stable. In addition, we present sufficient control conditions derived for the existence of stable tuberculosis disease.

1 Introduction

Viruses, bacteria, fungi and parasites are pathogenic microorganisms that cause infectious diseases. Such diseases can spread to others directly or indirectly from one person to another or from animals to humans and can result in deaths.

Tuberculosis, caused by the Mycobacterium tuberculosis bacterium, can become easily infectious and causes damage not only to the lungs but also to other organs like the brain, kidney, intestine, bone and skin [14, 23]. The World Health Organization report [25] reveals that approximately a third of the worlds population is infected with tuberculosis, with millions of deaths every year.

Mathematical models have shown significant importance in analyzing the dispersion and manipulation of an infectious disease [11]. The main objective of mathematical models, is to try to understand how a given disease spreads in the population and try to take it out in the future [21]. The consequences from mathematical models can assist in determining the possibility of epidemiologic explanations and predict the impact of changes on the system dynamics. It is substantial for clarifying the population transmission dynamics of the infectious agents and the potential impact of infectious disease control programs [1,11]. Several mathematical models on tuberculosis have been developed [15,16] which have a significant role controlling tuberculosis worldwide [15]. Most of tuberculosis models we find in the literature are the SIR [12,15] or SEIR model [3,5,7,8,15,18,19]. There are clusters (or class of individuals) each of which has a status that characterizes it as responsive, inseparable, infectious or recuperative [15].

At the present time, several optimal control methods for tuberculosis disease and other diseases have been analyzed and executed [4,22]. The optimal control method performs based on minimizing the cost function defined for the problem and the limiting factors and eventually determines the control inputs of the system to achieve the specified objective. The target factors or parameters defined in the cost function can be the time of action, state error [13], control effort [24], energy consumption [9] and so on. Athithan and Ghosh [2] considered a model for tuberculosis and used a detection strat-

egy to reduce the transmission of tuberculosis in a proposed optimal control strategy and used a model with 4 groups: susceptible, exposed, infectious and recovered. Denysiuk et al. [6] considered a model for TB-HIV/AIDS by proposing an optimal control problem for it where the objective is minimization of the number of HIV-infected individuals involved with prevention and treatment of disease. The optimal control approach is utilized for systems with known parameters which obtains the system inputs with the goal of minimizing the cost function. But if variable or unsteady parameters occur in a system, then the desired results by conventional control methods may not be possible. On the other hand, we can modernize the estimation of system parameters and assure the closed-loop stability of that system by using the adaptive control method [20,22].

In this paper, we study the characteristics of the model, fixed point and Basic Reproduction Ratio [23]. Then, we present the stability analysis of a fixed point, and we design the adaptive control for stabilizing the global tuberculosis disease. Finally, we present a conclusion.

2 Preliminaries

Consider a dynamical system which satisfies

$$\dot{x} = f(x,t), x(t_0) = x_0; x \in R^n, \tag{2.1}$$

where $f(x,t) : R^n \times R^+ \to R^n$ is continuous. A point $x_e \in R^n$ is a fixed point of the system if $f(x_e,t) = 0$.

Definition 2.1. *The fixed point x_e is stable if for $\epsilon > 0$, there exists $\delta > 0$ such that*

$$\| x(0) - x_e \| < \delta \Longrightarrow \| x(t) - x_e \| < \epsilon,$$

where $x(0)$ is a unique solution. The fixed point x_e is unstable if it is not stable. The fixed point x_e is asymptotically stable if it is stable and there exists $\delta > 0$ such that

$$\| x(0) - x_e \| < \delta \Longrightarrow x(t) \to x_e \text{ as } t \to \infty.$$

The fixed point x_e is globally asymptotically stable if it is stable and

$$x(t) \to x_e \text{ as } t \to \infty.$$

The following is a fundamental result of a Lyapunov stability theorem [10].

Theorem 2.2. *(Lyapunov global asymptotically stability theorem-G.A.S) Suppose there is a function $V : R^n \to R$ such that*
1. V is positive definite,
2. $\dot{V}(x) < 0$ for all $x \neq 0$, $\dot{V}(0) = 0$.
Then trajectories of $\dot{x} = f(x)$ converges to zero as $t \to \infty$;, that is, the system is globally asymptotically stable.

Theorem 2.3. *(Lasalle's theorem[17])*
Lasalle's theorem allows us to conclude G.A.S. of a system with only $\dot{V} \leqslant 0$, along with an observability type condition. We consider $\dot{x} = f(x)$. Suppose there is a function $V : R^n \to R$ such that
1. V is positive definite,
2. $\dot{V} \leqslant 0$,
3. the only solution of $\dot{\omega} = f(\omega), \dot{V}(\omega) = 0$ is $\omega(t) = 0$. for all t.
Then the system $\dot{x} = f(x)$ is globally asymptotically stable.

Next, we consider a nonlinear nonautonomous differential equation of the general form;

$$\begin{aligned} \dot{x}(t) &= f(t, x(t)); t \geq t_0 \in R, \\ x(t_0) &= x_0 \end{aligned} \tag{2.2}$$

where the state $x(t)$ takes values in X, $f(t, x) : R \times X \to X$ is a given nonlinear function and $f(t, 0) = 0$, for all $t \in R$.

An adaptive control is an active field in the design of control systems to deal with uncertainties. To design control laws that stabilize a chaotic system, the control system can be written as equation (2.3):

$$\dot{x}(t) = f(t, x(t), u(t)); t \geq 0, \tag{2.3}$$

where $u(t)$ is the external control input.

Definition 2.4. *The control system (2.3) is stabilizable if there exists the control $u(t) = k(x(t))$ such that the system*

$$\dot{x}(t) = f(t, x(t), k(x(t))); t \geq 0 \tag{2.4}$$

is asymtotically stable.

3 Model Description

In this section, we study a general SIR model for the spreading of Tuberculosis disease [23]. We separate the total population $N(t)$ into three distinct subgroups: susceptible $S(t)$, infected $I(t)$, and recovered $R(t)$. A group of susceptible individuals $S(t)$ increases because of the birth π, but $S(t)$ decreases because of the death μ and the direct contact with infected individual groups β. A group of individuals that infects the tuberculosis disease $I(t)$ increases related to the rate β but decreases because of the natural mortality μ, the mortality μ_t due to tuberculosis and the rate γ. A group of recovered individuals $R(t)$ increases because of the recovery rate γ and decreases because of natural mortality μ.

Based on the assumptions, the mathematical model for spreading of tuberculosis can be represented by the following system of differential equations

$$\begin{aligned} \dot{S}(t) &= \pi - \beta SI - \mu S \\ \dot{I}(t) &= \beta SI - (\mu + \mu_t + \gamma)I \\ \dot{R}(t) &= \gamma I - \mu R, \end{aligned} \tag{3.5}$$

with $N(t) = S(t) + I(t) + R(t)$ and the initial condition

$$S(0) \geqslant 0, I(0) \geqslant 0, R(0) \geqslant 0$$

By considering the total population density, we have $S(t) + I(t) + R(t) = 1$. So $R(t) = 1 - S(t) - I(t)$. Therefore, it is enough to consider

$$\begin{aligned} \dot{S}(t) &= \pi - \beta SI - \mu S \\ \dot{I}(t) &= \beta SI - (\mu + \mu_t + \gamma)I \end{aligned} \tag{3.6}$$

such that the region for the above system is written as

$$\Omega = \{(S(t), I(t)) \in R_+^2, S(t) + I(t) \leqslant 1\}$$

4 Mathematical Analysis

4.1 Fixed Point and Basic Reproduction Ratio

In this section, we study the fixed point and the basic reproduction ratio R_0 of the tuberculosis disease model [23]. The fixed point was obtained when we set the right hand side of the system (3.6) equal to zero or $\dot{S}(t) = 0$ and

$\dot{I}(t) = 0$. Thus the two fixed points are obtained as follows [23]:

1. The first fixed point is $E_0(S_0, I_0) = (\frac{\pi}{\mu}, 0)$. This is the disease free fixed point.

2. The second fixed point is $E_1(S_1, I_1) = (\frac{\mu+\mu_t+\gamma}{\beta}, \frac{\pi}{\mu+\mu_t+\gamma} - \frac{\mu}{\beta})$. This is the epidemic or infection occurred.

To handle infection, the following must be satisfied $S_1 \geqslant \frac{\pi}{\mu}$, where $S_1 = \frac{\mu+\mu_t+\gamma}{\beta}$. We conclude that

1. If $\frac{\mu+\mu_t+\gamma}{\beta} \geqslant \frac{\pi}{\mu}$, then an epidemic will not occur or the disease will not be transmitted. 2. If $\frac{\mu+\mu_t+\gamma}{\beta} < \frac{\pi}{\mu}$, then there will be an epidemic or an infection occurrence.

Next, the basic reproduction ratio R_0 can be found in [23]. If the disease is not transmitted, then $S_1 \geqslant \frac{\pi}{\mu}$ or $\frac{\pi}{\mu S_1} \leqslant 1$, or $\frac{\beta\pi}{\mu(\mu+\mu_t+\gamma)} - 1 \leqslant 0$ in order to obtain the basic reproduction ratio $R_0 = \frac{\beta\pi}{\mu(\mu+\mu_t+\gamma)}$ which can be used to measure the rate of spreading of the disease.

1. For $R_0 \leqslant 1$, the disease will disappear.
2. For $R_0 > 1$, the disease will be epidemic.

4.2 Stability Analysis of Fixed Point

In this section, we discuss the stability analysis of both fixed points. We use Lasalle's Theorem for both of the fixed points of the proposed model, the disease free fixed point and the epidemic fixed point. First, we give the globally asymptotically stability of the disease free fixed point.

Theorem 4.1. *If $R_0 \leqslant 1$, then the disease free fixed point $E_0(S_0, I_0) = (\frac{\pi}{\mu}, 0)$ of the system is globally asymptotically stable on Ω.*

Proof. To establish the globally asymptotically stability of the disease free fixed point E_0, we construct the Lyapunov function V. Let $V : \Omega \to R$, be defined by

$$V(S, I) = I(t).$$

Considering the time derivative of V along the solution of the system, we obtain

$$\begin{aligned}\dot{V}(t) &= \beta SI - (\mu + \mu_t + \gamma)I \\ &= (\mu + \mu_t + \gamma)[\frac{\beta SI}{\mu + \mu_t + \gamma} - I] \\ &= (\mu + \mu_t + \gamma)[\frac{\beta\pi}{\mu(\mu + \mu_t + \gamma)} - 1]I \\ &= (\mu + \mu_t + \gamma)(R_0 - 1)I,\end{aligned}$$

where R_0 is the basic reproduction ratio. We see that $\dot{V}(t) \leqslant 0$ for $R_0 \leqslant 1$. Therefore, by the Lasalle's Theorem [17], the disease free fixed point E_0 of the system is globally asymptotically stable on Ω.

Theorem 4.2. *The epidemic fixed point* $E_1(S_1, I_1) = (\frac{\mu+\mu_t+\gamma}{\beta}, \frac{\pi}{\mu+\mu_t+\gamma} - \frac{\mu}{\beta})$ *of the system is globally asymptotically stable on* Ω.

Proof. To establish the globally asymptotically stability of the epidemic fixed point $E_1 = (S^*, I^*)$, where $S^* = \frac{\mu+\mu_t+\gamma}{\beta}$ and $I^* = \frac{\pi}{\mu+\mu_t+\gamma} - \frac{\mu}{\beta}$, we construct the Lyapunov function $V : \Omega_+ \to R$, where

$$\Omega_+ = \{(S(t), I(t)) \in \Omega \mid S(t) > 0, I(t) > 0\}$$

is given by

$$V(S, I) = V_1[S - S^* ln(\frac{S}{S^*})] + V_2[I - I^* ln(\frac{I}{I^*})],$$

where V_1 and V_2 are positive constants to be chosen. By taking the derivative of the above function, we obtain

$$\begin{aligned}\dot{V} &= [V_1 S - V_1 S^*][\frac{\pi}{S} - \beta I - \mu] + [V_2 I - V_2 I^*][\beta S - (\mu + \mu_t + \gamma)] \\ &= V_1[S - S^*][\frac{\pi}{S} - \beta I - \mu] + V_2[I - I^*][\beta S - (\mu + \mu_t + \gamma)]\end{aligned}$$

Considering the fixed point, we have

$$-\mu = \beta I^* - \tfrac{\pi}{S^*} \text{ and } -(\mu + \mu_t + \gamma) = -\beta S^*$$

Hence, the above equation becomes

$$\begin{aligned}\dot{V} &= V_1[S-S^*]\pi[\frac{S^*-S}{SS^*}] - V_1(S-S^*)\beta(I-I^*) + V_2(I-I^*)\beta(S-S^*)\\ &= -V_1[S-S^*]\pi[\frac{S-S^*}{SS^*}] - V_1(S-S^*)\beta(I-I^*) + V_2(I-I^*)\beta(S-S^*)\\ &= -V_1\pi\frac{(S-S^*)^2}{SS^*} + \beta(V_2-V_1)(S-S^*)(I-I^*)\\ &= \beta(V_2-V_1)(S-S^*)(I-I^*) - V_1\pi\frac{(S-S^*)^2}{SS^*}.\end{aligned}$$

For $V_1 = V_2 = 1$, we have

$$\dot{V} = -\pi\frac{(S-S^*)^2}{SS^*} \leqslant 0.$$

We also obtain

$$\dot{V} = 0 \iff S = S^*$$

Therefore, by the Lasalle's Theorem [17], the epidemic fixed point E_1 of the system is globally asymptotically stable on Ω.

4.3 Adaptive Control of Tuberculosis Disease

In this section, we design the adaptive control for the global stabilized tuberculosis disease. The sufficient control conditions are derived by using Lyapunov stability theorem.

Theorem 4.3. *The tuberculosis disease is global stabilized for the initial conditions $S(t) > 0, I(t) > 0, R(t) > 0$ by the adaptive control u_1, u_2, where the estimate parameters are given by $\dot{\hat{\pi}} = S + K_3 e_\pi, \dot{\hat{\beta}} = SI^2 + K_4 e_\beta, \dot{\hat{\mu}} = -S^2 + K_5 e_\mu, \dot{\hat{\mu}}_t = -I^2 + K_6 e_{\mu_t}$, and $\dot{\hat{\gamma}} = -I^2 + K_7 e_\gamma$.*

Proof. We design the adaptive control for the tuberculosis disease as follows:

$$\begin{aligned}\dot{S}(t) &= \pi - \beta SI - \mu S + u_1\\ \dot{I}(t) &= \beta SI - (\mu + \mu_t + \gamma)I + u_2,\end{aligned} \tag{4.7}$$

where u_1 and u_2 are controllers to be designed by using the states and estimates of the parameters of the system. We consider the adaptive control

functions

$$\begin{aligned} u_1 &= -\hat{\pi} + \hat{\beta}SI + \hat{\mu}S - k_1 S \\ u_2 &= -\hat{\beta}SI + (\hat{\mu} + \hat{\mu_t} + \hat{\gamma})I - k_2 I, \end{aligned} \tag{4.8}$$

where $\hat{\pi}, \hat{\beta}, \hat{\mu}, \hat{\mu_t}$, and $\hat{\gamma}$ are estimates of the parameters and k_1 and k_2 are positive constants. Substituting the controllers, we have

$$\begin{aligned} \dot{S}(t) &= (\pi - \hat{\pi}) - (\beta - \hat{\beta})SI - (\mu - \hat{\mu})S - k_1 S \\ \dot{I}(t) &= (\beta - \hat{\beta})SI - (\mu - \hat{\mu})I - (\mu_t - \hat{\mu_t})I - (\gamma - \hat{\gamma})I - k_2 I. \end{aligned} \tag{4.9}$$

Define the parameter errors as

$$e_\pi = \pi - \hat{\pi}, e_\beta = \beta - \hat{\beta}, e_\mu = \mu - \hat{\mu}, e_{\mu_t} = \mu_t - \hat{\mu_t}, e_\gamma = \gamma - \hat{\gamma}. \tag{4.10}$$

Thus, equation (4.9) can be rewritten as

$$\begin{aligned} \dot{S}(t) &= e_\pi - e_\beta SI - e_\mu S - k_1 S \\ \dot{I}(t) &= e_\beta SI - e_\mu I - e_{\mu_t} I - e_\gamma I - k_2 I \end{aligned} \tag{4.11}$$

Consider the quadratic Lyapunov function

$$V = \frac{1}{2}(S^2 + I^2 + e_\pi^2 + e_\beta^2 + e_\mu^2 + e_{\mu_t}^2 + e_\gamma^2), \tag{4.12}$$

which is a positive definite function on R^7. So

$$\dot{e}_\pi = -\dot{\hat{\pi}}, \dot{e}_\beta = -\dot{\hat{\beta}}, \dot{e}_\mu = -\dot{\hat{\mu}}, \dot{e}_{\mu_t} = -\dot{\hat{\mu}}_t, \dot{e}_\gamma = -\dot{\hat{\gamma}}. \tag{4.13}$$

Differentiating V in equation (4.11) and using equation (4.13), we get

$$\begin{aligned} \dot{V}(t) = {} & Se_\pi - e_\beta S^2 I - e_\mu S^2 - k_1 S^2 + e_\beta SI^2 - e_\mu I^2 - e_{\mu_t} I^2 \\ & - e_\gamma I^2 - k_2 I^2 - e_\pi \dot{\hat{\pi}} - e_\beta \dot{\hat{\beta}} - e_\mu \dot{\hat{\mu}} - e_{\mu_t} \dot{\hat{\mu}}_t - e_\gamma \dot{\hat{\gamma}}. \end{aligned} \tag{4.14}$$

The estimated parameters of equation (4.14) are updated by the following

$$\begin{aligned} \dot{\hat{\pi}} &= S + k_3 e_\pi \\ \dot{\hat{\beta}} &= SI^2 + k_4 e_\beta \\ \dot{\hat{\mu}} &= -S^2 + k_5 e_\mu \\ \dot{\hat{\mu}}_t &= -I^2 + k_6 e_{\mu_t} \\ \dot{\hat{\gamma}} &= -I^2 + k_7 e_\gamma, \end{aligned} \tag{4.15}$$

where k_3, k_4, k_5, k_6, k_7 are positive constants. Substituting equation (4.15) into equation (4.14), we obtain

$$\dot{V}(t) = -e_\beta S^2 I - e_\mu I^2 - k_1 S^2 - k_2 I^2 - k_3 e_\pi^2 - k_4 e_\beta^2 - k_5 e_\mu^2 - k_6 e_{\mu_t}^2 - k_7 e_\gamma^2$$

which is a negative definite function on R^7. Therefore, by Lyapunov stability theorem [10], We obtain that the tuberculosis disease is global stabilized for the initial conditions $S(t) > 0, I(t) > 0, R(t) > 0$ by the adaptive control u_1, u_2, where the estimate parameters are given by $\dot{\hat{\pi}} = S + K_3 e_\pi, \dot{\hat{\beta}} = SI^2 + K_4 e_\beta, \dot{\hat{\mu}} = -S^2 + K_5 e_\mu, \dot{\hat{\mu}}_t = -I^2 + K_6 e_{\mu_t}$, and $\dot{\hat{\gamma}} = -I^2 + K_7 e_\gamma$.

5 Conclusions

In this paper, we studied a model of Tuberculosis disease, the fixed points and the reproduction ratio R_0. We analyzed the stability of fixed points and found the adaptive control of Tuberculosis disease. The mathematical analysis of the model showed that there are two fixed points, one is the disease free fixed point and the other is the epidemic fixed point. The proposed model was determined by the basic reproduction ratio R_0 which depends on the parameter values. Analyzing the stability of fixed points showed that the disease free fixed point of the system is globally asymptotically stable when $R_0 \leqslant 1$ and the epidemic fixed point of the system is globally asymptotically stable. Moreover, the tuberculosis disease is global stabilized for the initial conditions $S(t) > 0, I(t) > 0, R(t) > 0$ by using the obtained adaptive control and the parameter estimates.

Acknowledgment. We thank the Department of Mathematics, Faculty of Engineering, Rajamangala University of Technology Isan, Khon Kaen Campus, Thailand.

References

[1] J. L. Aron, Mathematical Modeling: The Dynamics of Infection, Jones and Bartlett Publishers, MA, 2006.

[2] S. Athithan, M. Ghosh, Optimal control of tuberculosis with case detection and treatment, World Journal of Modelling and Simulation, **11**, (2015), 111-122.

[3] S. Blower, P. Small, P. Hopewell, Control strategies for tuberculosis epidemics: new models for old problems, Science, **273**, no. 5274, (1996), 497–500.

[4] S. Bowong, Optimal control of the transmission dynamics of tuberculosis, Nonlinear Dynamics, **61**, no. 4, (2010), 729–748.

[5] C. Castillo Chavez, Z. Feng, To treat or not to treat: the case of tuberculosis, J. Math. Biol., **35**, no. 6, (1997), 629–656.

[6] R. Denysiuk, C. J. Silva, D. F. M. Torres, Multiobjective optimization to a TB-HIV/AIDS coinfection optimal control problem, Computational Applied Mathematics, **37**, no. 2, (2018), 2112–2128.

[7] Z. Feng, C. Castillo-Chavez, Mathematical Models for the Disease Dynamics of Tuberculosis, World Scientific, River Edge, 1998.

[8] Z. Feng, W. Huang, C. Castillo Chavez, On the role of variable latent periods in mathematical models for tuberculosis, J. Dyn. Differ. Eq., **13**, no. 2, (2001), 425–452.

[9] M. Geravand, P. Z. Korondi, A. Peer, Human sit-to-stand transfer modeling for optimal control of assistive robots. In 5th IEEE RAS/EMBS International Conference on Biomedical Robotics and Biomechatronics, (2014), 670–676.

[10] W. Hahn, The stability of Motion, Sringer, New York, 1967.

[11] N. A. Halim, Tuberculoss model: A mathematical analysis, M.S Thesis, Unversiti Malaya, 2013.

[12] T. Iskandar, N. A. Chaniago, S. Munzir, V. Halfiani, M. Ramli, Mathematical model of tuberculosis epidemic with recovery time delay, AIP Conf. Proc., (2017).

[13] S. Jamali, S. Taghvaei, S. A. Haghpanah, Optimal Strategy for Sit-to-Stand Movement Using Reinforcement Learning, Journal of Rehabilitation Sciences and Research, **4**, no. 3, (2018), 70–75.

[14] T. C. Jonesn, D. H. Ronald, Veterinary Pathology, Philadelphia: Lead and Febiger, 1993.

[15] S. K. Kabunga, E. F. Doungmo Goufo, V. H. Tuong, Analysis and simulation of a mathematical model of tuberculosis transmission in Democratic Republic of the Congo, Advances in Difference Equations, (2020), 642.

[16] S. Kim, A. A. de los Reyes V, E. Jung, Country specific intervention strategies for top three TB burden countries using mathematical model, PLoS ONE, **15**, no. 4, (2020).

[17] J. P. LaSalle, The stability of dynamical systems, Society for Industrial and Applied Mathematics, 1987.

[18] C. C. McCluskey, Global stability for a class of mass action systems allowing for latency in tuberculosis, J. Math. Anal. Appl., **338**, no. 1, (2008), 518–535.

[19] B. M. Murphy, B. H. Singer, S. Anderson, D. Kirschner, Comparing epidemic tuberculosis in demographically distinct heterogeneous populations, Math. Biosci., **180**, nos. 1-2, (2002), 161–185.

[20] K. S. Narendra, A. M. Annaswamy, Stable adaptive systems, Courier Corporation, 2012.

[21] A. Ndondo Mboma, Une analyse globale dun modèle mathématique de la trypanosomiase humaine africaine, Ph.D thesis, CRFMI, University of Kinshasa, 2017.

[22] M. H. Nematollahi, R. Vatankhah, M. Sharifi, Nonlinear adaptive control of tuberculosis with consideration of the risk of endogenous reactivation and exogenous reinfection, Journal of Theoretical Biology, **486**, (2020), 110081.

[23] E. Rohaeti, S. Wardatun, A. Andriyati, Stability Analysis Model of Spreading and Controlling of Tuberculosis, Applied Mathematical Sciences, **9**, no. 52, (2015), 2559–2566.

[24] M. Sharifi, H. Salarieh, S. Behzadipour, Nonlinear optimal control of planar musculoskeletal arm model with minimum muscles stress criterion, Journal of Computational and Nonlinear Dynamics, **12**, no. 1, (2017).

[25] World Health Organization: Global tuberculosis report, World Health Organization, 2018.