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Application of Fuzzy TOPSIS for Multiple criteria decision-making based on interval bipolar fuzzy sets

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Abstract

We present a new method for multiple criteria decision-making (MCDM) based on interval bipolar value sets using the positive and negative decisions of the decision-maker. Moreover, we assign the methodology name IBF-TOPSIS which considers the splitting of positive and negative data of interval bipolar value sets with the fuzzy TOPSIS method with score function. Every alternative has a rating that consists of two parts: positive and negative. The positive part represents positive thinking while the negative part represents negative thinking on the corresponding criterion. Furthermore, we illustrate our proposed methods with examples.

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1 Introduction

A successful decision-making process depends on choosing the best alternatives based on the available information characteristics and applying the appropriate proper method. Mostly, information is often extracted from incomplete data that is not clearly defined and the veracity of the information is doubtful. Examples of such information include economic performance indicators that may guide economic development, information derived for risk evaluation and analysis, and business decision-making and planning decisions. Mathematicians have been interested in decision-making processes and decision-making models for a long time. In 1965, Zadeh [1] introduced the concepts of Fuzzy Logic and Fuzzy Theory in which the decision-making model can be adapted automatically according to changes in environment and context, enabling more effective decisions to be made based on what is available. Solving new problems successfully is often based on prior knowledge or experience. Applying Fuzzy Logic or Fuzzy Theory principles to the decision-making process reflects usual human decision-making activities, which are based on the idea of future uncertainty, or lack of precise and concise information, that may be based on misunderstanding, decision maker errors, or incomplete or erroneous data. The difference between Fuzzy Logic and Boolean Logic is that Fuzzy Logic is more appropriate to the subjective analysis of partially correct or partially understood information rather than the more objective True/False dichotemy implied in Boolean Logic. This is an essential factor, relevant to multi-criteria decision-making (MCDM) when seeking the best option from among multiple feasible alternatives. MCDM refers to the screening, prioritizing, ranking, or selecting of a set of alternatives from usually independent, incommensurable, or conflicting attributes. One popular approach used in MCDM is the technique known as TOPSIS (the Technique for Order of Preference by Similarity to Ideal Solution), a multi-criteria decision analysis method originally published in 1981 by Hwang and Yoo [5]. TOPSIS has been applied in many decision-making arenas, including stock trade. Theoretically, to solve MCDM problems, TOPSIS is based on the idea of the selection of the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). In 2000, Chen [4] extended the concept of TOPSIS to the fuzzy environment usual in MCDM. In that approach, the rating of each alternative and the weight of each criterion are described by linguistic terms. In 2006, Jahanshahloo et al. [9] further extended the TOPSIS method for decision-making problems with fuzzy data. The TOPSIS approach has high accuracy when dealing with numerical values. However, data used in decision-making contains both numerical data and linguistic variables, and some information has both bipolarity, and positive and negative values that must be considered. So bipolar information is a factor in the effectiveness and efficiency of decision-making.

In 1994, Zhang [2] initiated and elaborated the concept of bipolar fuzzy sets and used bipolar fuzzy sets in the development of bipolar fuzzy logic. This concept has been widely applied to solve many real-world problems. In 1998, the notions of bipolar fuzziness and interval-based bipolar fuzzy logic were generalized to a real-valued bipolar fuzzy logic [3]. A membership degree range of the bipolar fuzzy set is in [-1, 0] ∪ [0, 1]. The membership degree 0 of an element means the element is irrelevant to the corresponding property. The membership degree in [-1, 0) and (0, 1] of an element indicates that the element somewhat satisfies the implicit counter-property and property. In 2018, Alghamdi et al. [7] introduced the idea of multi-criteria decision-making methods in a bipolar fuzzy environment. In 2020, Akram et al. [6] proposed bipolar fuzzy TOPSIS and bipolar fuzzy ELECTRE-I methods. In 2019, Mahmood et al. [8] introduced MCDM processes based on bipolar valued fuzzy sets which, using a score function, improved score functions and double improved score functions to solve MCDM problems. In concept, the score function improved the algorithm for use in MCDM problems [[10], [11], [12]].

In this paper, we present a new interval bipolar fuzzy (IBF) set in TOPSIS; namely, the IBF-TOPSIS method to solve MCDM problems that are equipped with interval bipolar fuzzy information. The outline of the paper is as follows: First, we describe the interval bipolar fuzzy set. Secondly, we show the design of the algorithms applied in the IBF-TOPSIS method. Finally, we illustrate our proposed methods with examples.

2 Preliminaries

In this section, we review the important definitions of fuzzy sets and interval bipolar fuzzy sets to be used in MCDM models.

Definition 2.1. *A fuzzy set A in a universe of discourse X is characterized by a membership function f that assigns to each element x in X a real number in the interval* $[0, 1]$*. The numeric value f stands for the grade of membership of x in X.*

We use $CS[0,1]$ to denote the set of all closed subintervals in $[0,1]$; i.e.,

$$CS[0,1] = \{\overline{f} := [f_l, f_u] \mid 0 \leq f_l \leq f_u \leq 1\}.$$

We note that $[f,f] = \{f\}$, for all $h \in [0,1]$. For $f = 0$ or 1, we use the notations $\overline{0} = [0,0] = \{0\}$ and $\overline{1} = [1,1] = \{1\}$.

Definition 2.2. *[13] Let X be a non-empty set. An interval valued fuzzy subset (briefly, IVF subset) of X is a function $\overline{A} : X \to CS[0,1]$.*

Definition 2.3. *Consider interval fuzzy sets $\bar{A}_1 = [l_1, u_1]$ and $\bar{A}_2 = [l_2, u_2]$. The operations with these are then defined as follows:*

1. $\bar{A}_1 \oplus \bar{A}_2 = [l_1, u_1] \oplus [l_2, u_2] = [l_1 + l_2, u_1 + u_2]$,
2. $\bar{A}_1 \ominus \bar{A}_2 = [l_1, u_1] \ominus [l_2, u_2] = [l_1 - u_2, u_1 - l_2]$,
3. $\bar{A}_1 \otimes \bar{A}_2 = [l_1, u_1] \otimes [l_2, u_2] = [l_1 \times l_2, u_1 \times u_2]$,
4. $\bar{A}_1 \oslash \bar{A}_2 = [l_1, u_1] \oslash [l_2, u_2] = [l_1 \div u_2, u_1 \div l_2]$,
5. $\bar{A_1}^{-1} = [l_1, u_1]^{-1} = [\frac{1}{u_1}, \frac{1}{l_1}]$,
6. $k\bar{A}_1 = k[l_1, u_1] = [kl_1, ku_1]$, *$k$ is a constant.*

Definition 2.4. *Consider interval fuzzy sets $\bar{A}_1 = [l_1, u_1]$ and $\bar{A}_2 = [l_2, u_2]$. Then the distance between them is calculated by*

$$d(\bar{A}_1, \bar{A}_2) = \sqrt{\frac{1}{2}[(l_1 - l_2)^2 + (u_1 - u_2)^2]}.$$

Next, we introduce the concept of interval bipolar fuzzy sets.

Definition 2.5. *Let X be a non-empty set. An interval bipolar fuzzy set (IBF set) f on X is an object of the form*

$$\bar{f} := \{(u, f^p(u), f^n(u)) \mid u \in X\},$$

where $f^p : X \to CS[0,1]$ and $f^n : X \to CS[-1,0]$.

Remark 2.6. *For simplicity, we use $\bar{f} = (X; f^p, f^n)$ for the IBF set $\bar{f} = \{(u, f^p(u), f^n(u)) \mid u \in X\}$.*

3 Algorithm for IBF set on score function in TOPSIS

In this section, we define the operations on the IBF set as follows:

Let $\bar{f} = (f^p, f^n)$ and $\bar{g} = (g^p, g^n)$ be IBF sets, where $f^p = [f_l^p, f_u^p]$, $f^n = [f_l^n, f_u^n]$, $g^p = [g_l^p, g_u^p]$ and $g^n = [g_l^n, g_u^n]$. The operation on the IBF set is defined by

$$\bar{f} \oplus \bar{g} = (f^p \oplus g^p, f^n \oplus g^n),$$

where $f^p \oplus g^p = [f_l^p + g_l^p, f_u^p + g_u^p]$ and $f^n \oplus g^n = [f_l^n + g_l^n, f_u^n + g_u^n]$.

$$\bar{f} \ominus \bar{g} = (f^p \ominus g^p, f^n \ominus g^n),$$

where $f^p \ominus g^p = [f_l^p - g_l^p, f_u^p - g_u^p]$ and $f^n \ominus g^n = [-|f_l^n - g_l^n|, -|f_u^n - g_u^n|]$.

$$\bar{f} \otimes \bar{g} = (f^p \otimes g^p, f^n \otimes g^n),$$

where $f^p \otimes g^p = [f_l^p \times g_l^p, f_u^p \times g_u^p]$ and $f^n \otimes g^n = [-(f_l^n \times g_l^n), -(f_u^n \times g_u^n)]$.

$$\bar{f} \oslash \bar{g} = (f^p \oslash g^p, f^n \oslash g^n),$$

where $f^p \oslash g^p = [f_l^p \div g_l^p, f_u^p \div g_u^p]$ and $f^n \oslash g^n = [-(f_l^n \div g_l^n), -(f_u^n \div g_u^n)]$.

$$k\bar{f} = [kf^p, kf^n],$$

where $kf^p = [kf_l^p, kf_u^p]$ and $kf^n = [kf_l^n, kf_u^n]$, k is a positive real number.

The distance IBF set is

$$d(\bar{f}, \bar{g}) = (d(f^p, g^p), d(f^n, g^n))$$

where

$$d(f^p, g^p) = \sqrt{\frac{1}{2}[(f_l^p - g_l^p)^2 + (f_u^p - g_u^p)^2]}$$

and

$$d(f^n, g^n) = -\sqrt{\frac{1}{2}[(f_l^n - g_l^n)^2 + (f_u^n - g_u^n)^2]}.$$

Next, we introduce the score function. In the following discussion $\psi := \{B_i = (f_{B_i}^p, f_{B_i}^n)|i \in I\}$ is a collection of BF sets.

Definition 3.1. *With ψ like before, the score function is a real valued function $S(B_i)$ defined by*

$$S(B_i) = f^p_{B_i} + f^n_{B_i}$$

where $\psi(B_i) \in [-1, 1]$ and $\psi(B_i) = 0$ if and only if $f^p_{B_i} = f^n_{B_i}$.

Definition 3.2. *The Improve score function $I(B_i)$ is a real valued function on ψ defined by*

$$I(B_i) = (f^p_{B_i})^2 S(B_i) + (f^n_{B_i})^2 S(B_i) - (f^p_{B_i} f^n_{B_i}) S(B_i).$$

Definition 3.3. *The Double Improve score function $D(B_i)$ is a real valued function defined by*

$$D(B_i) = S(B_i) + I(B_i).$$

Next, we apply the interval bipolar fuzzy set on the score function to the TOPSIS method (IBF-TOPSIS method) in order to solve some problems in the real world. Assume that D_k is a set of decision-maker $k = 1, 2, ..., K$, which is responsible for assessing alternatives A_i, $i = 1, 2, ..., m$, under each of criteria C_j, $j = 1, 2, ..., n$. The criteria are selected according to the investigation of the decision-maker.

Assuming that each alternative's suitability rating and criteria are assigned to interval bipolar fuzzy values and the weights, the steps of the IBF-TOPSIS method are as follows:

(i) For weights W of the IBF set assigned to each criterion by a decision maker, the normality condition is satisfied; that is, for

$$W = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix},$$

where $w_j = \dfrac{\bar{w_j}}{\bar{w}}$,$\bar{w_j} = \dfrac{1}{k}(w^1_j \oplus w^2_j \oplus ... \oplus w^k_j)$, $\bar{w} = \bar{w_1} \oplus \bar{w_2} \oplus ... \oplus \bar{w_n}$ and $\sum_{j=1}^n w_j = 1$.

(ii) Each alternative A_i is evaluated with respect to n criteria from k decision-maker. All the values assigned to the alternatives for each criterion form a decision matrix D:

$$D = [x_{ij}]_{m\times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

where $x_{ij} = (x_{ij}^p, x_{ij}^n) = \dfrac{1}{k}((x_{ij}^p, x_{ij}^n)^1 \oplus (x_{ij}^p, x_{ij}^n)^2 \oplus ... \oplus (x_{ij}^p, x_{ij}^n)^k)$. For each entry $x_{ij} = (x_{ij}^p, x_{ij}^n), x_{ij}^p \in [0, 1]$ represents the degree of satisfaction of alternative i under criteria j and $x_{ij}^n \in [-1, 0]$ represents the degree of satisfaction of the alternative i under criteria j.

Note that, if $x_{ij}^p = 1$, then the alternative i shows the maximum satisfaction behavior of the criteria j and in the case of $x_{ij}^n = -1$, we say that alternative i represents the maximum degree of dissatisfaction of the criteria j.

(iii) With the normalized fuzzy decision matrix $R = (R^p, R^n)$ as a IBF set

$$R^p = [r_{ij}^p]_{m\times n} \text{ and } R^n = [r_{ij}^n]_{m\times n}$$

where B is the set of benefit criteria,

$$r_{ij}^p = [\frac{a_{ij}^p}{\hat{c}_j}, \frac{c_{ij}^p}{\hat{c}_j}] \text{ and } r_{ij}^n = -[\frac{a_{ij}^n}{\check{c}_j}, \frac{c_{ij}^n}{\check{c}_j}], j \in B;$$

where $\hat{c}_j = \max\limits_i c_{ij}^p$ and $\check{c}_j = \min\limits_i c_{ij}^n$
and E is a cost criterion,

$$r_{ij}^p = [\frac{\hat{a}_j}{c_{ij}^p}, \frac{\hat{a}_j}{a_{ij}^p}] \text{ and } r_{ij}^n = -[\frac{\check{a}_j}{c_{ij}^n}, \frac{\check{a}_j}{a_{ij}^n}], j \in E;$$

where $\hat{a}_j = \min\limits_i a_{ij}^p$ and $\check{a}_j = \max\limits_i c_{ij}^n$.

(iv) We can construct the IBF set weighted fuzzy decision matrix V^p and V^n as

$$V^p = [v_{ij}^p]_{m\times n} \text{ and } V^n = [v_{ij}^n]_{m\times n}, i = 1, 2, ..., m, j = 1, 2, 3, ..., n$$

where $v_{ij}^p = r_{ij}^p \otimes w_j$ and $v_{ij}^n = r_{ij}^n \otimes w_j$.

(v) We can define the positive part of the IBF set of positive-ideal solution $\dot{A}^p$ and negative-ideal solution $\ddot{A}^p$ from V^p as follows:

$$\dot{A}^p = (\dot{v}_1^p, \dot{v}_2^p, ..., \dot{v}_n^p) \text{ and } \ddot{A}^p = (\ddot{v}_1^p, \ddot{v}_2^p, ..., \ddot{v}_n^p)$$

where $\dot{v}_j^p = \max\limits_j v_{ij}^p$ and $\ddot{v}_j^p = \min\limits_j v_{ij}^p$, $j = 1, 2, 3, ..., n$.

The negative part of the IBF set of positive-ideal solution $\dot{A}^n$ and negative-ideal solution $\ddot{A}^n$ from V^n are

$$\dot{A}^n = (\dot{v}_1^n, \dot{v}_2^n, ..., \dot{v}_n^n) \text{ and } \ddot{A}^n = (\ddot{v}_1^n, \ddot{v}_2^n, ..., \ddot{v}_n^n)$$

where $\dot{v}_j^n = \min_j v_{ij}^n$ and $\ddot{v}_j^n = \max_j v_{ij}^n$, $j = 1, 2, 3, ..., n$.

(vi) The distance between $D^p = (\dot{d}^p, \ddot{d}^p)$ and $D^n = (\dot{d}^n, \ddot{d}^n)$ of each alternative from $\dot{A}^p$ and $\dot{A}^n$ can be currently calculated as

$$\dot{d}_i^p = \sum_{j=1}^{n} d(v_{ij}^p, \dot{v}_j^p) \text{ and } \ddot{d}_i^p = \sum_{j=1}^{n} d(v_{ij}^p, \ddot{v}_j^p),\ i = 1, 2, ..., m,$$

and

$$\dot{d}_i^n = \sum_{j=1}^{n} d(v_{ij}^n, \dot{v}_j^n) \text{ and } \ddot{d}_i^n = \sum_{j=1}^{n} d(v_{ij}^n, \ddot{v}_j^n),\ i = 1, 2, ..., m,$$

where $d(\cdot,\cdot)$ is the distance measurement between two fuzzy sets.

(vii) A closeness coefficient IBF set $C_i = (C_i^p, C_i^n)$ of each alternative A_i, where

$$C_i^p = \frac{\ddot{d}_i^p}{\dot{d}_i^p + \ddot{d}_i^p} \text{ and } C_i^n = -\frac{\ddot{d}_i^n}{\dot{d}_i^n + \ddot{d}_i^n}.$$

(viii) Calculate the score function $S(C_i)$ as follows:

$$S(C_i) = C_i^p + C_i^n.$$

Improve the score function as

$$I(C_i) = (C_i^p)^2 S(C_i) + (C_i^n)^2 S(C_i) - (C_i^p C_i^n) S(C_i),$$

and the Double Improve score function $D(C_i)$ as follows:

$$D(C_i) = S(C_i) + I(C_i).$$

After obtaining these score function values $D(C_i)$, select the best alternative given by $best(A_i) = \max\{D(C_i) | i = 1, 2, ..., m\}$.

4 A numerical example and solution of decision

In this section, we simulate a situation of the decision on linguistic variables with bipolar information. For example, we consider stock trading under environment changes affected by external parameters of the stock exchange that is being analyzed. We define the external environment using PEST analysis (P-Political, E-Economic, S-Social, T-Technology). PEST analysis generally considers four parameters and additional V-Volatile parameters. The proposed method is applied to solve this problem and the computational procedure is summarized as follows:

Step 1: A committee of three decision-makers, D_1, D_2, and D_3 has been formed to select the most suitable stock. There are four stocks that they chose from the set of alternatives A_1, A_2, A_3, and A_4 using criteria C_1, C_2, C_3, C_4, and C_5 for evolution, as shown in Table 1.

Table 1: The criteria for evaluation defined by decision-makers

Criteria	Descriptions
C_1 : Political	Political factors & the effects of political and administrative problems including laws
C_2 : Economics	the effects of economic condition and economic factors
C_3 : Social	the effects of social, culture, and living conditions
C_4 : Technology	Technology factors & the effects of technology and innovation of the country
C_5 : Volatile	the risk of stock price volatility is expected to occur

Step 2: The decision-makers create the linguistic weighting variables in Table 2 to assess the importance of the criteria and the ratings of interval bipolar criterion in Table 3.

Step 3: The decision-makers use the linguistic weighting variables to assess the importance of the criterion and present it in Table 4 and use the linguistic rating variables to evaluate the rating of alternatives concerning each criterion and present it in Table 5.

Step 4: Convert the linguistic evaluation (shown in Tables 4 and 5) into interval bipolar fuzzy sets to construct the fuzzy decision matrix and determine the fuzzy weight of each criterion as Table 6.

Table 2: Linguistic variables for the importance weight of each criterion

Very low (VL)	[0.1, 0.2]
Low (L)	[0.2, 0.3]
Medium low (ML)	[0.3, 0.5]
Medium (M)	[0.5, 0.6]
Medium high (MH)	[0.6, 0.8]
High (H)	[0.8, 0.9]
Very high (VH)	[0.9, 1.0]

Table 3: Linguistic variables for the ratings of interval bipolar criterion

Positive		Negative	
Very low (PVL)	[0.1, 0.2]	Very high (NVH)	[-1.0, -0.9]
Low (PL)	[0.2, 0.3]	High (NH)	[-0.9, -0.8]
Medium low (PML)	[0.3, 0.5]	Medium high (NMH)	[-0.8, -0.6]
Medium (PM)	[0.5, 0.6]	Medium (NM)	[-0.6, -0.5]
Medium high (PMG)	[0.6, 0.8]	Medium low (NML)	[-0.5, -0.3]
High (PH)	[0.8, 0.9]	Low (NL)	[-0.3, -0.2]
Very high (PVH)	[0.9, 1.0]	Very low (NVL)	[-0.2, -0.1]

Table 4: The importance weight of the criteria

	D_1	D_2	D_3
C_1	H	VH	MH
C_2	VH	VH	VH
C_3	VH	H	H
C_4	VH	VH	VH
C_5	M	MH	MH

Step 5: Construct the normalized fuzzy decision matrix $R = (R^p, R^n)$ as Table 7.

Step 6: Construct the weighted normalized fuzzy decision matrix as Table 8.

Step 7: Determine the positive and negative part IBF set and the distance

Table 5: The ratings of the four stocks by decision makers under all criteria

Criteria	Candidates	Decision-makers		
		D_1	D_2	D_3
C_1	A_1	(PH,NVH)	(PMH,NL)	(PH,NL)
	A_2	(PMH,NL)	(PH,NM)	(PMH,NL)
	A_3	(PVH,NL)	(PVH,NM)	(PH,NL)
	A_4	(PMH,NL)	(PH,NM)	(PH,NM)
C_2	A_1	(PMH,NL)	(PM,NL)	(PH,NVL)
	A_2	(PVH,NL)	(PVH,NL)	(PVH,NVL)
	A_3	(PH,NL)	(PVH,NM)	(PVH,NM)
	A_4	(PVH,NML)	(PVH,NM)	(PH,NM)
C_3	A_1	(PH,NML)	(PVH,NML)	(PVH,NL)
	A_2	(PVH,NH)	(PVH,NH)	(PH,NVL)
	A_3	(PMH,NM)	(PVH,NL)	(PH,NVL)
	A_4	(PMH,NM)	(PVH,NL)	(PH,NL)
C_4	A_1	(PM,NM)	(PMH,NM)	(PMH,NVL)
	A_2	(PVH,NH)	(PMH,NL)	(PH,NL)
	A_3	(PVH,NL)	(PMH,NL)	(PVH,NL)
	A_4	(PMH,NL)	(PMH,NL)	(PVH,NL)
C_5	A_1	(PML,NL)	(PML,NL)	(PML,NL)
	A_2	(PH,NL)	(PML,NL)	(PL,NL)
	A_3	(PH,NL)	(PH,NL)	(PMH,NVL)
	A_4	(PH,NL)	(PH,NVL)	(PM,NVL)

Table 6: The fuzzy decision matrix and fuzzy weights of four alternatives

		C_1	C_2	C_3	C_4	C_5
x^p	A_1	[0.7333 , 0.8667]	[0.6333 , 0.7667]	[0.8667 , 0.9667]	[0.5667 , 0.7333]	[0.3000 , 0.5000]
	A_2	[0.6667 , 0.8333]	[0.9000 , 1.0000]	[0.8667 , 0.9667]	[0.6667 , 0.8333]	[0.4333 , 0.5667]
	A_3	[0.7333 , 0.8667]	[0.8667 , 0.9667]	[0.7667 , 0.9000]	[0.7000 , 0.8667]	[0.7000 , 0.8000]
	A_4	[0.8667 , 0.9667]	[0.8667 , 0.9667]	[0.7667 , 0.9000]	[0.8000 , 0.9333]	[0.7333 , 0.8667]
x^n	A_1	[-0.5333 , -0.4333]	[-0.2667 , -0.1667]	[-0.4333 , -0.2667]	[-0.4667 , -0.3667]	[-0.3000 , -0.2000]
	A_2	[-0.3667 , -0.2667]	[-0.2667 , -0.1667]	[-0.6667 , -0.5667]	[-0.5000 , -0.4000]	[-0.3000 , -0.2000]
	A_3	[-0.4000 , -0.3000]	[-0.5667 , -0.4333]	[-0.4667 , -0.3667]	[-0.3000 , -0.2000]	[-0.2667 , -0.1667]
	A_4	[-0.3667 , -0.2667]	[-0.5000 , -0.4000]	[-0.4667 , -0.3667]	[-0.3000 , -0.2000]	[-0.2667 , -0.1667]
	Weight	[0.1933 , 0.1971]	[0.2269 , 0.2190]	[0.2101 , 0.2044]	[0.2269 , 0.2190]	[0.1429 , 0.1606]

Table 7: The fuzzy normalized decision matrix

		C_1	C_2	C_3	C_4	C_5
R^p	A_1	[0.7586 , 0.8966]	[0.6333 , 0.7667]	[0.8966 , 1.0000]	[0.6071 , 0.7857]	[1.0000 , 0.6000]
	A_2	[0.6897 , 0.8621]	[0.9000 , 1.0000]	[0.8966 , 1.0000]	[0.7143 , 0.8929]	[0.6923 , 0.5294]
	A_3	[0.7586 , 0.8966]	[0.8667 , 0.9667]	[0.7931 , 0.9310]	[0.7500 , 0.9286]	[0.4286 , 0.3750]
	A_4	[0.8966 , 1.0000]	[0.8667 , 0.9667]	[0.7931 , 0.9310]	[0.8571 , 1.0000]	[0.4091 , 0.3462]
R^n	A_1	[-2.0000 , -1.6250]	[-1.6000 , -1.0000]	[-1.6250 , -1.0000]	[-2.3333 , -1.8333]	[-1.0000 , -1.5000]
	A_2	[-1.3750 , -1.0000]	[-1.6000 , -1.0000]	[-2.5000 , -2.1250]	[-2.5000 , -2.0000]	[-1.0000 , -1.5000]
	A_3	[-1.5000 , -1.1250]	[-3.4000 , -2.6000]	[-1.7500 , -1.3750]	[-1.5000 , -1.0000]	[-1.1250 , -1.8000]
	A_4	[-1.3750 , -1.0000]	[-3.0000 , -2.4000]	[-1.7500 , -1.3750]	[-1.5000 , -1.0000]	[-1.1250 , -1.8000]

Table 8: The fuzzy weighted normalized decision matrix

		C_1	C_2	C_3	C_4	C_5
V^p	A_1	[0.1466 , 0.1767]	[0.1437 , 0.1679]	[0.1884 , 0.2044]	[0.1378 , 0.1721]	[0.1429 , 0.0964]
	A_2	[0.1333 , 0.1699]	[0.2042 , 0.2190]	[0.1884 , 0.2044]	[0.1621 , 0.1955]	[0.0989 , 0.0850]
	A_3	[0.1466 , 0.1767]	[0.1966 , 0.2117]	[0.1666 , 0.1903]	[0.1702 , 0.2033]	[0.0612 , 0.0602]
	A_4	[0.1733 , 0.1971]	[0.1966 , 0.2117]	[0.1666 , 0.1903]	[0.1945 , 0.2190]	[0.0584 , 0.0556]
V^n	A_1	[-0.3866 , -0.3203]	[-0.3630 , -0.2190]	[-0.3414 , -0.2044]	[-0.5294 , -0.4015]	[-0.1429 , -0.2409]
	A_2	[-0.2658 , -0.1971]	[-0.3630 , -0.2190]	[-0.5252 , -0.4343]	[-0.5672 , -0.4380]	[-0.1429 , -0.2409]
	A_3	[-0.2899 , -0.2217]	[-0.7714 , -0.5693]	[-0.3676 , -0.2810]	[-0.3403 , -0.2190]	[-0.1607 , -0.2891]
	A_4	[-0.2658 , -0.1971]	[-0.6807 , -0.5255]	[-0.3676 , -0.2810]	[-0.3403 , -0.2190]	[-0.1607 , -0.2891]

$D^p = (\dot{d}^p, \ddot{d}^p)$ and $D^n = (\dot{d}^n, \ddot{d}^n)$ of each alternative from $\dot{A}^p$ and $\dot{A}^n$ in Table 9.

Table 9: Calculate the distance of each candidates

	Distance			
Candidates	$\dot{d}^p$	$\ddot{d}^p$	$\dot{d}^n$	$\ddot{d}^n$
A_1	0.1033	0.0876	0.2567	0.5042
A_2	0.0696	0.0939	0.3519	0.4623
A_3	0.0924	0.0764	0.4463	0.3334
A_4	0.0871	0.1020	0.3689	0.3536

Step 8: Calculate the score function.

After obtaining the score function, $D(C_i)$ the best alternative is given by $best(A_i) = \max\{D(C_i)|i = 1, 2, ..., 4\}$, where $best(A_i)$ is given in Table 10. So, selecting among the four available stocks, the best stock is A_4.

Table 10: The score function and selected best alternative

Candidates	C_i^p	C_i^n	$S(C_i)$	$I(C_i)$	$D(C_i)$	$best(A_i)$
A_1	0.4587	-0.6627	-0.2040	-0.1516	-0.3556	4
A_2	0.5744	-0.5678	0.0066	0.0043	0.0108	3
A_3	0.4526	-0.4276	0.0250	0.0094	0.0344	2
A_4	0.5392	-0.4894	0.0498	0.0251	0.0749	1

5 Conclusion

Bipolar information is essential for decisions or evolutions and is an appropriate and usually necessary tool. We applied the IBF-TOPSIS algorithm which incorporates fuzzy TOPSIS and score functions for interval bipolar fuzzy sets. It is the tool of choice to represent bipolar information as a decision matrix which is then used to calculate the positive and negative parts of the alternatives. Alternatives with a maximum importance value are the best choices. The enhanced tools that we developed for multiple criteria decision-making theory naturally leads to better decision-making.

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