International Journal of Mathematics and
Computer Science, **17**(2022), no. 2, 509–524

The Average Run Length Performance of Shewhart Control Chart when the Process Data are Sampled from Finite Population

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(Received September 7, 2021, Accepted October 12, 2021)

Abstract

A Shewhart control chart is an important control chart which is widely used to monitor mean process and control the quality characteristic of a manufacturing process in order to achieve quality improvement. In some situations, the data may be sampled from a finite distribution and using Shewhart chart can lead to erroneous conclusions. We apply a Shewhart control chart with finite population correction (FPC) to investigate the ARL performance based on uniform, symmetric, and skewed distribution. The results show that the Shewhart control chart with FPC is more effective against the Shewhart control chart without FPC. Moreover, we investigate adjusted k (the distance of the control limits from the center line) values which are suitable for getting the approximate in-control ARL 370.

1 Introduction

In all production processes nowadays, keeping products at superior quality has become a necessary condition. A control chart is an important statistical process control (SPC) tool that helps achieve that goal. It is used

Key words and phrases: Shewhart control chart, finite population correction, average run length.
AMS (MOS) Subject Classifications: 62D05, 62P30.
Sudarat Nidsunkid is the corresponding author.
ISSN 1814-0432, 2022, http://ijmcs.future-in-tech.net

to achieve and maintain process stability by showing whether the underlying process is in a state of statistical control or not. Process stability is a state in which a process has displayed a certain degree of consistency in the past and is expected to continue to do so in the future. This consistency is characterized by a stream of data falling within the control limits; namely, the lower control limit (LCL) and the upper control limits (UCL). In practice, there are two distinct phases of control charting, and each case has unique control limit specifications. Lowry and Montgomery [1] stated that Phase I consists of using the charts for retrospectively testing whether the process was in control when the first m preliminary subgroups were being drawn and the sample statistics computed. The objective is to obtain an in-control set of data to establish control limits for future monitoring purposes. These control limits are used in Phase II to test whether the process remains in control, when future subgroups are drawn during the second phase. Therefore, Phase II consists of using the control chart to detect any departure of the underlying process and relies on the assumption that the in-control parameters are known.

The most known and commonly used control chart is Shewhart control chart which is capable of quickly detecting shifts in the testing process that are larger than 1.5σ [2], but it is less likely to be effective in Phase II because it is not very sensitive to small and moderate size process shifts [3]. Two other types of charts, exponentially weighted moving average chart (EWMA) and cumulative sum control chart (CUSUM) [4], are designed to detect shifts in the process mean that are smaller than 1.5σ.

A population represents all objects or individuals of interest. The important assumption of a Normal Distribution is often made before applying control charts, especially the Shewhart chart. For a quality characteristic X, if a sample of size n is taken, the ideas of sample size that affect the control chart are as follows: Costa [5] studied the properties of the variable sample size $\bar{X}$ chart when the size of each sample depends on what is observed in the preceding sample and concluded that the variable sample size $\bar{X}$ chart is substantially quicker than the traditional $\bar{X}$ chart in detecting moderate shifts in the process. Reynolds and Stoumbos [6] showed that the variable sampling interval $\bar{X}$ chart which allows the sampling interval to be varied enables a substantial reduction in the expected times in detecting shifts in process parameters. Sim et al. [7] considered the occurrence of double assignable causes in a process, adopted the Markov chain approach to investigate the statistical properties of the variable sample size $\bar{X}$ chart and suggested a procedure to compute the optimal sample size. Lin and Chou [8] proposed two adaptive $\bar{X}$

charts; i.e., the variable sampling rate with sampling at fixed times $\bar{X}$ chart and the variable parameters with sampling at fixed times chart. Umar et al. [9] applied the expected ATS criterion based on the Markov chain approach to assess the efficiency of the variable sampling interval technique into the auxiliary information based EWMA chart when the precise size of the shift cannot be specified.

In terms of size, population can be classified as either finite or infinite. For a random sample of size n from infinite population, it is well known that the variance of the mean is σ^2/n, where σ^2 is the population variance. When the population is finite, Cochran [10] introduced the factor $(N-n)/N$ for the variance and $\sqrt{(N-n)/N}$ for the standard error which are called the finite population correction (FPC). If a population size is large compared to the sample size; namely, the population size is more than 20 times the sample size, or the sampling fraction (SF), $n/N < 0.05$, then the FPC can be ignored. The situations that data are sampled from a finite distribution and using Shewhart chart can lead to erroneous conclusions. Khoo [11] proposed the modified $\bar{X}$ control chart for samples drawn from finite populations and concluded that the modified $\bar{X}$ chart produced reliable in-control and out-of-control ARL values which are close to the standard $\bar{X}$ chart where sampling from infinite populations.

In this paper, the Shewhart control chart for mean with FPC will be applied for the quality characteristics which is sampled from finite population such as uniform, symmetric, skewed left and skewed right. The ARLs are computed and reported to study the effect of finite population on Shewhart control chart.

2 Shewhart Control Chart

The Shewhart control chart plots current subgroup statistics and does not retain any process history. It is assumed that the process to be monitored yields some quality characteristic values, X, that are normally distributed with in-control value of the mean μ and standard deviation σ. If a sample of n independent units is taken from that population, then $\overline{X}$, is normally distributed with the mean μ and standard deviation $\sigma/\sqrt{n}$. Suppose m samples are available, each containing n observations on the quality characteristics. It is usually necessary to monitor the mean value of the quality characteristic. Control of the process average or mean quality level is usually done with the control chart for mean. Typically, a lower control limit (LCL) or an upper control limit (UCL) for the quality characteristics is required to construct the control chart.

When a quality characteristic $X \sim N(\mu, \sigma^2)$, the Shewhart control chart for the mean has the following control limits:

$$\begin{aligned} LCL &= \mu - k\frac{\sigma}{\sqrt{n}} \\ Center\ line &= \mu \\ UCL &= \mu + k\frac{\sigma}{\sqrt{n}}, \end{aligned} \tag{2.1}$$

where k represents the distance of the control limits from the center line, expressed as multiples of the sample standard deviation $\sigma/\sqrt{n}$. It is traditional to choose k=3 or 3-sigma limits.

3 The Finite Population Correction

For the quality characteristic X when using simple random sampling (SRS), the mean of sample mean is the population mean, that is: $\mu_{\overline{x}} = \mu$. The standard deviation of sample mean called the standard error equals to population standard deviation divided by the square root of the sample mean, that is: $\sigma_{\overline{x}} = \sigma/\sqrt{n}$. That formula is suitable if the sample is selected from an extremely large population [10]. However, for a finite population that is not extremely large, and the SRS is taken without replacement, a finite population correction (FPC) should be adjusted to the standard error term.

Definition 3.1. *A finite population correction (FPC) is*

$$\frac{N-n}{N} = 1 - \frac{n}{N} = 1 - f, \tag{3.2}$$

where N is the population size and f is the sampling fraction (SF).

Therefore, the standard deviation of the sample mean from SRS taken from a finite population is

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N}} = \frac{\sigma}{\sqrt{n}}\sqrt{1-\frac{n}{N}}. \tag{3.3}$$

If SF is small, then the FPC will be close to 1. That is, the population size has very little or no effect on the standard deviation of sample mean. It has been suggested that the FPC can be ignored of the SF is less than 5% [10].

4 The Shewhart Control Chart with Finite Population Correction

If we sample quality characteristic values X from finite population without replacement such as in a conveyer belt system [11], the use of control chart in (2.1) can lead to erroneous conclusions as it will cause an inflated Type-II error. Therefore, the FPC should be combined with the standard error term. The Shewhart control chart for mean with FPC has the following control limits:

$$\begin{aligned} LCL &= \mu - k\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N}} \\ Center\ line &= \mu \\ UCL &= \mu + k\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N}}. \end{aligned} \tag{4.4}$$

where k is the distance of the control limits from the center line.

5 Distribution of Finite Population

For this research, finite population with size N from discrete distributions such as uniform, symmetric, skewed left and skewed right are considered for sampling without replacement. The probability density function of a discrete random variable X is described by the set of possible values of X and the probabilities assigned to each value of X. The properties of a discrete probability distribution are

1. $0 \leq f(x) \leq 1$
2. $\sum f(x) = 1$.

Recall that the mean and variance of a discrete random variable X are denoted by $E(x) = \sum x \cdot f(x)$ and $Var(x) = \sum [x - E(x)]^2 \cdot f(x)$, respectively.

The probability density function of a discrete uniform distribution is given by $f(x) = \frac{1}{N}$, where $x = 1, 2, , N$. The mean and variance for a uniform distribution are defined as $E(x) = \frac{N+1}{2}$ and $Var(x) = \frac{(N-1)(N+1)}{12}$, respectively [12]. Suppose a random variable X has possible values of $1, 2, ..., 160$. The probability density function of X is $f(x) = \frac{1}{160}$, where $x = 1, 2, , 160$. The mean and the variance are $E(x) = \frac{160+1}{2} = 80.5$ and $Var(x) = \frac{(160-1)(160+1)}{12} = 2,133.25$, respectively.

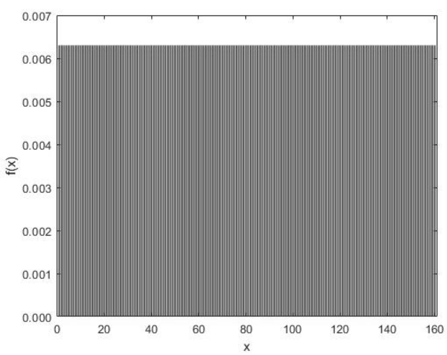
For discrete symmetric, skewed left, and skewed right distributions, the properties of discrete distribution are applied to define each of probability density functions. Table 1 shows the probability density function for discrete symmetric, skewed left, and skewed right distributions assuming population size $N = 160$ and the probability density function plot for each discrete distribution when assuming $N = 160$ are shown in Figure 1.

Table 1: The probability density function for discrete symmetric, skewed left, and skewed right distributions assuming population size $N = 160$.

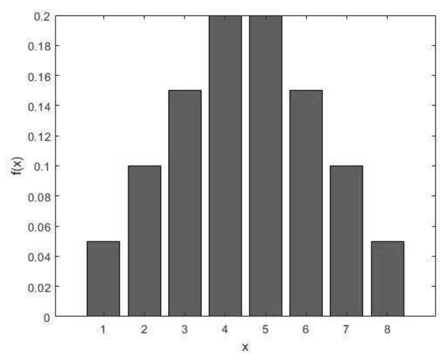
X	The probability density function: $f(x)$		
	symmetric	skewed left	skewed right
1	$\frac{8}{160}$	$\frac{8}{160}$	$\frac{16}{160}$
2	$\frac{16}{160}$	$\frac{8}{160}$	$\frac{24}{160}$
3	$\frac{24}{160}$	$\frac{16}{160}$	$\frac{32}{160}$
4	$\frac{32}{160}$	$\frac{24}{160}$	$\frac{32}{160}$
5	$\frac{32}{160}$	$\frac{32}{160}$	$\frac{24}{160}$
6	$\frac{24}{160}$	$\frac{32}{160}$	$\frac{16}{160}$
7	$\frac{16}{160}$	$\frac{24}{160}$	$\frac{8}{160}$
8	$\frac{8}{160}$	$\frac{16}{160}$	$\frac{8}{160}$
$\sum f(x)$	1	1	1
$E(x)$	4.5	5.1	3.9
$Var(x)$	3.25	3.49	3.49

6 Run Length Properties

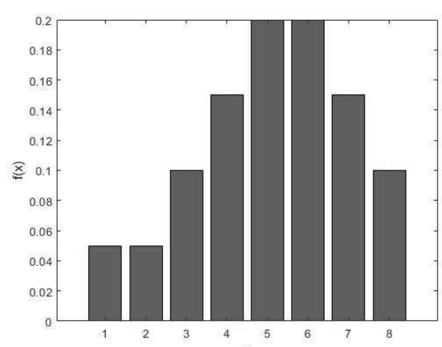
For a control chart, the run length is a random variable and is defined to be the number of subgroups, which must be collected (or equivalently, the number of charting statistics that must be plotted) until the first signal is observed suggesting a change from the in-control process. In practice, the performance of a control chart is considered in terms of certain measures



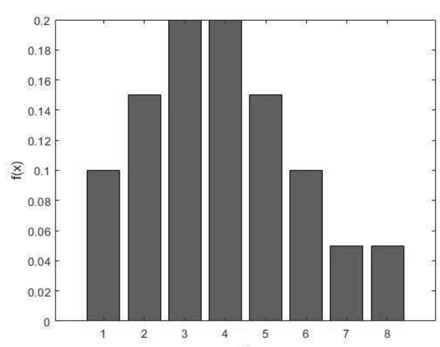
(a) Uniform



(b) Symmetric



(c) Skewed left



(d) Skewed right

Figure 1: The probability density function plot for each discrete distribution when assuming $N = 160$.

associated with its ARL. The ARL is an average of the number of subgroups sampled (or number of charting statistics needed to be plotted) before an out-of-control signal is detected, respectively.

7 Methodology and Simulation Study

In this article, we consider a Shewhart control chart which monitors the mean of the process based on sampling data from uniform, symmetric, skewed left and skewed right finite population of size $N = 80, 160, 320$ and 640. Data were simulated for SF $= 0.10, 0.15$ and 0.20 which correspond to sample size n. The multiple of the sample standard deviation $k = 3$ is specified to control limits. The shift in the mean such as $0, 0.1\sigma, 0.2\sigma$ and 0.5σ are chosen for variables. For each shift, this was repeated $50,000$ times generating sets of $50,000$ simulated run lengths. The ARL of a Shewhart chart with FPC and without FPC on control limits are compared to the ARL of Shewhart chart assuming sampling from normally infinite population.

8 Results

The results from this study concern the ARL values. Tables 2 to 5 summarized the estimated ARL of the Shewhart control chart when the quality characteristics are from normal infinite population and when the quality characteristics are samples from finite population. In cases of sampling from finite population, control chart with FPC and without FPC are applied. The in-control ARL for normal infinite population are approximately 370 for all cases of sample size. When the mean of process is changed, the out-of-control ARL varies inversely with size of the shift in mean. Therefore, the sizes of shift affect out-of-control ARLs.

If the quality characteristic is sampled from a finite population and a Shewhart control chart is run without FPC, there is a significant increase in both the in-control and out-of-control ARLs for all distributions. For each population size N and each distribution, when sample size n increases, the in-control ARL also increases. However, a reduction in out-of-control ARL occurs. Moreover, if the size of population is larger, the ARLs will decrease but still greater than the normal infinite ARLs.

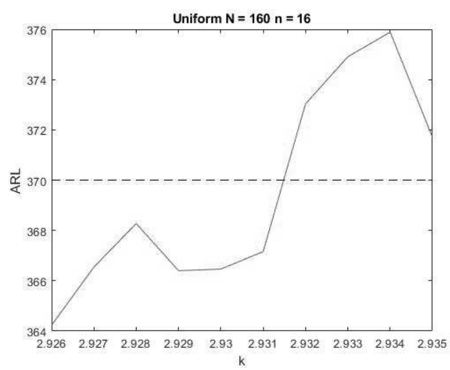
When sampling data is from a finite population and is detected by the process control with FPC, the in-control ARL are greater than 370 when the population sizes are small but close to 370 when the size of population is larger. In addition, the out-of-control ARL with FPC are smaller than the normal out-of-control ARL when the size of shift and the value of SF are larger. The sample size n also affects the ARL, at the same size of N, we can see that the sample size n yields more effective ARL than the smaller n. However, there are some cases that the in-control ARLs less than 370 such as when sampling $n = 64$ from $N = 320$, sampling $n = 96$ from $N = 640$ and sampling $n = 128$ from $N = 640$ from finite symmetric distribution.

Tables 2 to 5 also confirm what is expected as the shifts in mean become larger. That is, the ARL converge to zero for all cases as the magnitude in shifts increases. For each distribution, the large sizes of population produce reliable in-control and out-of-control ARL values which are close to that of the ARLs where sampling is made from an infinite population.

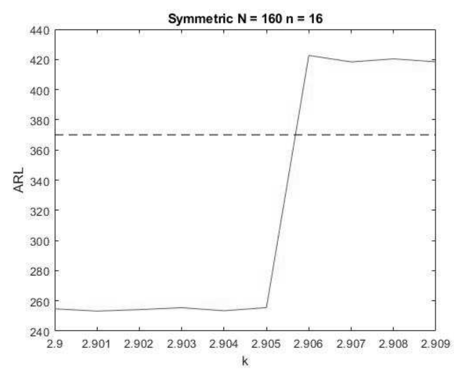
Moreover, we can adjust values of k for in-control process of a Shewhart chart with FPC that provide the same approximate in-control ARL as the normally infinite population. The most of adjusted k values should be ranged from 2.85 to 2.99 exclude when sampling $n = 64$ from $N = 320$, sampling $n = 96$ from $N = 640$ and sampling $n = 128$ from $N = 640$ from symmetric distribution which have the in-control ARL less than 370. Figure 2 shows an example of plots between values of k and ARL when sampling $n = 16$ from finite population size $N = 160$.

9 Conclusions

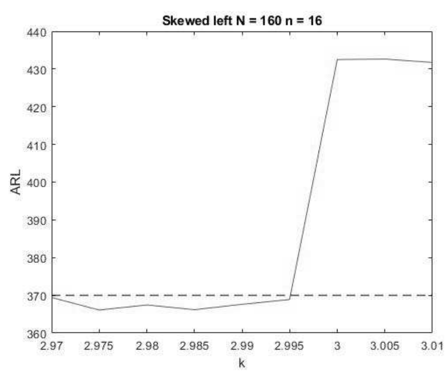
Because of the ARL performance, when the sampling data are from finite population, the Shewhart control chart with FPC is more effective against the Shewhart control chart without FPC. For all finite distribution such as uniform, symmetry, skewed left and skew right, the differences in ARLs between the Shewhart chart with FPC and the Shewhart chart with normally infinite population will become smaller as the sample size increases due to the Central Limit Theorem. The Shewhart control chart with FPC using the distance of the control limits from the center line: k which is slightly less than 3 yields similar approximate in-control ARL as the normally infinite population.



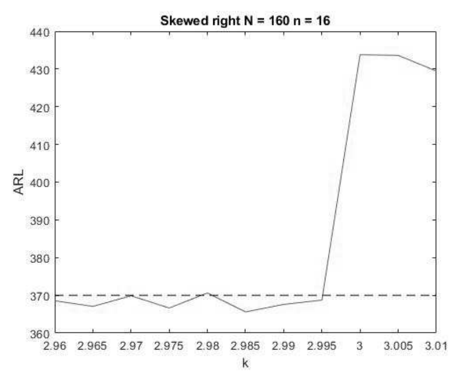
(a) Uniform



(b) Symmetric



(c) Skewed left



(d) Skewed right

Figure 2: Plots between values of and ARL when sampling n = 16 from finite populations size N = 160 (or SF = 0.10) for each distribution.

Table 2: The ARL of the Shewhart control chart for monitoring the mean when the quality characteristics are from normally infinite population with size n and when sampling from different finite populations of size $N = 80$.

			Shift in Mean			
			0	0.10σ	0.20σ	0.50σ
Sampling $n = 8$ from normally infinite population			369.72	264.47	129.76	17.67
Sampling $n = 8$ from finite population (SF = 0.10)	Uniform	w/out FPC	1275.26	662.61	231.91	21.12
		with FPC	670.32	378.58	152.29	15.65
	Symmetric	w/out FPC	1147.06	430.69	229.98	24.87
		with FPC	503.46	428.22	122.15	16.10
	Skewed left	w/out FPC	1060.41	1037.08	292.77	26.01
		with FPC	603.54	446.34	284.51	16.57
	Skewed right	w/out FPC	1059.46	364.78	204.09	18.76
		with FPC	599.75	201.28	118.05	12.89
Sampling $n = 12$ from normally infinite population			370.39	225.96	93.70	9.79
Sampling $n = 12$ from finite population (SF = 0.15)	Uniform	w/out FPC	1393.40	612.38	179.93	11.86
		with FPC	551.63	273.38	91.03	7.79
	Symmetric	w/out FPC	1029.22	544.20	180.35	10.17
		with FPC	520.92	290.68	106.97	7.55
	Skewed left	w/out FPC	1268.17	859.50	273.53	12.26
		with FPC	434.95	443.06	93.52	9.03
	Skewed right	w/out FPC	1268.50	544.30	125.07	13.19
		with FPC	436.82	196.57	80.62	7.45
Sampling $n = 16$ from normally infinite population			370.80	200.63	71.62	6.28
Sampling $n = 16$ from finite population (SF = 0.20)	Uniform	w/out FPC	1813.99	644.46	154.48	7.46
		with FPC	483.72	209.00	60.45	4.63
	Symmetric	w/out FPC	1569.43	533.04	128.06	7.99
		with FPC	472.29	192.61	54.56	5.00
	Skewed left	w/out FPC	1546.78	1046.61	221.01	8.29
		with FPC	421.19	210.10	58.40	4.22
	Skewed right	w/out FPC	1546.91	408.65	111.73	6.81
		with FPC	425.42	164.85	52.38	4.44

Table 3: The ARL of the Shewhart control chart for monitoring the mean when the quality characteristics are from normally infinite population with size n and when sampling from different finite populations of size $N = 160$.

			Shift in Mean			
			0	0.10σ	0.20σ	0.50σ
Sampling $n = 16$ from normally infinite population			370.80	200.63	71.62	6.28
Sampling $n = 16$ from finite population (SF = 0.10)	Uniform	w/out FPC	833.77	358.08	102.52	6.85
		with FPC	476.82	217.6	67.54	5.42
	Symmetric	w/out FPC	710.54	302.71	86.65	7.25
		with FPC	422.32	192.32	59.20	5.74
	Skewed left	w/out FPC	700.67	536.72	146.02	7.49
		with FPC	431.45	322.64	95.87	6.01
	Skewed right	w/out FPC	700.80	235.53	76.98	6.23
		with FPC	432.70	327.62	54.33	5.11
Sampling $n = 24$ from normally infinite population			370.64	160.22	45.98	3.45
Sampling $n = 24$ from finite population (SF = 0.15)	Uniform	w/out FPC	1071.12	328.90	71.39	3.61
		with FPC	435.01	154.97	38.71	2.75
	Symmetric	w/out FPC	1016.51	359.35	63.58	3.43
		with FPC	418.66	168.56	34.95	2.63
	Skewed left	w/out FPC	1036.36	411.77	70.03	3.61
		with FPC	442.12	187.91	37.60	2.78
	Skewed right	w/out FPC	1048.83	263.02	72.62	3.56
		with FPC	444.88	133.56	41.24	2.75
Sampling $n = 32$ from normally infinite population			370.35	130.64	32.34	2.31
Sampling $n = 32$ from finite population (SF = 0.20)	Uniform	w/out FPC	1504.95	327.73	55.05	2.35
		with FPC	424.48	117.01	24.42	1.78
	Symmetric	w/out FPC	1386.75	283.02	61.45	2.30
		with FPC	417.96	108.39	22.09	1.77
	Skewed left	w/out FPC	1551.83	456.43	68.05	2.45
		with FPC	421.25	119.66	24.21	1.74
	Skewed right	w/out FPC	1540.89	277.38	50.71	2.35
		with FPC	423.08	111.96	24.77	1.81

Table 4: The ARL of the Shewhart control chart for monitoring the mean when the quality characteristics are from normally infinite population with size n and when sampling from different finite populations of size $N = 320$.

			Shift in Mean			
			0	0.10σ	0.20σ	0.50σ
Sampling $n = 32$ from normally infinite population			370.36	130.99	32.29	2.32
Sampling $n = 32$ from finite population (SF = 0.10)	Uniform	w/out FPC	723.79	200.95	41.15	2.34
		with FPC	416.86	126.35	28.55	2.04
	Symmetric	w/out FPC	671.76	177.02	45.83	2.29
		with FPC	468.69	132.44	28.41	2.08
	Skewed left	w/out FPC	643.82	202.29	39.02	2.42
		with FPC	370.51	147.30	30.63	2.03
	Skewed right	w/out FPC	643.81	168.57	38.35	2.33
		with FPC	372.58	99.70	24.73	2.12
Sampling $n = 48$ from normally infinite population			369.18	93.64	18.69	1.47
Sampling $n = 48$ from finite population (SF = 0.15)	Uniform	w/out FPC	963.91	166.57	24.97	1.45
		with FPC	400.48	83.13	14.87	1.29
	Symmetric	w/out FPC	941.96	151.68	26.33	1.46
		with FPC	383.88	74.72	15.57	1.30
	Skewed left	w/out FPC	987.97	175.34	24.89	1.42
		with FPC	408.56	85.77	14.59	1.28
	Skewed right	w/out FPC	993.36	165.10	26.20	1.47
		with FPC	412.99	83.82	15.71	1.31
Sampling $n = 64$ from normally infinite population			370.68	71.44	12.32	1.19
Sampling $n = 64$ from finite population (SF = 0.20)	Uniform	w/out FPC	1368.23	146.84	16.91	1.15
		with FPC	395.28	57.61	8.85	1.08
	Symmetric	w/out FPC	1433.29	137.99	17.79	1.15
		with FPC	366.93	61.08	8.69	1.08
	Skewed left	w/out FPC	1392.75	166.17	17.50	1.15
		with FPC	374.66	58.75	8.73	1.07
	Skewed right	w/out FPC	1384.56	135.63	16.64	1.15
		with FPC	373.42	51.84	8.49	1.07

Table 5: The ARL of the Shewhart control chart for monitoring the mean when the quality characteristics are from normally infinite population with size n and when sampling from different finite populations of size $N = 640$.

			Shift in Mean			
			0	0.10σ	0.20σ	0.50σ
Sampling $n = 64$ from normally infinite population			370.95	71.35	12.49	1.18
Sampling $n = 64$ from finite population (SF = 0.10)	Uniform	w/out FPC	682.14	99.41	14.21	1.17
		with FPC	393.71	65.40	10.60	1.13
	Symmetric	w/out FPC	707.26	93.03	14.94	1.17
		with FPC	427.44	64.11	11.36	1.13
	Skewed left	w/out FPC	684.70	111.31	14.72	1.18
		with FPC	421.51	75.43	11.30	1.14
	Skewed right	w/out FPC	690.28	91.58	13.91	1.17
		with FPC	423.10	63.95	13.91	1.17
Sampling $n = 96$ from normally infinite population			370.71	46.09	6.72	1.03
Sampling $n = 96$ from finite population (SF = 0.15)	Uniform	w/out FPC	923.83	70.73	7.75	1.02
		with FPC	386.06	38.39	5.25	1.01
	Symmetric	w/out FPC	1013.53	68.36	7.83	1.02
		with FPC	352.41	37.49	5.34	1.01
	Skewed left	w/out FPC	916.94	73.24	7.60	1.02
		with FPC	408.29	40.01	5.30	1.01
	Skewed right	w/out FPC	923.21	68.78	7.76	1.02
		with FPC	407.24	38.95	5.40	1.01
Sampling $n = 128$ from normally infinite population			370.58	32.38	4.37	1.00
Sampling $n = 128$ from finite population (SF = 0.20)	Uniform	w/out FPC	1305.56	54.51	4.89	1.00
		with FPC	385.54	24.24	3.16	1.00
	Symmetric	w/out FPC	1381.92	56.96	5.00	1.00
		with FPC	362.12	23.61	3.13	1.00
	Skewed left	w/out FPC	1305.65	57.64	4.91	1.00
		with FPC	398.16	24.22	3.08	1.00
	Skewed right	w/out FPC	1314.03	51.91	4.77	1.00
		with FPC	398.28	25.22	3.23	1.00

Acknowledgment. This work was supported by a research grant from International SciKU Branding, Faculty of Science, Kasetsart University.

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