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The solitonic electron acoustic electrostatic waves in two dimensions

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Abstract

We theoretically explore Electron Acoustic (EA) features in a superthermal two ion plasma by reducing the basic fluid model to a KP equation. Using the physical characteristics of KP equation solitons, we examine the two dimension electrostatic EA nonlinear existence. The present investigation could be valuable in explaining the Satellite observations of electrostatic bipolar field pulses in EMBLPS.

1 Introduction

The nonlinear solitary structures in fluid models in space and laboratories is one of the most important studies in recent years [1-2]. The electrostatic electron-acoustic nonlinear evolutions (EEANs) in plasma fluids of two different temperature species of ions or electrons is a significant tool for investigating wave nature in laboratory and space [3-4]. Broadband Electrostatic Noise (BEN) in plasma sheet boundary layer has been established by measurements of Satellites [1-4]. On the other hand, the particles that have excess superthermal kappa statistical forms play a development and effective factor in confirming the theoretical studies with the observations [4-5].

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El-Shewy studied the spectral kappa parameter effect on the electrostatic structure of higher order solitons [6]. Moreover, Kakad et al. [7] studied the EEANs generations in a plasma of four components without relativistic speeds between hot and cold ion components. The aim of this work is to study the ionic temperatures and superthermal effects on the properties of solitonic EA nonlinear waves in two-dimensional model investigated by the KP equation.

2 Basic equations and the KP equation

Consider a collisionless unmagnetized homogeneous plasma that consists of a cold fluid of electrons and lower-higher temperature ions obeying superthermal distributions. This mode is governed in two dimensions by normalized equations [1]:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_{ex}) + \frac{\partial}{\partial y}(n_e u_{ey}) = 0, \tag{2.1}$$

$$\frac{\partial u_e}{\partial t} + u_{ex}\frac{\partial u_{ex}}{\partial x} + u_{ey}\frac{\partial u_{ex}}{\partial y} - \frac{\partial \varphi}{\partial x} = 0, \tag{2.2}$$

$$\frac{\partial u_{ey}}{\partial t} + u_{ex}\frac{\partial u_{ey}}{\partial x} + u_{ey}\frac{\partial u_{ey}}{\partial y} - \frac{\partial \varphi}{\partial y} = 0, \tag{2.3}$$

and Poisson's equation,

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = (n_e - \mu\left(1 + \frac{\varphi}{(\mu+\nu\sigma)(\kappa_c - 3/2)}\right)^{-\kappa_c + 1l2} - \nu\left(1 + \frac{\sigma\varphi}{(\mu+\nu\sigma)(\kappa_h - 3/2)}\right)^{-\kappa_h + 1l2}). \tag{2.4}$$

In equations (2.1) - (2.2), the cold density of electrons n_e is normalized by equilibrium density $n_e^{(0)}$, the fluid velocities $u_{ex,y}$ in the x- and y-directions are normalized by $C_{eff} = (T_{eff}/m_e)^{1/2}$ (effective acoustic speed of electrons, where $T_{eff} = \frac{T_c}{\mu+\nu\sigma}$, the $x(y), t$ and potential φ are normalized by $\lambda_{Deff} = \left(T_{eff}/4\pi n_e^{(0)} e^2\right)^{1/2}$, $\omega_{ce}^{-1}\left(\omega_{ce} = \left(4\pi n_e^{(0)} e^2/m_e\right)^{1/2}\right)$ and T_{eff}/e. Here, $\nu = n_{ih}^{(0)}/n_e^{(0)}$, $\mu = n_{ic}^{(0)}/n_e^{(0)}$ where, $n_{ih}^{(0)}$ and $n_{ic}^{(0)}$ are the initial densities of higher- and lower temperature ions with condition of neutrality $\mu+\nu = 1$. Also, $\sigma =$

T_c/T_h is the temperature ratio of ions and $\kappa_c\,(\kappa_h)$ is superthermal parameter. To derive nonlinear KP equation in finite and small amplitude EA waves, we use the stretched coordinate forms:

$$\tau = \epsilon^3 t\,, \qquad \chi = \epsilon(x-\lambda t)\,, \qquad \text{and} \qquad Y = \epsilon^2 y, \tag{2.5}$$

where ϵ is a dimensionless small parameter and λ is EA phase velocity.

All quantities in (2.1) - (2.2) are expanded in ϵ about the values of equilibrium as:

$$\begin{aligned} n_e &= 1+\epsilon^2 n_e^{(1)} + \epsilon^4 n_e^{(2)} + \epsilon^6 n_e^{(3)} + ..., \\ u_{ex} &= \epsilon^2 (u_{ex}^{(1)} + \epsilon^2 u_{ex}^{(2)} + \epsilon^4 u_{ex}^{(3)} + ...), \\ u_{ey} &= \epsilon^3 (u_{ey}^{(1)} + \epsilon^2 u_{ey}^{(2)} + \epsilon^4 u_{ey}^{(3)} + ...), \\ \varphi &= \epsilon^2\ (\varphi^{(1)} + \epsilon^2 \varphi^{(2)} + \epsilon^4 \varphi^{(3)} + ...\ ). \end{aligned} \tag{2.6}$$

Boundary conditions are imposed as $|\chi| \to \infty,\ u_{ex} = u_{ey} = 0, n_e = 1$. Using (2.3) and (2.4) in (2.1) - (2.2), equating like power coefficients of ϵ. The linear dispersion equation:

$$\lambda = \sqrt{\frac{(2\kappa_c-3)(2\kappa_h-3)(\mu+\nu\sigma)}{(2\kappa_c-1)(2\kappa_h-3)\mu + (2\kappa_c-3)(2\kappa_h-1)\nu\sigma}}. \tag{2.7}$$

And the KP equation in $\varphi^{(1)}$ has the form:

$$\frac{\partial}{\partial\chi}\left(\frac{\partial\varphi^{(1)}}{\partial\tau} + R\,\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\chi} + S\frac{\partial^3\varphi^{(1)}}{\partial\chi^3}\right) + \frac{\lambda}{2}\frac{\partial^2\varphi^{(1)}}{\partial Y^2} = 0, \tag{2.8}$$

where,

$$\begin{aligned} R &= \lambda^3\left(\frac{(2\kappa_c-1)(2\kappa_c+1)\mu}{2(2\kappa_c-3)^2(\mu+\nu\sigma)^2} + \frac{(2\kappa_h-1)(2\kappa_h+1)\nu\sigma^2}{2(\mu+\nu\sigma)^2}\right) - \frac{3}{2\lambda}, \\ S &= \frac{\lambda^3}{2}. \end{aligned} \tag{2.9}$$

Using the transformations:$\varphi^{(1)}(\chi,\tau) = \Phi(\zeta)$ and $\zeta = L\chi + MY - \tau\vartheta$,where L and M are cosines in χ and Y directions, respectively, ϑ is acoustic speed. The solution of equation (2.6) take the form:

$$\begin{aligned} \Phi(\zeta) &= \frac{3h_0}{G1}\text{sech}^2\left(\zeta\sqrt{h_0}/2\sqrt{G2}\right), \\ G1 &= L\ R, G2 = L^3\ S, h_0 = -\frac{\lambda M^2}{2L} + v. \end{aligned} \tag{2.10}$$

Established on these results, the model can support two potential structure solitons; namely, compressive potential and rarefactive potential that rely on the nonlinear coefficient sign. Compressive potential exists if $\mu > \mu_c$ while rarefactive potential exist for $\mu < \mu_c$, where μ_c is the hot ions critical density that make the nonlinear coefficient vanishes. When $\mu = \mu_c$,

$$\mu_c = \frac{\lambda^4 \left(4\kappa_c^2 - (3-2\kappa_c)^2 \left(4\kappa_h^2 - 1\right)\sigma^2 - 1\right) + 6(3-2\kappa_c)^2(\sigma-1)\sigma}{6(3-2\kappa_c)^2(\sigma-1)^2} \pm$$
$$\frac{\lambda^2}{6(3-2\kappa_c)^2(\sigma-1)^2}(\lambda^4 \left(-4\kappa_c^2 + (3-2\kappa_c)^2\left(4\kappa_h^2 - 1\right)\sigma^2 + 1\right)^2 - 12(3-2\kappa_c)^2$$
$$\sigma(\sigma-1)\left(-4\kappa_c^2 + (3-2\kappa_c)^2\left(4\kappa_h^2-1\right)\sigma + 1\right))^{\frac{1}{2}}, \qquad (2.11)$$

the coefficient $R = 0$ and KP equation has only a dispersion properties which demolish the soliton.

3 Results and Discussion

The propagated electrostatic EA nonlinear wave existence and energy in two dimensions have been discussed for superthermal ion plasma. Numerical results in this work used parameter values appropriate for Earth's magnetotail boundary layer plasma sheets [8-9]. The model of equations (2.1)-(2.4) reduced to the KP equation (2.8) which have all the nonlinear and dispersion properties of this model and produce soliton solution in forms of rarefactive and compressive wave (2.10) with critical density μ_c. For $n_{ic}^{(0)} = 0.035\ cm^{-3}$, $n_{ih}^{(0)} = 0.065\ cm^{-3}$, $n_e^{(0)} = 0.1\ cm^{-3}$, $T_c = 50\ eV, T_h = 100\ eV$ and $\theta = 31.8°$ this model agree with [7].

4 Conclusions

The properties of solitonic EA nonlinear two-dimensional superthermal ion plasma have been obtained by obtaining the KP equation and its related features. Also, the applications of these results might be very interesting in the broadband of earth's plasmas sheets. On the other hand, for $\kappa_h\,(\kappa_c) \to \infty$, this model results agree with Kakad et al. in one dimension [1] and Elwakil et al. in two dimensions [9].

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