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Almost (Λ, sp)-continuous multifunctions

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Abstract

Our purpose is to introduce the concept of almost (Λ, sp)-continuous multifunctions. In particular, we investigate some characterizations of almost (Λ, sp)-continuous multifunctions.

1 Introduction

In 1968, Singal and Singal [6] introduced and studied the notion of almost continuous functions. In 1982, Popa [5] extended the concept of almost continuous functions to multifunctions and introduced the notions of upper and lower continuous multifunctions. In 1983, Abd El-Monsef et al. [1] introduced a weak form of open sets called β-open sets. In 2004, Noiri and Hatir [4] defined Λ_{sp}-sets in terms of the concept of β-open sets and investigated the notion of Λ_{sp}-closed sets by using Λ_{sp}-sets. Boonpok [2] introduced the concepts of (Λ, sp)-closed sets and (Λ, sp)-open sets which are defined by utilizing the notions of Λ_{sp}-sets and β-closed sets. The purpose of the present paper is to introduce the notion of almost (Λ, sp)-continuous multifunctions. Moreover, we discuss several characterizations of almost (Λ, sp)-continuous multifunctions.

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2 Preliminaries

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is called *β-open* [1] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$. The complement of a β-open set is called *β-closed*. The family of all β-open sets of a topological space (X, τ) is denoted by $\beta(X, \tau)$. A subset $\Lambda_{sp}(A)$ [4] is defined as follows:

$$\Lambda_{sp}(A) = \cap\{U \mid A \subseteq U, U \in \beta(X, \tau)\}.$$

A subset A of a topological space (X, τ) is called a *Λ_{sp}-set* [4] if $A = \Lambda_{sp}(A)$. A subset A of a topological space (X, τ) is said to be *(Λ, sp)-closed* [2] if $A = T \cap C$, where T is a Λ_{sp}-set and C is a β-closed set. The complement of a (Λ, sp)-closed set is called *(Λ, sp)-open*. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, sp)-cluster point* [2] of A if $A \cap U \neq \emptyset$ for every (Λ, sp)-open set U of X containing x. The set of all (Λ, sp)-cluster points of A is called the *(Λ, sp)-closure* [2] of A and is denoted by $A^{(\Lambda,sp)}$. The union of all (Λ, sp)-open sets contained in A is called the *(Λ, sp)-interior* [2] of A and is denoted by $A_{(\Lambda,sp)}$.

By a multifunction $F : (X, \tau) \to (Y, \sigma)$, following [3], we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and

$$F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}.$$

In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$ and for each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$. Let $\mathcal{P}(Y)$ be the collection of all nonempty subsets of Y. For any (Λ, sp)-open set V of a topological space (Y, σ), we denote $V^+ = \{B \in \mathcal{P}(Y) \mid B \subseteq V\}$ and $V^- = \{B \in \mathcal{P}(Y) \mid B \cap V \neq \emptyset\}$.

3 Almost (Λ, sp)-continuous multifunctions

In this section, we introduce the notion of almost (Λ, sp)-continuous multifunctions. Moreover, we discuss several characterizations of almost (Λ, sp)-continuous multifunctions.

Definition 3.1. *A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be almost (Λ, sp)-continuous if, for each $x \in X$ and each (Λ, sp)-open sets V_1, V_2 of Y*

such that $F(x) \in V_1^+ \cap V_2^-$*, there exists a* (Λ, sp)*-open set* U *of* X *containing* x *such that* $F(U) \subseteq [V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}$ *and* $F(z) \cap [V_2^{(\Lambda,sp)}]_{(\Lambda,sp)} \neq \emptyset$ *for every* $z \in U$.

Theorem 3.2. *For a multifunction* $F : (X, \tau) \to (Y, \sigma)$*, the following properties are equivalent:*

(1) F *is almost* (Λ, sp)*-continuous;*

(2) $F^+(V_1) \cap F^-(V_2) \subseteq [F^+([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}$ *for every* (Λ, sp)*-open sets* V_1, V_2 *of* Y*;*

(3) $[F^-([[K_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cup F^+([[K_2]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(K_1) \cup F^+(K_2)$ *for every* (Λ, sp)*-closed sets* K_1, K_2 *of* Y*;*

(4)

$$[F^-([[B_1^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cup F^+([[B_2^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)}$$
$$\subseteq F^-(B_1^{(\Lambda,sp)}) \cup F^+(B_2^{(\Lambda,sp)})$$

for every subsets B_1, B_2 *of* Y*;*

(5)

$$F^+([B_1]_{(\Lambda,sp)}) \cap F^-([B_2]_{(\Lambda,sp)})$$
$$\subseteq [F^+([[[B_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([[[B_2]_{(\Lambda,sp)})]^{(\Lambda,sp)}]_{(\Lambda,sp)}]_{(\Lambda,sp)}$$

for every subsets B_1, B_2 *of* Y.

Proof. (1) ⇒ (2): Let V_1, V_2 be any (Λ, sp)-open sets of Y such that

$$x \in F^+(V_1) \cap F^-(V_2).$$

Then, $F(x) \in V_1^+ \cap V_2^-$ and hence there exists a (Λ, sp)-open set U of X containing x such that $F(U) \subseteq [V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}$ and $F(z) \cap [V_2^{(\Lambda,sp)}]_{(\Lambda,sp)} \neq \emptyset$ for each $z \in U$. Thus, $U \subseteq F^+([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})$ and hence

$$x \in [F^+([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}.$$

Therefore, $F^+(V_1) \cap F^-(V_2) \subseteq [F^+([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}$.

(2) ⇒ (3): Let K_1, K_2 be any (Λ, sp)-closed sets of Y. Then, $Y - K_1$ and $Y - K_2$ are (Λ, sp)-open sets of Y, by (2),

$$\begin{aligned}
&X - [F^-(K_1) \cup F^+(K_2)]\\
&= F^+(Y - K_1) \cap F^-(Y - K_2)\\
&\subseteq [F^+([[Y - K_1]^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([[Y - K_2]^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}\\
&= [(X - F^-([[K_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cap (X - F^+([[K_2]_{(\Lambda,sp)}]^{(\Lambda,sp)})]_{(\Lambda,sp)}\\
&= X - [F^-([[K_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cup F^+([[K_2]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)}
\end{aligned}$$

and hence

$$[F^-([[K_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cup F^+([[K_2]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(K_1) \cup F^+(K_2).$$

(3) ⇒ (4): Let B_1, B_2 be any subsets of Y. Then, $B_1^{(\Lambda,sp)}$ and $B_2^{(\Lambda,sp)}$ are (Λ, sp)-closed in Y, by (3),

$$\begin{aligned}
&[F^-([[B_1^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cup F^+([[B_2^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)}\\
&\subseteq F^-(B_1^{(\Lambda,sp)}) \cup F^+(B_2^{(\Lambda,sp)}).
\end{aligned}$$

(4) ⇒ (5): Let B_1, B_2 be any subsets of Y. Thus, by (4),

$$\begin{aligned}
&F^-([B_1]_{(\Lambda,sp)}) \cap F^+([B_2]_{(\Lambda,sp)})\\
&= X - [F^+([Y - B_1]^{(\Lambda,sp)}) \cup F^-([Y - B_2]^{(\Lambda,sp)})]\\
&\subseteq X - [F^+([[[Y - B_1]^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cup F^-([[[Y - B_2]^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)}\\
&= X - [F^+(Y - [[[B_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^-(Y - [[[B_2]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)}\\
&= X - [(X - F^-([[[B_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup (X - F^+([[[B_2]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)}\\
&= [F^-([[[B_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^+([[[B_2]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}.
\end{aligned}$$

(5) ⇒ (2): The proof is obvious.

(2) ⇒ (1): Let V_1, V_2 be any (Λ, sp)-open sets of Y such that

$$x \in F^+(V_1) \cap F^-(V_2).$$

By (2), $x \in F^+(V_1) \cap F^-(V_2) \subseteq [F^+([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}$. Then, there exists a (Λ, sp)-open set U of X such that

$$x \in U \subseteq F^+([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)}).$$

Thus, $F(U) \subseteq [V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}$ and $F(z) \cap [V_2^{(\Lambda,sp)}]_{(\Lambda,sp)} \neq \emptyset$ for every $z \in U$. This shows that F is almost (Λ, sp)-continuous. □

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