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A topological representation of matroids using graphs

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Abstract

Matroids arise in combinatorial optimization since the greedy method runs for structures. In this article, we study some fundamental matroid characteristics. Also, we build topologies generated by matroids with an equivalence relation. and we study the connectedness on matroids through their topological structures. Furthermore, we investigate simple graphs and matroid connectivity.

1 Introduction and preliminaries

Whitney [16] used matroids to capture the essence of dependence. Several terminologies from graph theory are used in matroid theory by Oxley [2, 11, 13]. The connectedness of matroids is strongly linked to the connectedness of graphs [10, 12]. A matroid cycle generated by a graph has the same interrelationship [14, 15]. Both topology and graphs are used to represent structures such as fractals [4, 5, 6, 7, 11] and their applications [4, 8, 9].
In this work, we turn our attention to linked matroids and graphs connected

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by upper covering approximation where we cover approximation space and graphs. In Section 2, we focus our attention on the characteristics of matroid, the foundation of a submatroid, and some instances. In Section 3, we investigate the relationship between a matroid's connectivity and the topology it generates. In Section 4, we include a graph to inspire a covering and use the upper covering approximation to investigate graph and matroid connectivity.

Definition 1.1. *[13] A matroid $\mathcal{M}$ is a pair $(E, \mathfrak{I})$ consisting of a ground set E and a collection, say independent sets, satisfying the following conditions:*

(1) $\phi \in \mathfrak{I}$

(2) If $A \in \mathfrak{I}$ *and* $B \subseteq A$*,then* $B \in \mathfrak{I}$.

(3) Let $A, B \in \mathfrak{I}$ *and* $|A| < |B|$*, then* $\exists a \in B - A$ *such that* $A \cup \{a\} \in \mathfrak{I}$.

Definition 1.2. *[13] Let $\mathcal{M} = (E, \mathfrak{I})$ be a matroid. $\mathfrak{B}$ is a basis of $\mathfrak{I}$ if $\mathfrak{B}$ is a maximal independent set such that*

(1) $\mathfrak{B} \neq \phi$

(2) If $B_1, B_2 \in \mathfrak{B}$ *and* $B_1 \subseteq B_2$*, then* $B_1 = B_2$ *or equivalently if* $B_1, B_2 \in \mathfrak{B}$*, then* $|B_1| = |B_2|$.

(3) If $B_1, B_2 \in \mathfrak{B}$ *and* $x \in B_1 - B_2$*, then* $\exists\ y \in B_2 - B_1 \ni B_1 - x \cup \{y\} \in \mathfrak{B}$.

Definition 1.3. *[13] The members not belonging to $\mathfrak{I}$ are called dependent sets. The minimal dependent sets are called circuits and is denoted by $\mathcal{C}(M)$.*

Lemma 1.4. *[3, 13] Let $\mathcal{M}$ be a matroid and $C_1, C_2 \in \mathcal{C}(\mathcal{M})$. If $e_1 \in C_1 - C_2, e_2 \in C_2 - C_1$ and $C_1 \cap C_2 \neq \phi$, then $\exists\ C_3 \in \mathcal{C}(\mathcal{M})$ such that $e_1, e_2 \in C_3 \subseteq C_1 \cup C_2$.*

Proposition 1.5. *[13] The matroid $\mathcal{M}$ is connected iff for every pair of distinct elements of E, there exists a circuit containing these elements.*

Definition 1.6. *[15] Let $\mathfrak{C}$ be a family of a ground set E. If $\cup\mathfrak{C} = E$, then $\mathfrak{C}$ is called a covering of E and $(E, \mathfrak{C})$ is a covering approximation.*

Definition 1.7. *[15] Let $(E, \mathfrak{C})$ be a covering approximation and $Y \subseteq E$. The covering upper (resp. lower) approximation of Y, denoted by $\bar{\mathfrak{C}}(Y)$ (resp. $\underline{\mathfrak{C}}(Y)$)is defined by $\bar{\mathfrak{C}}(Y) = \cup\{K \in \mathfrak{C} : K \cap Y \neq \phi\}$, $\underline{\mathfrak{C}}(Y) = (\bar{\mathfrak{C}}(Y^c))^c$, where Y^c is the complement of Y w.r.to E.*

2 Some topological properties on matroids

Definition 2.1. *Let $\mathcal{M} = (E, \Im)$ be a matroid and $A \subseteq E$. $A^{'} = \{p \in E :$ for each independent set J and $p \in J$ implies $J \cap (A - \{p\}) \neq \phi\}$. $A^{'}$ is said to be accumulation points of A.*

Lemma 2.2. *$A^{'} = \phi$, for any finite subset A in $\mathcal{M} = (E, \Im)$.*

Proof. Let $A = \{e_1, e_2, \cdots, e_n\}$ be an independent set in $\mathcal{M}$ which implies the singleton sets $\{e_i\}_{i=1}^{n}$ are independent sets, for each e_i. Then $\forall\ p \in E$, $p \in A \Longrightarrow \{p\} \cap A - \{p\} = \phi$. On the other hand, if $p \notin A$, $\{p\} \in \Im \Longrightarrow \{p\} \cap (A - \{p\}) = \phi$, then $A^{'} = \phi$. □

Definition 2.3. *Interior (resp. exterior, boundary) points of any A in $\mathcal{M} = (E, \Im)$ are defined by $int_{\mathcal{M}}(A) = \cup\{\Im_i \in \Im : \Im_i \subseteq A\}$ (resp. $ext_{\mathcal{M}}(A) = int_{\mathcal{M}}(E - A)$, $bd_{\mathcal{M}}(A) = E - (int_{\mathcal{M}}(A) \cup ext_{\mathcal{M}}(A))$).*

Remark 2.4. *(1) $int_{\mathcal{M}}(\phi) = \phi$, $int_{\mathcal{M}}(E) \neq E$.*

(2) If $A \subseteq B \Rightarrow int_{\mathcal{M}}(A) \subseteq int_{\mathcal{M}}(B)$.

(3) $int_{\mathcal{M}}(A) = A \Leftrightarrow A$ is independent.

Theorem 2.5. *(1) $int_{\mathcal{M}}(A \cap B) = int_{\mathcal{M}}(A) \cap int_{\mathcal{M}}(B)$.*

(2) $int_{\mathcal{M}}(A \cup B) \supseteq int_{\mathcal{M}}(A) \cup int_{\mathcal{M}}(B)$.

Proof. (1) Since $A \cap B \subseteq A$, $B \Longrightarrow int_{\mathcal{M}}(A \cap B) \subseteq int_{\mathcal{M}}(A)$, $int_{\mathcal{M}}(B) \Longrightarrow int_{\mathcal{M}}(A \cap B) \subseteq int_{\mathcal{M}}(A) \cap int_{\mathcal{M}}(B)$ since $A \supseteq int_{\mathcal{M}}(A)$, $B \supseteq int_{\mathcal{M}}(B) \Longrightarrow A \cap B \supseteq int_{\mathcal{M}}(A) \cap int_{\mathcal{M}}(B) \Longrightarrow int_{\mathcal{M}}(A \cap B) \supseteq int_{\mathcal{M}}(A) \cap int_{\mathcal{M}}(B) \Longrightarrow int_{\mathcal{M}}(A \cap B) = int_{\mathcal{M}}(A) \cap int_{\mathcal{M}}(B)$.

(2) $A \subseteq A \cup B$, $A \subseteq A \cup B \Longrightarrow int_{M}(A) \subseteq int_{\mathcal{M}}(A \cup B)$, $int_{\mathcal{M}}(B) \subseteq int_{\mathcal{M}}(A \cup B) \Longrightarrow int_{\mathcal{M}}(A) \cup int_{\mathcal{M}}(B) \subseteq int_{\mathcal{M}}(A \cup B)$. □

Definition 2.6. *A submatroid $(A, \Im_A)$ on A in a matroid $(E, \Im)$ is defined by $\Im_A = \{B \cap A : B \in \Im\}$.*

Proposition 2.7. *The submatroid $(A, \Im_A)$ is a matroid on A.*

Proof. (1) Since $\phi \in \Im$, $\phi \cap A = \phi \in \Im_A$.

(2) If $s_1^{'} \in \Im_A$ and $s_2^{'} \subseteq s_1^{'} \Rightarrow \exists\ s_1 \in \Im \ni s_1 \cap A = s_1^{'}$. Since $s_2^{'} \subseteq s_1^{'}$, $s_2^{'} \subseteq s_1 \cap A \Rightarrow s_2^{'} \subseteq s_1$ and $s_2^{'} \subseteq A \Rightarrow s_2^{'} \in I$, by Definition 1.1, we get $s_2^{'} \in I_A$, since $s_2^{'} \subset A$.

(3) Let $s_1^{'}$, $s_2^{'} \in \Im_A$ and $|s_1^{'}| < |s_2^{'}| \Rightarrow \exists\ a \in s_2^{'} - s_1^{'}$, where $s_1^{'}$, $s_2^{'} \subset A$. By Definition 1.1 $\Im_A$ so $a \in A \Rightarrow \exists\ s_1$, $s_2 \in I \ni s_1 \cap A = s_1^{'}$, $s_2 \cap A = s_2^{'}$, $|s_1| < |s_2|$ and $a \in s_2 - s_1 \Rightarrow s_1 \cup \{a\} \in \Im \Rightarrow s_1 \cap A \cup \{a\} = s_1^{'} \cup \{a\} \in \Im_A$. □

Example 2.8. *Let $E = \{a, b, c, d, e, f\}$ with $\Im$ consisting of ϕ, all singletons, all all sets with two elements, and $\{a, b, c\}$, $\{b, c, d\}$, $\{c, d, e\}$, $\{d, e, f\}$. Define $A = \{a, b\} \subseteq E$. Then, $\Im_A = \{\phi, \{a, b\}, \{a\}, \{b\}\}$.*

Definition 2.9. *For a submatroid $(A, \Im_A)$, $\mathfrak{B}_A$ is defined by $\mathfrak{B}_A = max\{B_i \cap A : B_i \in \mathfrak{B}\}$.*

Proposition 2.10. *$\mathfrak{B}_A$ is a basis for $(A, \Im_A)$.*

Proof. (1) $\mathfrak{B}_A \neq \phi$ since it has the maximal independent set $(A, \Im_A)$.

(2) If $s_1^{'}$, $s_2^{'} \in \mathfrak{B}_A$ and $s_1^{'} \subseteq s_2^{'}$, $\exists$ s_1, $s_2 \in \mathfrak{B}$ such that $s_1 \cap A = s_1^{'}$ and $s_2 \cap A = s_2^{'}$ implies $s_1 \subseteq s_2$ or $s_2 \subseteq s_1$. Then, $s_1 = s_2$, by Definition 1.2. Hence $s_1^{'} = s_2^{'}$.

(2) If $s_1^{'}$, $s_2^{'} \in \mathfrak{B}_A$ and $x \in s_1^{'} - s_2^{'}$, then $\exists$ $s_1, s_2 \in \mathfrak{B}$ such that $s_1 \cap A = s_1^{'}$ and $s_2 \cap A = s_2^{'}$ and $x \in s_1 - s_2$, where s_1, $s_2 \in \mathfrak{B}$. By Definition 1.2(3), $\exists$ an element $y \in s_2 - s_1$ such that $s_1 - x \cup \{y\} \in \mathfrak{B} \Rightarrow s_1^{'} - x \cup \{y\} \in \mathfrak{B}_A$. □

Example 2.11. *(Continuation for Example 2.8). Let $\mathfrak{B} = \{\{a, b, c\}, \{b, c, d\}, \{c, d, e\}, \{d, e, f\}\}$ be a basis for matroid $\mathcal{M} = (E, \Im)$. Then, $\mathfrak{B}_A = \{\{a, b\}\}$*

3 Topologies via relations on matroids

Definition 3.1. *Let $\mathcal{C}(\mathcal{M})$ be circuits of $\mathcal{M} = (E, \Im)$. A relation $\mathfrak{R}$ on $\mathcal{M}$ is defined by $a\mathfrak{R}b$ iff $a = b$ or $\exists$ $C \in \mathcal{C}(\mathcal{M})$ such that $\{a, b\} \subseteq C$. $[a]_{\mathfrak{R}} = \{a\} \cup \{b : a\mathfrak{R}b\}$ $\forall$ $a \in E$ forms a subbasis for a topology on E which will be denoted by $\tau(\mathcal{M})$.*

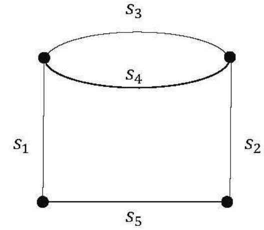

Figure 1: Non simple graph

Example 3.2. *In Figure 1, we deduce that the matroid $\mathcal{M} = (E, \Im)$ with $E = \{s_1, s_2, s_3, s_4, s_5\}$ and $\Im = \{J \subseteq E : J$ is a subgraph with no cycles $\}$. Hence, $\Im = \{\phi, \{s_1\}, \{s_2\}, \{s_3\}, \{s_4\}, \{s_5\}, \{s_1, s_2\}, \{s_1, s_3\}, \{s_1, s_4\}, \{s_1, s_5\}, \{s_2, s_3\}, \{s_2, s_4\}, \{s_2, s_5\}, \{s_3, s_5\}, \{s_4, s_5\}, \{s_1, s_2, s_4\}, \{s_1, s_2, s_3\}, \{s_1, s_3, s_5\}, \{s_1, s_4, s_5\}\}$. $\mathcal{C}(\mathcal{M}) = \{s_3, s_4,\ s_1, s_3, s_2, s_5,\ s_1, s_2, s_4, s_5\}$ is circuits of $\mathcal{M}$. Now, $[s_1]_{\mathfrak{R}} = \{s_1, s_2, s_3, s_4, s_5\}$, $[s_2]_{\mathfrak{R}} = \{s_1, s_2, s_3, s_4, s_5\}$,*

$[s_3]_{\mathfrak{R}} = \{s_1, s_2, s_3, s_4, s_5\}$, $[s_4]_{\mathfrak{R}} = \{s_1, s_2, s_3, s_4, s_5\}$, and $[s_5]_{\mathfrak{R}} = \{s_1, s_2, s_3, s_4, s_5\}$. So, $\{\{s_1, s_2, s_3, s_4, s_5\}\}$ is a subbasis, $\{\phi, E\}$ is a basis and and $\tau(\mathcal{M}) = \{\phi, E\}$

Proposition 3.3. *The $\mathfrak{R}$ is an equivalence relation on E.*

Proof. (1) Since $a = a$, $\forall\, a \in E \Rightarrow a\mathfrak{R}a$.

(2) Let $aRb \Rightarrow \exists\, C \in \mathcal{C}(\mathcal{M})$ such that $\{a, b\} \subseteq C \Rightarrow \{b, a\} \subseteq C \Rightarrow b\mathfrak{R}a$.

(3) If $a\mathfrak{R}b$ and $b\mathfrak{R}z \Rightarrow \exists\, C_1 \in \mathcal{C}(\mathcal{M})$ such that $\{a, b\} \subseteq C_1$ $\exists\, C_2 \in \mathcal{C}(M)$ such that $\{b, z\} \subseteq c_2$ and $c_1 \cap C_2 = \{b\} \neq \phi \Rightarrow \exists\, C_3 \in \mathcal{C}(\mathcal{M}) \ni c_3 \subseteq C_1 \cup C_2 - \{b\}$ $\Rightarrow a\mathfrak{R}c$. □

Proposition 3.4. *The family $\{[a]_{\mathfrak{R}} : a \in E\}$ is a subbasis for some topology on E.*

Proof. Consider $\tau(\mathcal{M}) = \{G : G = \bigcup_{i\in\Delta}(\bigcap_{j=1}^{n} s_{ij})\}$. We need to prove that a finite intersection of elements of $\{[a]_{\mathfrak{R}} : a \in E\}$ is a basis for $\tau(\mathcal{M})$.

(1) $\bigcup\{[a]_{\mathfrak{R}} : a \in E\} = E$.

(2) Since $[a_1]_{\mathfrak{R}} \cap [a_2]_{\mathfrak{R}} = \phi$, then ϕ is an element of the base. Also, the finite intersection of $\{[a]_{\mathfrak{R}} : a \in E\}$ is a base for $\tau(\mathcal{M})$ □

Proposition 3.5. *If a matroid contains loops and $\mathcal{C}(\mathcal{M})$ is a covering of $U(\mathcal{M})$, then each of $\tau(\mathcal{M})$ and matroid is disconnected.*

Proof. Since matroid has loops, then $\exists$ a circuit $C = \{u\} \in \tau(\mathcal{M})$ and $\mathcal{C}(\mathcal{M})$ is a covering of $U(\mathcal{M})$. Then, $\exists\, H = \{a_1, a_2, a_3, \cdots, a_n\} \subseteq \tau(\mathcal{M})$ which does not contain u and so $U(\mathcal{M}) = \{u\} \cup \{a_1, a_2, a_3, \cdots, a_n\} \neq \phi$. Furthermore, $\tau(\mathcal{M})$ is disconnected, for a matroid $\exists\, v \in U(\mathcal{M}) \ni \{u, v\} \nsubseteq C$ and $\{u\}$ is a loop. Continuing in the same process, we get $\mathcal{M}$ is disconnected. □

4 Connectedness on matroids vs graphs

Definition 4.1. *For a matroid $\mathcal{M}$, an undirected simple graph $G(\mathcal{M}) = (V, E)$ is defined by*

(1) the vertices $V = U(\mathcal{M})$, where $U(M)$ is a ground set of $\mathcal{M}$.

(2) the edges $\forall\, u, v \in V$ and $u \neq v$, $uv \in E \Leftrightarrow \exists\, C \in \mathcal{C}(\mathcal{M})$ and $\{u, v\} \subseteq C$.

The family of sets with cardinality 2 in a graph G for $\mathcal{M}$ such that $\exists\, C \in \mathcal{C}(\mathcal{M})$ and $\{u, v\} \subseteq C$ $\forall\, u, v \in U(\mathcal{M})$, $u \neq v$ is denoted by $F(G(M))$.

Proposition 4.2. *If $\mathcal{M}$ has no loops and $\mathcal{C}(\mathcal{M})$ is a covering of $U(\mathcal{M})$, then $F(G(\mathcal{M}))$ is a covering of $U(\mathcal{M})$.*

Proof. Assume that $F(G(\mathcal{M}))$ is not cover $U(\mathcal{M})$. Then, $\exists\ a \in U(\mathcal{M})$ such that $b \in U(\mathcal{M}) - \{a\}$, but there is no a circuit $C \in \mathcal{C}(\mathcal{M})$ such that $C = \{a\}$ or $a \in C$. This contradict with the loopless of $\mathcal{M}$. Conversely, $\phi \notin \mathcal{C}(\mathcal{M})$, $\forall\ a \in U(\mathcal{M})\ \exists\ b \in U(\mathcal{M})$ and $a \neq b$ and a is connected with b. Then, $C_a \in \mathcal{C}(\mathcal{M})$ such that $a \in C_a$. Thus, $U(\mathcal{M}) = \bigcup_{a\in U(\mathcal{M})}\{a\} \subseteq \bigcup_{a\in U(\mathcal{M})} C_a \subseteq U(\mathcal{M})$. This means $U(\mathcal{M}) = \bigcup \mathcal{C}(\mathcal{M})$. Therefore, $\mathcal{C}(\mathcal{M})$ is a covering of $U(\mathcal{M})$. □

Lemma 4.3. *Let $\mathcal{M}$ be a matroid without loops and $\mathcal{C}(\mathcal{M})$ is a covering of $U(\mathcal{M})$. Then, $\forall\ Y \subseteq U(\mathcal{M})$, $\overline{F(G(\mathcal{M}))}(Y) = \overline{\mathcal{C}(\mathcal{M})}(Y)$.*

Proof. Since $\mathcal{M}$ is loopless and $\mathcal{C}(\mathcal{M})$ is a covering of $U(\mathcal{M})$, then $F(G(\mathcal{M}))$ is a covering of V by Proposition 4.2. Let $x \in \overline{\mathcal{C}(\mathcal{M})}(Y) \Rightarrow \exists\ C \in \mathcal{C}(\mathcal{M})$ such that $C \cap Y \neq \phi$ and $x \in C$. There are two cases: if $x \in Y$, then $x \in \overline{F(G(\mathcal{M}))}(Y)$ because $Y \subset \overline{F(G(\mathcal{M}))}(Y)$. If $x \notin Y$, pitch $y \in C \cap Y$, then $x \neq y$ and $\{x, y\} \in F(G(\mathcal{M}))$ which implies $x \in \overline{F(G(\mathcal{M}))}(Y)$. Consequently, $\overline{\mathcal{C}(\mathcal{M})}(Y) \subset \overline{F(G(\mathcal{M}))}(Y)$. Now, we prove that $\overline{F(G(\mathcal{M}))}(Y) \subset \overline{\mathcal{C}(\mathcal{M})}(Y) \Rightarrow x \in \overline{F(G(\mathcal{M}))}(Y) \Rightarrow \exists\ C \in F(G(\mathcal{M}))$ such that $C \cap Y \neq \phi$ and $x \in C$. Here, there are two cases: if $x \in Y \Rightarrow x \in \overline{\mathcal{C}(\mathcal{M})}(Y)$ because $Y \subset \overline{\mathcal{C}(\mathcal{M})}(Y)$. If $x \notin Y$, pitch $y \in C \cap Y$, then x is connected to y and $\exists\ C \in \mathcal{C}(\mathcal{M})$ such that $\{x, y\} \subseteq C \Rightarrow x \in \overline{\mathcal{C}(\mathcal{M})}(Y)$. Consequently, $\overline{F(G(\mathcal{M}))}(Y) \subset \overline{\mathcal{C}(\mathcal{M})}(Y)$. □

Theorem 4.4. *Let $G(\mathcal{M})$ be a simple undirected graph with no isolated vertices. Then, the following are equivalent:*

(1) $G(\mathcal{M})$ is connected.

(2) $\overline{\mathcal{C}(\mathcal{M})}(Y) \neq Y$, $\forall\ Y \subseteq U(\mathcal{M})$.

(2) $\overline{\mathcal{C}(\mathcal{M})}(Y) = U(\mathcal{M})$.

Proof. $G(\mathcal{M})$ is connected iff $\forall\ Y \subseteq U(\mathcal{M})\ \overline{\mathcal{C}(\mathcal{M})}(Y) \neq Y$. Also, $G(\mathcal{M})$ is connected iff $G(\mathcal{M})$ is a complete graph iff $\overline{\mathcal{C}(\mathcal{M})}(Y) = U(\mathcal{M})$. □

Corollary 4.5. *For a covering $F(G(\mathcal{M}))$ of $U(\mathcal{M})$, the following are equivalent:*

(1) $G(\mathcal{M})$ is connected.

(2) $\overline{F(G(\mathcal{M}))}(Y) \neq Y$, $\forall\ Y \subseteq U(\mathcal{M})$.

(3) $\overline{F(G(\mathcal{M}))}(Y) = U(M)$.

5 Conclusion

Throughout our study, we presented some topological structures in the viewpoint of matroids and graphs. We studied some topological properties, such as connectedness, on these new structures. These new types of topological structures can be used to establish new types of matroid separation axioms.

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