International Journal of Mathematics and
Computer Science, **17**(2022), no. 3, 1277–1286

An Application of Weighted Similarity on Intuitionistic Fuzzy Soft Matrices in Medical Diagnostics

Raden Sulaiman[1], **Abdul Ghafur Ahmad**[2],
A'yunin Sofro[1], **Dwi Nur Yunianti**[1], **Rudianto Artiono**[1]

[1]Department of Mathematics
Universitas Negeri Surabaya
Surabaya, Indonesia

[2]Department of Mathematical Sciences
Universiti Kebangsaan Malaysia
Bangi, Malaysia

email: radensulaiman@unesa.ac.id

(Received November 28, 2021, Accepted April 14, 2022)

Abstract

In this paper, we discuss the concept of intuitionistic fuzzy set (IFS), distance and similarity measure two IFS which was developed on distance and similarity measures on the intuitionistic fuzzy soft matrix (IFSM). We give some formulae of distance and similarity measure on IFS and study Gandhimathi's approach for medical diagnosis, but by using weighed similarity measure from Song et al. [1]. Finally, we present a case study for the simulation of this method.

1 Introduction and Motivation

Medical diagnosis determines a person's current health condition as a basis for making medical decisions about treatment. This determination is made based on the symptoms that arise from a patient. Often, several diseases give

Key words and phrases: Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Soft Matrix, Similarity Measure.
AMS (MOS) Subject Classifications: 03E72, 08A72, 94D05.
ISSN 1814-0432, 2022, http://ijmcs.future-in-tech.net

the same symptoms to determine a person's disease can be seen through the proximity of the disease's symptoms. The closeness of the disease symptoms can then be viewed as a measure of the closeness between two objects. In this study, we carry out the development of applications in fuzzy theory in the health sector. The concept of "fuzzy" can provide an excellent approach to detect a person's disease based on the symptoms. It can be done using the concept of distance. Two close objects will have a small distance measure. The smaller the distance than the closer the two objects are. Moreover, object proximity based on the concept of distance was developed on the concept of an intuitionistic fuzzy set. In [1], many concepts of distance due to several researchers were described. In the same article, the authors formulated the concept of distance between two intuitionistic fuzzy sets and showed that the numerical concept of distance was smoother than those of other researchers.

Meanwhile, the concept of similarity is complementary to the concept of distance. Based on the concept of distance, the researcher can formulate the similarity of two intuitionistic fuzzy sets. It is often represented in the form of matrices for the sake of ease of computational operations. In its current development, it has been expanded to become the intuitionistic fuzzy soft matrix concept. Gandhimathi [2] has developed the application of intuitionistic fuzzy soft matrix similarity in medical diagnosis. However, Gandhimathi still used the concept of similarity based on the "Hamming" distance concept.

In this study, the application of intuitionistic fuzzy soft matrix similarity in medical diagnosis will be applied to the concept of distance based on [1]. This modification is predicted to obtain a smoother result compared to the numerical analysis results that Song et al. have carried out. Thus, the conclusion about detecting the name of the disease suffered by the patient based on the symptoms that appear will be more accurate.

2 Fuzzy Soft Set and Fuzzy Soft Matrix

Fuzzy set was introduced by Zadeh in 1965 [3]. A fuzzy set is a generalization of the crisp set or classic set. Then, in 1983, Atanassov [4-6] generalized the concept of a fuzzy set and defined the concept of intuitionistic fuzzy set. Every fuzzy set can be viewed as an intuitionistic fuzzy set. Many researchers [7-10] discussed applications of intuitionistic fuzzy sets. Currently, the concepts of intuitionistic fuzzy soft matrix, intuituinistic fuzzy soft matrix (IFSM), and their application aree been developed.

Following are the definition of fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy soft set and intuitionistic fuzzy soft matrix.

Definition 2.1. *Let U be an initial universe of discourse and E be a set of parameters. Let $P(U)$ denote the power set of U. The function $F : E \to P(U)$ is called a soft set over U.*

Definition 2.2. *Let X be a crisp set. The set $A = \{(x, \mu_A(x)) : x \in X\}$, where $0 \leq \mu_A(x) \leq 1$, for all $x \in X$ is called a Fuzzy Set (FS) of X. The function μ_A is called the membership function and $\mu_A(x)$ is a grade of membership of x in A.*

Example 2.3. *Let $X = \{x, y, z\}$. Then $A = \{(x, 0.7), (y, 0.6), (z, 0.5)\}$ is an example of fuzzy set of X.*

Definition 2.4 (9). *Let X be a crisp set. An Intuitionistic Fuzzy Set (IFS) $\mathcal{A}$ of X is defined as $\mathcal{A} = \{(x, \mu_A(x), v_A(x)) : x \in X\}$, where*
$0 \leq \mu_A(x) \leq 1, 0 \leq v_A(x) \leq 1$, and $0 \leq \mu_A(x) + v_A(x) \leq 1$, for all $x \in X$.

The function μ_A is called the membership function and v_A is called the the non-membership function, while $\mu_A(x)$ and $v_A(x)$ are called the degree of membership and non-membership of x to the set $\mathcal{A}$, respectively.
The amount $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is called the degree of indeterminacy or hesitation part, which may cater to either membership value or non-membership value or both.

Example 2.5. *Let $X = \{a_1, a_2, a_3\}$ and $Y = \{b_1, b_2, b_3, b_4\}$. The following are two examples of IFS of X and Y, respectively.*
(1) $\mathcal{A} = \{(a_1, 0.7, 0.2), (a_2, 0.6, 0.4), (a_3, 0.8, 0.1)\}$,
(2) $\mathcal{B} = \{(b_1, 0.3, 0.6), (b_2, 0.8, 0.2), (b_3, 0.2, 0.8), (b_4, 0.7, 0.2)\}$.
We have $\pi_A(a_1) = 0.1, \pi_A(a_3) = 0.1, \pi_B(b_1) = 0.1, \pi_B(b_3) = 0$.

Definition 2.6. *Let $U = \{u_1, u_2, ..., u_n\}$. An intuitionistic fuzzy subset of S, denoted by IF^U is defined as a function from U to $\{[a, b]\}$ where $0 \leq a, b \leq 1$ and $0 \leq a + b \leq 1$.*

Definition 2.7. *Let $U = \{x_1, x_2, ..., x_n\}$ be a universal set and*
$P = \{e_1, e_2, ..., e_m\}$ be parameters. An intuitionistic fuzzy soft set (G, U) over P is defined as a function G from S to IF^U, where
for $i \in \{1, 2, 3, ..., n\}$, $G(e_i) = \{(x_j, [\mu_i(x_j), v_i(x_j)]) : j = 1, 2, 3, ..., m\}$, where $\mu_i(x_j)$ represents the membership of x_j in the parameter e_i, while $v_i(x_j)$ represents the non-membership of x_j in the parameter e_i.

Definition 2.8. *Let* $U = \{x_1, x_2, ..., x_n\}$ *be a universal set and* $P = \{e_1, e_2, ..., e_m\}$. *Let* $A \subset P$ *and* (G, U) *be an intuitionistic fuzzy soft set over* U. *Then the intuitionistic fuzzy soft set* (G, U) *can be represented in a matrix form as:* $A_{m\times n} = [a_{ij}]_{m\times n}$, *where*

$$a_{ij} = \begin{cases} (\mu_j(x_i), v_j(x_i)), & x_j \in A \\ (0,0), & x_j \notin A \end{cases}.$$

This matrix is called the Intuitionistic Fuzzy Soft Matrix (IFSM).

Example 2.9. *Suppose that there are five cars under consideration; namely,* $U = \{x_1, x_2, x_3, x_4\}$ *and the set of parameters* $E = \{e_1, e_2, e_3, e_4\}$, *where* e_1 *stands for "completeness of features",* e_2 *stands for "resale value",* e_3 *stands for "maintenance cost", and* e_4 *stands for "insurance", respectively. Consider the function* F *from a parameter set* $A = \{e_1, e_2, e_4\} \subsetneq E$ *to all intuitionistic fuzzy subsets of the power set* S. *Consider an intuitionistic fuzzy soft set* (G, A) *which describes the "perfect car" that is being considered for purchase. The intuitionistic fuzzy soft set* (G, A) *is* $U = \{G(e_1), G(e_2), G(e_4)\}$, *where*
$G(e_1) = \{(x_1, [0.6, 0.4]), (x_2, [0.4, 0.5]), (x_3, [0.3, 0.7]), (x_4, [0.7, 0.2])\}$.
$G(e_2) = \{(x_1, [0.3, 0.6]), (x_2, [0.4, 0.6]), (x_3, [0.5, 0.4]), (x_4, [0.8, 0.2])\}$.
$G(e_4) = \{(x_1, [0.6, 0.3]), (x_2, [0.4, 0.6]), (x_3, [0.2, 0.7]), (x_4, [0.8, 0.2])\}$.
The matrix form of the intuitionistic fuzzy soft set is represented as:

$$\begin{bmatrix} [0.6, 0.4] & [0.3, 0.6] & [0.0, 0.0] & [0.6, 0.3] \\ [0.4, 0.5] & [0.4, 0.6] & [0.0, 0.0] & [0.4, 0.6] \\ [0.3, 0.7] & [0.5, 0.4] & [0.0, 0.0] & [0.2, 0.7] \\ [0.7, 0.2] & [0.8, 0.2] & [0.0, 0.0] & [0.8, 0.2] \end{bmatrix}.$$

3 Similarity Intuitionistic Fuzzy Soft Matrix

The concept of similarity and distance are complementary. Similarity measures can be obtained from the distance formula and vice versa.
In this section, we begin by explaining the definition of distance, followed by similarity.

Definition 3.1. *Let* A, B *be two* $IFSs$ *and* D *denote the function,* $D : IFS \times IFS \to [0, 1]$. *The mapping* D *is called a distance if for every* IFS A *and* B *it satisfies the following conditions:*
(i) $0 \leq D(A, B) \leq 1$

(ii) $D(A,B)$=0, if and only if A=B.
(iii) $D(A,B)$=$D(B,A)$
(iv) If $A \subseteq B \subseteq C$, then $D(A,B) \leq D(A,C)$, and $D(B,C) \leq D(A,C)$.

Definition 3.2. *Let $A, B \in IFS$ and D denote the function, $S : IFS \times IFS \to [0,1]$. The mapping S is called a similarity measure if for every IFS A and B it satisfies the the following conditions:*
(i) $0 \leq S(A,B) \leq 1$
(ii) $S(A,B)$=1, if and only if A=B.
(iii) $S(A,B)$=$S(B,A)$
(iv) If $A \subseteq B \subseteq C$, then $S(A,B) \geq S(A,C)$, and $S(B,C) \geq S(A,C)$.

Many researchers have constructed the formulae of similarity measure on IFS. In the following, several formulae of similarity measure that have been developed are presented.
Let $X = \{x_1, x_2, ..., x_n\}$ be a universe of discourse and $\mathcal{A}, \mathcal{B}$ be IFS, where $\mathcal{A} = \{(x, \mu_A(x), v_A(x)) : x \in X\}$ and $\mathcal{B} = \{(x, \mu_B(x), v_B(x)) : x \in X\}$, respectively.
Chen [11] proposed the formulae of similarity measure as:

$$S_C(A,B) = 1 - \frac{\sum_{i=1}^{n} |(\mu_A(x_i) - v_A(x_i)) - (\mu_B(x_i) - v_B(x_i))|}{2n}.$$

Hong and Kim [12] formulates similarity measure as:

$$S_H(A,B) = 1 - \frac{\sum_{i=1}^{n} |(\mu_A(x_i) - \mu_B(x_i) - (v_A(x_i)) - v_B(x_i))|}{2n}.$$

Li and Xu [13] proposed similarity measure as:

$$S_L(A,B) = 1 - \frac{\sum_{i=1}^{n} |(\mu_A(x_i) - v_A(x_i)) - (\mu_B(x_i) - v_B(x_i))|}{4n} -$$

$$\frac{\sum_{i=1}^{n} |(\mu_A(x_i) - \mu_B(x_i))| + |(v_A(x_i) - v_B(x_i))|}{4n}.$$

Song et al. [1] proposed a new semilarity measure as:

$$S_\gamma(A,B) = \frac{1}{2n} \sum_{i=1}^{n} (\sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{v_A(x_i)v_B(x_i)} +$$

$$\sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1 - v_A(x_i))(1 - v_B(x_i))})$$

In addition, Song et al. [1] proposed a similarity measure formula by giving weights for each x_j. This formula is:

$$S_\alpha(A,B) = \frac{1}{2}\sum_{i=1}^{n} w_i(\sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{v_A(x_i)v_B(x_i)} +$$

$$\sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1 - v_A(x_i))(1 - v_B(x_i))}),$$

where $w_i \in [0,1]$ is the weight factor of the features x_i, and $\sum_{i=1}^{n} w_i = 1$. This formula is more transparent and comparable with the similarity measure proposed earlier by other authors. An application follows.

4 Application IFSM in Medical Diagnosis

Several applications of fuzzy concepts in health have been discussed. In 2001, Kumar et al. [14] proposed to diagnose diseases using intuitionistic fuzzy relations (IFR). In 2012, Samuel [7] introduced the application of IFR to diagnose diseases using the Max-min composition. Moreover, in 2018, Gandhimathi [2] discussed the application of intuitionistic "fuzzy soft matrix" as a representation of IFS in medical diagnosis. Furthermore, in 2019, Thaoa, Alib, and Smarandachec [15] discussed the application of "fuzzy clustering" in disease diagnosis. The latest research on the application of fuzzy concepts in the health sector is a study conducted by El-Badie [16]; namely, the application of fuzzy concepts to diagnose liver disease.

In this section, we propose a new approach of diagnosing disease using Intuitionistic Fuzzy Soft Matrices which is different from Gandhimaththi's approach. This method uses a weighted similarity formula of IFSM as modification of similarity measure of Intuitionistic Fuzzy Set that has been studied by Song et al. [1].

We will diagnose the patient's disease name based on the symptoms that appear. Symptoms are obtained by direct observation or the results of interviews with patients. We consider the following sets:
Let the set of patients be denoted by $P = \{p_1, p_2, ..., p_m\}$, the set of symptoms by $S = \{s_1, s_2, ..., s_n\}$, and $D = \{d_1, d_2, ..., d_k\}$ the set of diseases.

To diagnose a patient's disease based on the symptoms that appear, we first construct an intuitionistic fuzzy soft set (G, P) over S. Intuitionistic Fuzzy Soft Matrix of this IFSS gives a relation between set of patients to set of symptoms. Then, construct intuitionistic fuzzy soft set (G, S) over D. This matrix form of IFSS gives a relation between the set of symptoms

and the set of diseases. Compute distance between of symptom and diseases. Finally, write the smallest distance of diseases. If the entry of this smallest is a_{ij}, then we conclude that the patient p_i is suffering from disease d_j.
The following is an algorithm to diagnose a patient by using IFSM. This algorithm is a modification of an algorithm by Gandhimathi [2].

Algorithm:

Step 1: Write the set of Universe.
Step 2: List the set of parameters.
Step 3: Construct IFSM for each sets of parameters.
Step 4: Calculate distance of IFSM by using "weighted similarity".
Step 5: Find the smallest distance for each patient. Then make conclusion.

Case Study: Let us consider the same data as in [7,22,23,33-36]. Suppose that there are four patients: Al, Bob, Joe, Ted. The set of patients is $P = \{p_1 = Al, p_2 = Bob, p_3 = Joe, p_4 = Ted\}$. The set of symptoms is $S = \{s_1 = Temperature, s_2 = Headache, s_3 = Stomachpain(S.pain), s_4 = Cough, s_5 = Chestpain(C.pain)\}$.
The set of diagnosis $D = \{d_1 = viralfever, d_2 = Malaria, d_3 = Typhoid, d_4 = StomachProblem, d_5 = Chestproblem\}$. Suppose that the intuitionistic fuzzy soft set (G, S) over P is $(G, S) = \{G(s_1), G(s_2), G(s_3), G(s_4), G(s_5)\}$, where:
$G(s_1) = \{(p_1, [.8, .1]), (p_2, [0, .8]), (p_3, [.8, .1]), (p_4, [.6, .1])\}$,
$G(s_2) = \{(p_1, [.6, .1]), (p_2, [.4, .4]), (p_3, [.8, .1]), (p_4, [.5, .4])\}$,
$G(s_3) = \{(p_1, [.2, .8]), (p_2, [.6, .1]), (p_3, [0, .6]), (p_4, [.3, .4])\}$,
$G(s_4) = \{(p_1, [.6, .1]), (p_2, [.1, .7]), (p_3, [.2, .7]), (p_4, [.7, .2])\}$,
$G(s_5) = \{(p_1, [.1, .6]), (p_2, [.1, .8]), (p_3, [0, .5]), (p_5, [.3, .4])\}$.
The intuitionistic fuzzy soft matrix is shown in Table 1.

Table 1: IFSM Patient-Symptom

	Temperature	Headache	S. pain	Cough	C. pain
Al	[.8, .1]	[.6, .8]	[.2, .8]	[.6, .1]	[.1, .6]
Bob	[0, .8]	[.4, .4]	[.6, .1]	[.7, .1]	[.1, .8]
Joe	[.8, .1]	[.8, .1]	[0, .6]	[.2, .7]	[0, .5]
Ted	[.6, .1]	[.5, .4]	[.3, .4]	[.7, .2]	[.3, .4]

Consider the intuitionistic fuzzy soft set (G, D) over S; that is, $(G, D) = \{G(d_1), G(d_2), G(d_3), G(d_4), G(d_5)\}$, where:
$G(d_1) = \{(s_1, [.4, 0]), (s_2, [.7, 0]), (s_3, [.3, .3]), (s_4, [.1, .7]), (s_4, [.1, .8])\}$,
$G(d_2) = \{(s_1, [.3, .5]), (s_2, [.2, .6]), (s_3, [.6, .1]), (s_4, [.2, .4]), (s_4, [0, .8])\}$,
$G(d_3) = \{(s_1, [.1, .7]), (s_2, [0, .9]), (s_3, [.2, .7]), (s_4, [.8, 0]), (s_4, [.2, .8])\}$,

$G(d_4) = \{(s_1, [.4, .3]), (s_2, [.7, 0]), (s_3, [.2, .6]), (s_4, [.2, .7]), (s_4, [.2, .8])\}$,
$G(d_5) = \{(s_1, [.1, .7]), (s_2, [.1, .8]), (s_3, [.1, .9]), (s_4, [.2, .7]), (s_4, [.8, .1])\}$.
The intuitionistic fuzzy soft matrix is shown in Table 2.

Table 2: IFSM Symptoms-Diagnoses

	Viral fever	Malaria	Typhoid	S.problem	C. problem
Temp.	$[.4, 0]$	$[.7, 0]$	$[.3, .3]$	$[.1, .7]$	$[.1, .8]$
Headache	$[.3, .5]$	$[.2, .6]$	$[.6, .1]$	$[.2, .4]$	$[0, .8]$
Stomach pain	$[.1, .7]$	$[0, .9]$	$[.2, .7]$	$[.8, 0]$	$[.2, .8]$
Cough	$[.4, .3]$	$[.7, 0]$	$[.2, .6]$	$[.2, .7]$	$[.2, .8]$
Chest pain	$[.1, .7]$	$[.1, .8]$	$[.1, .9]$	$[.2, .7]$	$[.8, .1]$

To calculate the degree of similarity between two IFSMs, we determine w_i for each symptom s_i and diagnoses d_j as Table 3.

Table 3: Quantity of w_i

	Viral fever	Malaria	Typhoid	S.problem	C. problem
Temp.	0.4	0.4	0.3	0.2	0.1
Headache	0.2	0.2	0.2	0.1	0.1
Stomach pain	0.1	0.2	0.3	0.5	0.1
Cough	0.2	0.1	0.1	0.1	0.3
Chest pain	0.1	0.1	0.1	0.1	0.4
Total	1	1	1	1	1

Similarity degree S_α between each patient's symptoms and the considered of possible diagnoses as Table 4.

Table 4: Similarity S_α

	Viral fever	Malaria	Typhoid	S.problem	C. problem
Al	0.917	0.918	**0.938**	0.627	0.740
Bob	0.716	0.594	0.864	**0.962**	0.805
Joe	0.900	0.869	**0.925**	0.653	0.768
Ted	**0.954**	0.913	0.927	0.808	0.833

Table 4 shows the index of similarity between patients and diseases. The largest similarity index means the smallest distance between patients and diseases. Therefore, the diseases is most likely diagnosed by that index. It is a number that is used to decide a disease that most possible has been suffered by the patient based on the symptoms. From this table, we conclude that Al and Joe suffer from Typhoid, Bob suffers from a stomach problem, and Ted suffers from a Viral fever.

5 Conclusion

Similarity measure on intuitionistic soft matrix can be used in medical diagnosis. There are many existing formulae on distance or similarity to express the degree of proximity of two IFSMs. The formula on weighed similarity gives a smoother result. The method used in this paper was a combination of Gandhimathi's method and Song et al. formula.

References

[1] Yafie Song, Xiondan Wang, Lei Lei, Aijun Xue, A New Similarity Measure between Intuitionistic Fuzzy Sets and Its Application to Pattern Recognition, Abstract and Applied Analysis, (2014), 1–9.

[2] T. Gandhimathi, An Application of intuitionistic fuzzy Soft Matrix in Medical Diagnosis, Journal of Computational and Theoretical Nanoscience, **15,** (2018), 1–4.

[3] L. A. Zadeh, Fuzzy sets, Fuzzy Inf. Eng., **6**, (1965), 338–353.

[4] K. Atanassov, Intuitionistic fuzzy sets, Sci-Techn., Library of Bulgarian Academy, 389–410.

[5] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems **20,** (1986), 87–96.

[6] K. Atanassov, G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems **31,** (1989), 343–349.

[7] A. E. Samuel, M. Balamurugani, Fuzzy Max-Min Composition Technique in Medical Diagnosis, Applied Mathematical Sciences, **35,** no. 6, (1983), 1741–1746.

[8] H. J. Zimmermann, Fuzzy Set Theory and Its Applications, 1996.

[9] Timothy J. Ross, Fuzzy Logic with Engineering Applications, 2004.

[10] Z. Zhang, A rough set approach to intuitionistic fuzzy soft set based decision making, Applied Mathematical Modeling, **36,** (2012), 4605–4633.

[11] S. M. Chen, Measures of Similarity between vague sets, Fuzzy Sets and Systems, **74,** no. 2, (1995), 217–223.

[12] D. H. Hong, C. Kim, A note on similarity measures between vague sets and between elements, Information Sciences, **115,** nos. 1–4, (1999), 83–96.

[13] F. Li, Z. Xu, Similarity measures between vague sets, Journal of Software, **12,** no. 6, (2001), 922–927.

[14] S. Kumar De, R. Biswas, A. Ranjan Royi, An Application of intuitionistic fuzzy sets in medical diagnosis, Fuzzy sets and systems, **117,** (2001), 209-213.

[15] Nguyen Xuan Thaoa, M. Alib, F. Smarandachec, An intuitionistic fuzzy clustering algorithm based on a new correlation coefficient with application in medical diagnosis, Journal of Intelligent and Fuzzy Systems, **36,** no. 1, (2019), 189–198.

[16] A. A. El-Badie, A. A. Kamel, F. A. El-Sattar, Z. El-Mogi, A Fuzzy Decision Support for Diagnosis of Some Disease in Educational Medical Instiutions, International Journal of Fuzzy Logic and Intelligent Systems, **20,** no. 4, (2020), 358–368.