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The complement of the complementary prism

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Abstract

In this paper we investigate some properties of the complement $\overline{G\overline{G}}$ of the complementary prism of a graph G of order n. We show that $\overline{G\overline{G}}$ is connected except for $n = 1$, with diameter 2 when $n \geq 3$. Furthermore, we prove that for $n \geq 3$, the graph $\overline{G\overline{G}}$ is Hamiltonian and self-centered, but it is never diametrical. Finally, we characterize $\overline{G\overline{G}}$ that are self-complementary, divisor graphs, regular and Eulerian.

1 Introduction

The complementary prisms form a special class of the complementary products of graphs [14]. The complementary prism of a graph G, denoted by $G\overline{G}$, is obtained from the disjoint union of G and $\overline{G}$ by adding the perfect matching between corresponding vertices of G and $\overline{G}$. For example, the Petersen graph is just $C_5\overline{C_5}$.

Complementary prisms have received much attention. The hamiltonicity of complementary prisms was studied in [17]. Roman domination in complementary prisms was considered in [3] and [7]. Other domination parameters

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of complementary prisms were studied in [4], [9]-[13], [15] and [16]. Independence saturation in complementary prisms was considered in [5].

In this paper, we study the complements of complementary prisms. We will investigate hamiltonicity, regularity of $\overline{G\overline{G}}$ and when $\overline{G\overline{G}}$ is: a divisor graph, Eulerian, self-complementary, self-centered and diametrical.

It is obvious that $\overline{G\overline{G}}$ consists of a copy of $\overline{G}$ and a copy of G, such that every vertex x_i in the copy of $\overline{G}$ is adjacent to all vertices in the copy of G except the vertex y_i corresponding to x_i. Thus, for a graph G with vertex set $V(G) = \{y_1, y_2, \cdots, y_n\}$, the complement $\overline{G\overline{G}}$ of the complementary prism of G is the graph whose vertex set is the disjoint union $V(\overline{G\overline{G}}) = \{x_1, x_2, \cdots, x_n\} \cup \{y_1, y_2, \cdots, y_n\}$ and edge set

$$E(\overline{G\overline{G}}) = \{x_i x_j : y_i y_j \notin E(G)\} \cup \{y_i y_j : y_i y_j \in E(G)\} \cup \{x_i y_j : i \neq j\}.$$

Throughout this paper, n represents the order of G, and we will denote the vertex set of the copy of $\overline{G}$ in $\overline{G\overline{G}}$ by $X = \{x_1, x_2, \cdots, x_n\}$ while the vertex set of the copy of G will be denoted by $Y = \{y_1, y_2, \cdots, y_n\}$, where for each i, y_i is the vertex corresponding to x_i. So, the edge set of $\overline{G\overline{G}}$ equals $E(\overline{G}) \cup E(G) \cup \{x_i y_j : i \neq j\}$, where the vertex set of $\overline{G}$ is X while the vertex set of G is Y. For each i, $\deg_{\overline{G}} x_i$ will denote the degree of x_i in the copy of $\overline{G}$ induced by X and $\deg_G y_i$ will denote the degree of y_i in the copy of G induced by Y.

Before starting our study, we need some more terminology. A graph H is a *divisor graph* if the vertices of H can be labeled by positive integers such that two distinct vertices i and j are adjacent if and only if one of the integers i and j divides the other. For example, the vertices of a graph of size 0 can be labeled by distinct primes. Thus nK_1 is a divisor graph. Another class of graphs known to be divisor graphs is that of bipartite graphs [8]. A vertex v of a digraph D with positive indegree and positive outdegree such that $(u, w) \in E(D)$ whenever both (u, v) and (v, w) belong to $E(D)$ is a *transitive vertex*. A *divisor orientation* D of a graph H is an orientation of H in which every vertex is a transmitter, a receiver, or a transitive vertex [1]. The following characterization of divisor graphs was proved in [8]: divisor graphs are precisely the comparability graphs.

Lemma 1.1. *A graph H is a divisor graph if and only if H has a divisor orientation.*

The *converse* of a digraph D is obtained from D by reversing the directions of all arcs of D. The following fact was given in [1]:

Lemma 1.2. *If D is a divisor orientation of a graph H, then the converse of D is also a divisor orientation of H.*

The 3-*sun* is the graph obtained from the complete graph on the three vertices u_1, u_2, u_3 by adding three new vertices w_1, w_2, w_3 and adding the edges w_iu_i, w_iu_{i+1} (modulo 3), for $i = 1, 2, 3$. The following three results will be referred to in this paper. The first two of them were proved in [8], while the third was shown in [2].

Lemma 1.3. *Every induced subgraph of a divisor graph is a divisor graph.*

Lemma 1.4. *If H is a divisor graph, then H contains no induced odd cycle of length greater than* 3*.*

Lemma 1.5. *The* 3*-sun is not a divisor graph.*

A graph all of whose vertices have the same degree k is k-*regular* or *regular* with *valency* k. The *girth* of a graph H containing a cycle is the length of its shortest cycle. The set of neighbors of a vertex v in a graph is denoted by $N(v)$, while $N[v]$ represents the closed neighborhood $N(v) \cup \{v\}$ of the vertex v. For undefined notions, we refer the reader to [6]. In this paper, all graphs we consider contain neither loops nor multiple edges.

2 Diameter and radius

We will determine when $\overline{G\overline{G}}$ is connected and, for $n > 1$, we compute the diameter of $\overline{G\overline{G}}$.

Theorem 2.1. *The complement $\overline{G\overline{G}}$ of the complementary prism is connected except when G is the trivial graph. Moreover, for any graph G of order $n > 1$, we have*

$$diam(\overline{G\overline{G}}) = \begin{cases} 3 & \text{if } n = 2 \\ 2 & \text{if } n \geq 3 \end{cases}$$

Proof. For the trivial graph K_1, we have $\overline{K_1\overline{K_1}} = 2K_1$ which is disconnected. If $n = 2$, then $\overline{G\overline{G}} = P_4$ which is connected and has diameter 3. So, assume that $n \geq 3$. Let u and w be two different vertices of $\overline{G\overline{G}}$. We distinguish two cases:

Case (1) $u, w \in X$ (the case that $u, w \in Y$ is similar.)

Thus $u = x_i$ and $w = x_j$ for some i, j with $i \neq j$. Since $n \geq 3$, take $k \in \{1, 2, \cdots, n\} - \{i, j\}$. Then $x_i y_k x_j$ is a path in $\overline{G\overline{G}}$ and hence $d(x_i, x_j) \leq 2$.

Case (2) u belongs to one of the two sets X, Y and w belongs to the other.

Say that $u = x_i$ and $w = y_j$ for some $i, j \in \{1, 2, \cdots, n\}$. If $i \neq j$, then $d(x_i, y_j) = 1$. So assume that $i = j$. But $\deg_{\overline{G}} x_i + \deg_G y_i = n - 1 > 1$. This implies that either x_i has a neighbor x_s in $\overline{G}$ or y_i has a neighbor y_k in G. Then either $x_i x_s y_i$ or $x_i y_k y_i$ is a path in $\overline{G\overline{G}}$, respectively. Thus, since $x_i y_i \notin E(\overline{G\overline{G}})$, we have $d(x_i, y_i) = 2$.

Therefore, for $n \geq 3$, we have $diam(\overline{G\overline{G}}) = 2$. □

For a graph G of order 2, we have $\overline{G\overline{G}} = P_4$ which has radius 2. For $n \geq 3$, the radius of $\overline{G\overline{G}}$ cannot be 1 because there is no vertex adjacent to all other vertices. For each i, we have $x_i y_i \notin E(\overline{G\overline{G}})$. But $rad(\overline{G\overline{G}}) \leq diam(\overline{G\overline{G}})$. Therefore, the following result follows by Theorem 2.1.

Theorem 2.2. *For any nontrivial graph G, we have $rad(\overline{G\overline{G}}) = 2$.*

A graph is *self-centered* if its center consists of all vertices or, equivalently, if its radius and diameter are equal.

Corollary 2.3. *For any graph G of order $n \geq 3$, the graph $\overline{G\overline{G}}$ is self-centered.*

A connected graph H is *diametrical* if each vertex x of H has a unique vertex $\overline{x}$ such that $d(x, \overline{x}) = diam(H)$ [18]. Diametrical graphs form a special class of self-centered graphs.

Theorem 2.4. *For any graph G of order $n \geq 2$, the graph $\overline{G\overline{G}}$ is not diametrical.*

Proof. For $n = 2$, the graph $\overline{G\overline{G}} = P_4$ is even not self-centered. Assume that $n \geq 3$. Obviously, at least one of G and $\overline{G}$ must contain an edge. Thus either there exist two nonadjacent vertices x_i,x_j in $\overline{G}$ or there exist two nonadjacent vertices y_i,y_j in G. Then either $d(x_j, x_i) = d(y_i, x_i) = 2 = diam(\overline{G\overline{G}})$ or $d(y_j, y_i) = d(x_i, y_i) = 2 = diam(\overline{G\overline{G}})$, respectively. Therefore, $\overline{G\overline{G}}$ is not diametrical. □

3 Some characterizations concerning $\overline{G\overline{G}}$

In this section, we will investigate when $\overline{G\overline{G}}$ is self-complementary, Hamiltonian, Eulerian, regular and a divisor graph.

A graph is called *self-complementary* if it is isomorphic to its complement.

Theorem 3.1. *Let G be a graph of order n. Then $\overline{G\overline{G}}$ is self-complementary if and only if $n = 2$.*

Proof. For $n = 2$, we have $\overline{G\overline{G}} = P_4$ which is self-complementary.

Conversely, assume that $\overline{G\overline{G}}$ is self-complementary. Then the size of $\overline{G\overline{G}}$ equals the size of $G\overline{G}$. But the size of $\overline{G\overline{G}}$ equals $\left|E(\overline{G})\right| + |E(G)| + n(n-1)$ while the size of $G\overline{G}$ equals $|E(G)| + \left|E(\overline{G})\right| + n$. Thus $n(n-1) = n$, which implies that $n = 2$. □

Clearly, $\overline{G\overline{G}}$ has no cycle when $n < 3$. The following result shows that $\overline{G\overline{G}}$ has girth 3 for any graph G of order at least 3.

Proposition 3.2. *Let G be a graph of order $n \geq 3$. Then girth$(\overline{G\overline{G}}) = 3$.*

Proof. At least one of G and $\overline{G}$ has an edge, say $x_ix_j \in E(\overline{G\overline{G}})$, for some i, j. Take $k \in \{1, 2, \cdots, n\} - \{i, j\}$. Then $x_iy_kx_jx_i$ is a triangle in $\overline{G\overline{G}}$. □

For $n \geq 3$, Proposition 3.2 shows that the length of a shortest cycle in $\overline{G\overline{G}}$ is 3, while the next result assures that the length of a longest cycle in $\overline{G\overline{G}}$ is $2n$.

Theorem 3.3. *Let G be a graph of order n. Then $\overline{G\overline{G}}$ is Hamiltonian if and only if $n \geq 3$.*

Proof. Both $\overline{K_1\overline{K_1}}$ and $\overline{K_2\overline{K_2}}$ have no cycle. So assume that $n \geq 3$. Then $x_1y_2x_3 \cdots y_{n-1}x_ny_1x_2y_3 \cdots x_{n-1}y_nx_1$ is a spanning cycle of $\overline{G\overline{G}}$ when n is odd, while $x_1y_2x_3y_4 \cdots x_{n-1}y_nx_2y_1x_4y_3 \cdots x_ny_{n-1}x_1$ is a spanning cycle of $\overline{G\overline{G}}$ when n is even. Therefore, $\overline{G\overline{G}}$ is Hamiltonian. □

For every i, the degree of the vertex x_i in $\overline{G\overline{G}}$ equals $\deg_{\overline{G}} x_i + (n-1)$ and the degree of the vertex y_i in $\overline{G\overline{G}}$ equals $\deg_G y_i + (n-1)$. It is well known that a nontrivial connected graph is Eulerian if and only if each of its vertices has even degree. On the other hand, for any graph G, we have at least one of G or $\overline{G}$ is connected, and obviously $G\overline{G}$ is isomorphic to $\overline{G}G$.

Theorem 3.4. *Let G be a connected graph of order $n > 1$. Then $\overline{G\overline{G}}$ is Eulerian if and only if n is odd and G is Eulerian.*

Proof. Assume that $\overline{G\overline{G}}$ is Eulerian. Then, for each i, we have $\deg x_i = \deg_{\overline{G}} x_i + (n-1)$ is even. But $\deg_{\overline{G}} x_i = (n-1) - \deg_G y_i$. Thus, for each i, $2(n-1) - \deg_G y_i$ is even. Consequently, $\deg_G y_i$ is even which implies that G is Eulerian. Again, since $\overline{G\overline{G}}$ is Eulerian, we have $\deg y_i = \deg_G y_i + (n-1)$ is even. Then $(n-1)$ must be even since $\deg_G y_i$ is even. Thus n is odd.

Conversely, assume that n is odd and G is Eulerian. Then, for each i, $\deg_G y_i$ is even and hence $\deg_{\overline{G}} x_i = (n-1) - \deg_G y_i$ is also even. Thus, for each i, both $\deg x_i = \deg_{\overline{G}} x_i + (n-1)$ and $\deg y_i = \deg_G y_i + (n-1)$ are even. By Theorem 2.1, the graph $\overline{G\overline{G}}$ is connected. Therefore, $\overline{G\overline{G}}$ is Eulerian. □

Next, we consider regularity.

Theorem 3.5. *Let G be a graph of order n. Then $\overline{G\overline{G}}$ is regular if and only if $n \equiv 1 \pmod 4$ and G is $(\frac{n-1}{2})$-regular. Moreover, if $\overline{G\overline{G}}$ is regular, then it has valency $\frac{3}{2}(n-1)$.*

Proof. Assume that $\overline{G\overline{G}}$ is regular. Then, for each i, we have $\deg x_i = \deg y_i$ which means that $\deg_{\overline{G}} x_i + (n-1) = \deg_G y_i + (n-1)$. But $\deg_{\overline{G}} x_i = (n-1) - \deg_G y_i$. Thus $(n-1) - \deg_G y_i = \deg_G y_i$. Then $\deg_G y_i = \frac{(n-1)}{2}$ for each i, which implies that G is $(\frac{n-1}{2})$-regular and $\overline{G\overline{G}}$ is $\frac{3}{2}(n-1)$-regular. Then n is odd and the sum of the degrees in G of all vertices of G equals $n\frac{n-1}{2}$. But the sum of the degrees of all vertices in any graph is even. Hence $\frac{n-1}{2}$ must be even. Thus $n-1$ is a multiple of 4 and so $n \equiv 1 \pmod 4$.

Conversely, assume that $n \equiv 1 \pmod 4$ and G is $(\frac{n-1}{2})$-regular. Then, for each i, we have $\deg y_i = \deg_G y_i + (n-1) = \frac{n-1}{2} + (n-1) = \frac{3}{2}(n-1)$ and $\deg x_i = \deg_{\overline{G}} x_i + (n-1) = (n-1) - \deg_G y_i + (n-1) = 2(n-1) - \frac{n-1}{2} = \frac{3}{2}(n-1)$. Therefore, $\overline{G\overline{G}}$ is $\frac{3}{2}(n-1)$-regular. □

The graphs $\overline{K_1\overline{K_1}}$ and $\overline{C_5\overline{C_5}}$ are 0-regular and 6-regular, respectively. Note that, for $n \equiv 1 \pmod 4$, the integer $\frac{3}{2}(n-1)$ is a multiple of 6. Thus we get the following result:

Corollary 3.6. *If $\overline{G\overline{G}}$ is m-regular, then $m \equiv 0 \pmod 6$.*

The rest of this section is devoted to determine when $\overline{G\overline{G}}$ is a divisor graph. Let us start with the following two lemmas, the first of which is a direct observation that follows from the definition of the divisor orientation.

Lemma 3.7. *Let D be a divisor orientation of a graph H with $(x, y) \in E(D)$. Then*

(1) for every vertex $w \in N(x) - N[y]$, we must have $(x, w) \in E(D)$, and

(2) for every vertex $z \in N(y) - N[x]$, we must have $(z, y) \in E(D)$.

Lemma 3.8. *Each of the graphs $\overline{P_3\overline{P_3}}, \overline{P_4\overline{P_4}}, \overline{C_4\overline{C_4}}$ and $\overline{C_5\overline{C_5}}$ is not a divisor graph.*

Proof. Obviously, $\overline{C_5\overline{C_5}}$ contains an induced 5-cycle. Thus, by Lemma 1.4, the graph $\overline{C_5\overline{C_5}}$ is not a divisor graph. Consider the graph $\overline{P_3\overline{P_3}}$ and assume that $y_1y_2y_3$ is the path P_3 in $\overline{P_3\overline{P_3}}$. Assume to the contrary that $\overline{P_3\overline{P_3}}$ is a divisor graph. According to Lemmas 1.1 and 1.2, we can assume that (y_2, y_3) is an arc of a divisor orientation D of $\overline{P_3\overline{P_3}}$. By Lemma 3.7, we get $(y_2, y_1), (y_2, x_3), (x_2, y_3) \in E(D)$. Now, since $(y_2, y_1) \in E(D)$, applying Lemma 3.7 gives $(y_2, x_1), (x_2, y_1) \in E(D)$. Since $(x_2, y_3), (x_2, y_1) \in E(D)$, by Lemma 3.7 we must have $(x_1, y_3), (x_3, y_1) \in E(D)$. If $(x_1, x_3) \in E(D)$, then $(x_1, x_3), (x_3, y_1) \in E(D)$ but $(x_1, y_1) \notin E(D)$, a contradiction. Similarly, if $(x_3, x_1) \in E(D)$, then $(x_3, x_1), (x_1, y_3) \in E(D)$ but $(x_3, y_3) \notin E(D)$, a contradiction. Therefore, $\overline{P_3\overline{P_3}}$ is not a divisor graph.

Finally, each of $\overline{P_4\overline{P_4}}$ and $\overline{C_4\overline{C_4}}$ contains an induced subgraph isomorphic to $\overline{P_3\overline{P_3}}$. Therefore, by Lemma 1.3, the graphs $\overline{P_4\overline{P_4}}$ and $\overline{C_4\overline{C_4}}$ are not divisor graphs. □

It is well known that, for any graph H of order greater than 5, at least one of H and $\overline{H}$ contains a triangle. Indeed, for any vertex x of a graph H of order greater than 5, we have $max\{\deg_H x, \deg_{\overline{H}} x\} \geq 3$. Note that if there exists a vertex x of a graph H (either G or $\overline{G}$) with $\deg_H x \geq 3$, then we have a triangle in H whenever two of the neighbors of x are adjacent in H, otherwise we have a triangle in $\overline{H}$.

Lemma 3.9. *Let H be a graph of order m. Then neither H nor $\overline{H}$ contains a triangle if and only if H or $\overline{H}$ belongs to $\{C_5, C_4, P_4, P_3, K_2, K_1\}$.*

Proof. If $m \geq 6$, then either H or $\overline{H}$ contains a triangle. Trivially, a graph of order at most 2 contains no triangle. If $m = 3$, then H or $\overline{H}$ contains a triangle if and only if the size of H is either 3 or 0. The graphs of order 3 and size 1 or 2 are precisely $\overline{P_3}$ and P_3. So assume that $m \in \{4, 5\}$. If there exists a vertex x of H such that $max\{\deg_H x, \deg_{\overline{H}} x\} \geq 3$, then H or $\overline{H}$ contains a triangle as illustrated in the paragraph preceding this lemma. Thus, assume that the maximum degrees $\triangle(H)$ and $\triangle(\overline{H})$ are both less than 3. Then, for $m = 4$, every vertex of H must have a positive degree less than 3 because $\deg_H x + \deg_{\overline{H}} x = 3$ for any $x \in V(H)$. Therefore, H must be one of the three graphs P_4, C_4 and $\overline{C_4}$. Clearly, P_4, C_4 and $\overline{C_4}$ (whose complements are $P_4, \overline{C_4}$ and C_4 , respectively) contain no triangle. Finally, assume that $m = 5$. Then every vertex of H must have degree 2 because $max\{\triangle(H), \triangle(\overline{H})\} \leq 2$ and $\deg_H x + \deg_{\overline{H}} x = 4$ for any $x \in V(H)$. Therefore, H must be the 5-cycle. Clearly, C_5 (which is self-complementary) contains no triangle. □

Now we are in a position to determine when $\overline{G\overline{G}}$ is a divisor graph.

Theorem 3.10. *Let G be a graph of order n. Then $\overline{G\overline{G}}$ is a divisor graph if and only if $n \leq 2$.*

Proof. We consider the two exclusive cases:

Case (1) One of the graphs G or $\overline{G}$ contains a triangle.

Say that $\overline{G}$ contains a triangle. Without loss of generality we can assume that $x_1x_2x_3x_1$ is a triangle in $\overline{G\overline{G}}$. Then $\{x_1, x_2, x_3, y_1, y_2, y_3\}$ induces a 3-sun in $\overline{G\overline{G}}$, and hence by Lemmas 1.5 and 1.3, $\overline{G\overline{G}}$ is not a divisor graph.

Case (2) Neither G nor $\overline{G}$ contains a triangle.

Then, by Lemma 3.9, G or $\overline{G}$ belongs to $\{C_5, C_4, P_4, P_3, K_2, K_1\}$. But $\overline{G\overline{G}}$ and $\overline{\overline{G}G}$ are isomorphic. Therefore, the result follows by Lemma 3.8 and the fact that the graphs $\overline{K_1\overline{K_1}} = 2K_1$ (having size 0) and $\overline{K_2\overline{K_2}} = P_4$ (bipartite) are divisor graphs. □

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