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Positive Integer Solutions of the Diophantine Equation $\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=\frac{1}{3}$

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Abstract

In this paper, we find positive integer solutions of the Diophantine equation $\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=\frac{1}{3}$.

1 Introduction and Preliminaries

A Diophantine equation is an equation, usually involving two or more unknowns, such that the only solutions of interest are the integer ones. The Erdös-Straus conjecture (Does $1/x+1/y+1/z=4/n$ have a positive integer solution for every integer $n \geq 2$?) was an open problem in number theory (see [2],[3]). The conjecture was formulated in 1948 by Erdös and

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Straus, and published by Erdös [1]. Later, many researchers studied and solved this Diophantine equation. In 2013, Rabago and Tagle [5] studied the areas and volume of a certain regular solid and the Diophantine equation $1/x + 1/y + 1/z = 1/2$. Later, Sándor showed that the Diophantine equation $1/x + 1/y + 1/z = p/q$ has a finite number of positive integer solutions. Recently, Pakapongpun [4] found positive integer solutions of the Diophantine equation $1/x + 2/y + 3/z = 1/2$. In this paper, we find positive integer solutions of the Diophantine equation $1/x + 2/y + 3/z = 1/3$.

2 Method of Analysis

From the Diophantine equation $1/x + 2/y + 3/z = 1/3$, we get $x \geq 4, y \geq 7$ and $z \geq 10$. We are going to consider three cases.

Case I: If $x \leq y \leq z$ or $x \leq z \leq y$, then $1/3 = 1/x + 2/y + 3/z \leq 6/x$. Hence $4 \leq x \leq 18$. In this case, we will consider 15 subcases ((1)-(15)).

If $x = 4$, then $2/y + 3/z = 1/12$ that is $(y - 24)(z - 36) = 864$. (1)

If $x = 5$, then $2/y + 3/z = 2/15$ that is $(y - 15)(2z - 45) = 675$. (2)

If $x = 6$, then $2/y + 3/z = 1/6$ that is $(y - 12)(z - 18) = 216$. (3)

If $x = 9$, then $2/y + 3/z = 6/27$ that is $(y - 9)(6z - 81) = 729$. (4)

If $x = 12$, then $2/y + 3/z = 1/4$ that is $(y - 8)(z - 12) = 96$. (5)

For (1), the following cases are only possible:
$y - 24 = 1, z - 36 = 864$;
$y - 24 = 2, z - 36 = 432; y - 24 = 3, z - 36 = 288; y - 24 = 4, z - 36 = 216$;
$y - 24 = 6, z - 36 = 144; y - 24 = 8, z - 36 = 108; y - 24 = 9, z - 36 = 96$;
$y - 24 = 12, z - 36 = 72; y - 24 = 16, z - 36 = 54; y - 24 = 18, z - 36 = 48$;
$y - 24 = 24, z - 36 = 36; y - 24 = 27, z - 36 = 32; y - 24 = 32, z - 36 = 27$;
$y - 24 = 36, z - 36 = 24; y - 24 = 48, z - 36 = 18; y - 24 = 54, z - 36 = 16$;
$y - 24 = 72, z - 36 = 12; y - 24 = 96, z - 36 = 9; y - 24 = 108, z - 36 = 8$;
$y - 24 = 144, z - 36 = 6; y - 24 = 216, z - 36 = 4; y - 24 = 288, z - 36 = 3$;
$y - 24 = 432, z - 36 = 2; y - 24 = 864, z - 36 = 1$,
leading to the solutions:
$(x, y, z) = (4, 25, 900), (4, 26, 468), (4, 27, 324)$,
$(4, 28, 252), (4, 30, 180), (4, 32, 144), (4, 33, 132), (4, 36, 108), (4, 40, 90)$,
$(4, 42, 84), (4, 48, 72), (4, 51, 68), (4, 56, 63), (4, 60, 60), (4, 72, 54), (4, 78, 52)$,
$(4, 96, 48), (4, 120, 45), (4, 132, 44), (4, 168, 42), (4, 240, 40), (4, 312, 39)$,
$(4, 456, 38), (4, 888, 37)$.

In the same manner,

(2) leads to $(x, y, z) = (5, 16, 360), (5, 18, 135), (5, 20, 90), (5, 24, 60)$,

$(5,30,45),(5,40,36),(5,42,35),(5,60,30),(5,90,27),(5,150,25),(5,240,24),$ $(5,690,23)$.

(3) leads to $(x,y,z)=(6,13,234),(6,14,126),(6,15,90),(6,16,72),$ $(6,18,54),(6,20,45),(6,21,42),(6,24,36),(6,30,30),(6,36,27),(6,39,26),$ $(6,48,24),(6,66,22),(6,84,21),(6,120,20),(6,228,19)$.

(4) leads to $(x,y,z)=(9,10,135),(9,12,54),(9,18,27),(9,36,18),$ $(9,90,15),(9,252,14)$.

(5) leads to $(x,y,z)=(12,9,108),(12,10,60),(12,11,44),(12,12,36),$ $(12,14,28),(12,16,24),(12,20,20),(12,24,18),(12,32,16),(12,40,15),$ $(12,56,14),(12,104,13)$.

If $x=7$, then $2/y+3/z=4/21$. (6)

If $y\le z$, then (6) gives, $4/21=2/y+3/z\le 5/y$ that is, $7\le y\le 26$, However, only $y\in\{12,14,24\}$ gives positive integer of z. Hence, $(x,y,z)=$ $(7,12,126),(7,14,63),(7,24,28)$.

If $z\le y$, then (6) gives, $4/21=2/y+3/z\le 5/z$ that is, $7\le z\le 26$, However, only $z\in\{16,18,21\}$ gives positive integer of y. Hence, $(x,y,z)=$ $(7,672,16),(7,84,18),(7,42,21)$.

If $x=8$, then $2/y+3/z=5/24$. (7)

If $y\le z$, then (7) gives, $5/24=2/y+3/z\le 5/y$ that is, $8\le y\le 24$, However, only $y\in\{10,12,15,16,24\}$ gives positive integer of z. Hence, $(x,y,z)=(8,10,360),(8,12,72),(8,15,40),(8,16,36),(8,24,24)$.

If $z\le y$, then (7) gives, $5/24=2/y+3/z\le 5/z$ that is, $8\le z\le 24$, However, only $z\in\{15,16,18,24\}$ gives positive integer of y. Hence, $(x,y,z)=$ $(8,240,15),(8,96,16),(8,48,18),(8,24,24)$.

If $x=10$, then $2/y+3/z=7/30$. (8)

If $y\le z$, then (8) gives, $7/30=2/y+3/z\le 5/y$ that is, $10\le y\le 21$, However, only $y\in\{10,12,15\}$ gives positive integer of z. Hence, $(x,y,z)=$ $(10,10,90),(10,12,45),(10,15,30)$.

If $z\le y$, then (8) gives, $7/30=2/y+3/z\le 5/z$ that is, $10\le z\le 21$, However, only $z\in\{13,14,15,18,20\}$ gives positive integer of y. Hence, $(x,y,z)=(10,780,13),(10,105,14),(10,60,15),(10,30,18),(10,24,20)$.

If $x=11$, then $2/y+3/z=8/33$. (9)

If $y\le z$, then (9) gives, $8/33=2/y+3/z\le 5/y$ that is, $11\le y\le 20$, However, all values of y do not give positive integer of z. Hence, there is no solutions for (9).

If $z\le y$, then (9) gives, $8/33=2/y+3/z\le 5/z$ that is, $11\le z\le 20$, However, all values of z do not give positive integer of y. Hence, there is no solution for (9).

If $x=13$, then $2/y+3/z=10/39$. (10)

If $y \leq z$, then (10) gives, $10/39 = 2/y + 3/z \leq 5/y$ that is, $13 \leq y \leq 19$, However, all values of y do not give positive integer of z. Hence, there is no solutions for (10).
If $z \leq y$, then (10) gives $10/39 = 2/y + 3/z \leq 5/z$ that is, $13 \leq z \leq 19$, However, only $z \in \{13\}$ gives positive integer of y. Hence, $(x, y, z) = (13, 78, 13)$.

If $x = 14$, then $2/y + 3/z = 11/42$. (11)

If $y \leq z$, then (11) gives, $11/42 = 2/y + 3/z \leq 5/y$ that is, $14 \leq y \leq 19$, However, all values of y do not give positive integer of z. Hence, there is no solutions for (11).
If $z \leq y$, then (11) gives, $11/42 = 2/y + 3/z \leq 5/z$ that is, $14 \leq z \leq 19$, However, only $z \in \{14, 18\}$ gives positive integer of y. Hence, $(x, y, z) = (14, 42, 14), (14, 21, 18)$.

If $x = 15$, then $2/y + 3/z = 12/45$. (12)

If $y \leq z$, then (12) gives, $12/45 = 2/y + 3/z \leq 5/y$ that is, $15 \leq y \leq 18$, However, all values of y do not give positive integer of z. Hence, there is no solutions for (12).
If $z \leq y$, then (12) gives, $12/45 = 2/y + 3/z \leq 5/z$ that is, $15 \leq z \leq 18$, However, only $z \in \{15, 18\}$ gives positive integer of y. Hence, $(x, y, z) = (15, 30, 15), (15, 20, 18)$.

If $x = 16$, then $2/y + 3/z = 13/48$. (13)

If $y \leq z$, then (13) gives, $13/48 = 2/y + 3/z \leq 5/y$ that is, $16 \leq y \leq 18$, However, all values of y do not give positive integer of z. Hence, there is no solutions for (13).
If $z \leq y$, then (13) gives, $13/48 = 2/y + 3/z \leq 5/z$ that is, $16 \leq z \leq 18$, However, only $z \in \{16\}$ gives positive integer of y. Hence, $(x, y, z) = (16, 24, 16)$.

If $x = 17$, then $2/y + 3/z = 14/51$. (14)

If $y \leq z$, then (14) gives, $14/51 = 2/y + 3/z \leq 5/y$ that is, $17 \leq y \leq 18$, However, all values of y do not give positive integer of z. Hence, there is no solutions for (14).
If $z \leq y$, then (14) gives, $14/51 = 2/y + 3/z \leq 5/z$ that is, $17 \leq z \leq 18$, However, all values of z do not give positive integer of y. Hence, there is no solutions for (14).

If $x = 18$, then $2/y + 3/z = 15/54$. (15)

If $y \leq z$, then (15) gives, $15/54 = 2/y + 3/z \leq 5/y$ that is, $y = 18$, Hence, $(x, y, z) = (18, 18, 18)$.
If $z \leq y$, then (15) gives, $15/54 = 2/y + 3/z \leq 5/z$ that is, $z = 18$, Hence, $(x, y, z) = (18, 18, 18)$.

Case II: If $y \le x \le z$ or $y \le z \le x$, then $1/3 = 1/x + 2/y + 3/z \le 6/y$. Hence $7 \le y \le 18$. In this case, we will consider 12 subcases ((16)-(27)).

If $y = 7$, then $1/x + 3/z = 1/21$ that is $(x-21)(z-63) = 1323$. (16)

If $y = 8$, then $1/x + 3/z = 1/12$ that is $(x-12)(z-36) = 432$. (17)

If $y = 9$, then $1/x + 3/z = 1/9$ that is $(x-21)(z-63) = 1323$. (18)

If $y = 12$, then $1/x + 3/z = 1/6$ that is $(x-6)(z-18) = 108$. (19)

If $y = 15$, then $1/x + 3/z = 1/5$ that is $(x-5)(z-15) = 75$. (20)

For (16), the following cases are only possible: $x-21=1, z-63=1323; x-21=3, z-63=441; x-21=7, z-63=189; x-21=9, z-63=147; x-21=21, z-63=63; x-21=27, z-63=49; x-21=49, z-63=27; x-21=63, z-63=21; x-21=147, z-63=9; x-21=189, z-63=7; x-21=441, z-63=3; x-21=1323, z-63=1$,
leading to the solutions:
$(x,y,z) = (22,7,1386), (24,7,504), (28,7,252), (30,7,210), (42,7,126), (48,7,112), (70,7,90), (84,7,84), (168,7,72),$
$(210,7,70), (462,7,66), (1344,7,64)$.

In the same manner,

(17) leads to $(x,y,z) = (13,8,468), (14,8,252), (15,8,180), (16,8,144), (18,8,108), (20,8,90), (21,8,24), (24,8,72), (28,8,63), (30,8,60), (36,8,54), (39,8,52), (48,8,48), (60,8,45), (66,8,44), (84,8,42), (120,8,40), (156,8,39), (228,8,38), (444,8,37)$.

(18) leads to $(x,y,z) = (10,9,270), (12,9,108), (18,9,54), (36,9,36), (90,9,30), (252,9,28)$.

(19) leads to $(x,y,z) = (7,12,126), (8,12,72), (9,12,54), (10,12,45), (12,12,36), (15,12,30), (18,12,27), (24,12,24), (33,12,22), (42,12,21), (60,12,20), (114,12,19)$,

(20) leads to $(x,y,z) = (6,15,90), (8,15,40), (10,15,30), (80,15,16), (30,15,18), (6,15,90)$.

If $y = 10$, then $1/x + 3/z = 2/15$. (21)

If $x \le z$, then (21) gives, $2/15 = 1/x + 3/z \le 4/x$ that is, $10 \le x \le 30$, However, only $x \in \{10,12,15,20,21,30\}$ gives positive integer of z. Hence, $(x,y,z) = (10,10,90), (12,10,60), (15,10,45), (20,10,36), (21,10,35), (30,10,30)$.

If $z \le x$, then (21) gives, $2/15 = 1/x + 3/z \le 4/z$ that is, $10 \le z \le 30$, However, only $z \in \{23,24,25,27,30\}$ gives positive integer of x. Hence, $(x,y,z) = (345,10,23), (120,10,24), (75,10,25), (45,10,27), (30,10,30)$.

If $y = 11$, then $1/x + 3/z = 5/33$. (22)

If $x \le z$, then (22) gives, $5/33 = 1/x + 3/z \le 4/x$ that is, $11 \le x \le 26$, However, only $x \in \{12\}$ gives positive integer of z. Hence, $(x,y,z) =$

$(12, 11, 44)$.
If $z \leq x$, then (22) gives, $5/33 = 1/x + 3/z \leq 4/z$ that is, $11 \leq z \leq 26$, However, only $z \in \{20, 22\}$ gives positive integer of x. Hence, $(x, y, z) = (660, 11, 20), (66, 11, 22)$.

If $y = 13$, then $1/x + 3/z = 7/39$. (23)
If $x \leq z$, then (23) gives, $7/39 = 1/x + 3/z \leq 4/x$ that is, $13 \leq x \leq 22$, However, all values of x do not give positive integer of z. Hence, there is no solutions for (23).
If $z \leq x$, then (23) gives, $7/39 = 1/x + 3/z \leq 4/z$ that is, $13 \leq z \leq 22$, However, only $z \in \{18\}$ gives positive integer of x. Hence, $(x, y, z) = (78, 13, 18)$.

If $y = 14$, then $1/x + 3/z = 4/21$. (24)
If $x \leq z$, then (24) gives, $4/21 = 1/x + 3/z \leq 4/x$ that is, $14 \leq x \leq 21$, However, only $x \in \{21\}$ gives positive integer of z. Hence, $(x, y, z) = (21, 14, 21)$.
If $z \leq x$, then (24) gives, $4/21 = 1/x + 3/z \leq 4/z$ that is, $14 \leq z \leq 21$, However, only $z \in \{16, 18, 21\}$ gives positive integer of x. Hence, $(x, y, z) = (336, 14, 16), (42, 14, 18), (21, 14, 21)$.

If $y = 16$, then $1/x + 3/z = 10/48$. (25)
If $x \leq z$, then (25) gives, $10/48 = 1/x + 3/z \leq 4/x$ that is, $16 \leq x \leq 19$, However, all values of x do not give positive integer of z. Hence, there is no solutions for (25).
If $z \leq x$, then (25) gives, $10/48 = 1/x + 3/z \leq 4/z$ that is, $16 \leq z \leq 19$, However, only $z \in \{16, 18\}$ gives positive integer of x. Hence, $(x, y, z) = (48, 16, 16), (24, 16, 18)$.

If $y = 17$, then $1/x + 3/z = 11/51$. (26)
If $x \leq z$, then (26) gives, $11/51 = 1/x + 3/z \leq 4/x$ that is, $17 \leq x \leq 18$, However, both of them do not give positive integer of z. Hence, there is no solution for (26).
If $z \leq x$, then (26) gives, $11/51 = 1/x + 3/z \leq 4/z$ that is, $17 \leq z \leq 18$, However, both of them do not give positive integer of x. Hence, there is no solution for (26).

If $y = 18$, then $1/x + 3/z = 12/54$. (27)
If $x \leq z$, then (27) gives, $12/54 = 1/x + 3/z \leq 4/x$ that is, $x = 18$. Hence, $(x, y, z) = (18, 18, 18)$.
If $z \leq x$, then (27) gives, $12/54 = 1/x + 3/z \leq 4/z$ that is, $z = 18$. Hence, $(x, y, z) = (18, 18, 18)$.

Case III: If $z \leq x \leq y$ or $z \leq y \leq x, 1/3 = 1/x + 2/y + 3/z \leq 6/z$. Hence $10 \leq z \leq 18$. In this case, we will consider 9 subcases ((28)-(36)).

If $z = 10$, then $1/x + 2/y = 1/30$ that is $(x - 30)(y - 60) = 1800$. (28)
If $z = 11$, then $1/x + 2/y = 2/33$ that is $(2x - 33)(y - 33) = 1089$. (29)
If $z = 12$, then $1/x + 2/y = 1/12$ that is $(x - 12)(y - 24) = 288$. (30)
If $z = 15$, then $1/x + 2/y = 2/15$ that is $(2x - 15)(y - 15) = 225$. (31)
If $z = 18$, then $1/x + 2/y = 1/6$ that is $(x - 6)(y - 12) = 72$. (32)

In the same manner,

(28) leads to $(x, y, z) = (31, 1860, 10), (32, 960, 10), (33, 660, 10), (34, 510, 10), (35, 420, 10), (36, 360, 10), (38, 285, 10), (39, 260, 10), (40, 240, 10), (42, 210, 10), (45, 180, 10), (48, 160, 10), (50, 150, 10), (54, 135, 10), (55, 132, 10), (60, 120, 10), (66, 110, 10), (70, 105, 10), (75, 100, 10), (80, 96, 10), (90, 90, 10), (102, 85, 10), (105, 84, 10), (120, 80, 10), (130, 78, 10), (150, 75, 10), (180, 72, 10), (210, 70, 10), (230, 69, 10), (255, 68, 10), (330, 66, 10), (390, 65, 10), (480, 64, 10), (630, 63, 10), (930, 32, 10), (1830, 61, 10)$.

(29) leads to $(x, y, z) = (17, 1122, 11), (18, 396, 11), (21, 154, 11), (22, 132, 11), (33, 66, 11), (66, 44, 11), (77, 42, 11), (198, 36, 11), (561, 34, 11)$.

(30) leads to $(x, y, z) = (13, 312, 12), (14, 168, 12), (15, 120, 12), (16, 96, 12), (18, 72, 12), (20, 60, 12), (21, 56, 12), (24, 48, 12), (28, 42, 12), (30, 40, 12), (36, 36, 12), (44, 33, 12), (48, 32, 12), (60, 30, 12), (84, 28, 12), (108, 27, 12), (156, 26, 12), (300, 25, 12)$

(31) leads to $(x, y, z) = (8, 240, 15), (9, 90, 15), (10, 60, 15), (12, 40, 15), (15, 30, 15), (20, 24, 15), (30, 20, 15), (45, 18, 15), (120, 16, 15)$.

(32) leads to $(x, y, z) = (7, 84, 18), (8, 48, 18), (9, 36, 18), (10, 30, 18), (12, 24, 18), (14, 21, 18), (15, 20, 18), (18, 18, 18), (24, 16, 18), (30, 15, 18), (42, 14, 18), (78, 13, 18)$.

If $z = 13$, then $1/x + 2/y = 4/39$. (33)

If $x \le y$, then (33) gives, $4/39 = 1/x + 2/y \le 3/x$ that is, $13 \le x \le 29$, However, only $x \in \{13\}$ gives positive integer of y. Hence, $(x, y, z) = (13, 78, 13)$.

If $y \le x$, then (33) gives, $4/39 = 1/x + 2/y \le 3/y$ that is, $13 \le y \le 29$, However, only $y \in \{20, 24, 26\}$ gives positive integer of x. Hence, $(x, y, z) = (390, 20, 13), (52, 24, 13), (39, 26, 13)$.

If $z = 14$, then $1/x + 2/y = 5/42$. (34)

If $x \le y$, then (34) gives, $5/42 = 1/x + 2/y \le 3/x$ that is, $14 \le x \le 25$, However, only $x \in \{14, 21\}$ gives positive integer of y. Hence, $(x, y, z) = (14, 42, 14), (21, 28, 14)$.

If $y \le x$, then (34) gives, $5/42 = 1/x + 2/y \le 3/y$ that is, $14 \le y \le 25$, However, only $y \in \{17, 18, 21, 24\}$ gives positive integer of x. Hence, $(x, y, z) = (714, 17, 14), (126, 18, 14), (42, 21, 14), (28, 24, 14)$.

If $z = 16$, then $1/x + 2/y = 7/48$. (35)

If $x \leq y$, then (35) gives, $7/48 = 1/x + 2/y \leq 3/x$ that is, $16 \leq x \leq 20$, However, only $x \in \{16\}$ gives positive integer of y. Hence, $(x, y, z) = (16, 24, 16)$.
If $y \leq x$, then (35) gives, $7/48 = 1/x + 2/y \leq 3/y$ that is, $16 \leq y \leq 20$, However, only $y \in \{16\}$ gives positive integer of x. Hence, $(x, y, z) = (48, 16, 16)$.

If $z = 17$, then $1/x + 2/y = 8/51$. (36)
If $x \leq y$, then (36) gives, $8/51 = 1/x + 2/y \leq 3/x$ that is, $17 \leq x \leq 19$, However, all values of x do not give positive integer of y. Hence, there is no solutions for (36).
If $y \leq x$, then (36) gives, $8/51 = 1/x + 2/y \leq 3/y$ that is, $17 \leq y \leq 19$, However, all values of y do not give positive integer of x. Hence, there is no solution for (36).

Consequently, the Diophantine equation $1/x + 2/y + 3/z = 1/3$ has 236 solutions as follows :
$(x, y, z) = (4, 25, 900), (4, 26, 468), (4, 27, 324), (4, 28, 252), (4, 30, 180), (4, 32, 144),$
$(4, 33, 132), (4, 36, 108), (4, 40, 90), (4, 42, 84), (4, 48, 72), (4, 51, 68), (4, 56, 63),$
$(4, 60, 60), (4, 72, 54), (4, 78, 52), (4, 96, 48), (4, 120, 45), (4, 132, 44), (4, 168, 42),$
$(4, 240, 42), (4, 312, 39), (4, 456, 38), (4, 888, 37), (5, 16, 360), (5, 18, 135), (5, 20, 90),$
$(5, 24, 60), (5, 30, 45), (5, 40, 36), (5, 42, 35), (5, 60, 30), (5, 90, 27), (5, 150, 25),$
$(5, 240, 24), (5, 690, 23), (6, 13, 234), (6, 14, 126), (6, 15, 90), (6, 16, 72), (6, 18, 54),$
$(6, 20, 45), (6, 21, 42), (6, 24, 36), (6, 30, 30), (6, 36, 27), (6, 39, 26), (6, 48, 24),$
$(6, 66, 22), (6, 84, 21), (6, 120, 20), (6, 228, 19), (7, 12, 126), (7, 14, 63), (7, 24, 28),$
$(7, 42, 21), (7, 84, 18), (7, 672, 16), (8, 10, 360), (8, 12, 72), (8, 15, 40), (8, 16, 36),$
$(8, 24, 24), (8, 48, 18), (8, 96, 16), (8, 240, 15), (9, 10, 135), (9, 12, 54), (9, 18, 27),$
$(9, 36, 18), (9, 90, 15), (9, 252, 14), (10, 9, 270), (10, 10, 90), (10, 12, 45),$
$(10, 15, 30), (10, 24, 20), (10, 30, 18), (10, 60, 15), (10, 105, 14), (10, 780, 13),$
$(12, 9, 108), (12, 10, 60), (12, 11, 44), (12, 12, 36), (12, 14, 28), (12, 16, 24),$
$(12, 20, 20), (12, 24, 18), (12, 32, 16), (12, 40, 15), (12, 56, 14), (12, 104, 13),$
$(13, 8, 468), (13, 78, 13), (13, 312, 12), (14, 8, 252), (14, 21, 18), (14, 42, 14),$
$(14, 168, 12), (15, 8, 180), (15, 10, 45), (15, 12, 30), (15, 20, 18), (15, 30, 15),$
$(15, 120, 12), (16, 8, 144), (16, 24, 16), (16, 96, 12), (17, 1122, 11), (18, 8, 108),$
$(18, 9, 54), (18, 12, 27), (18, 18, 18), (18, 72, 12), (18, 396, 11), (20, 8, 90),$
$(20, 10, 36), (20, 24, 15), (20, 60, 12), (21, 8, 84), (21, 10, 35), (21, 14, 21),$
$(21, 28, 14), (21, 56, 12), (21, 154, 11), (22, 7, 1386), (22, 132, 11), (24, 7, 504),$
$(24, 8, 72), (24, 12, 24), (24, 16, 18), (24, 48, 12), (28, 7, 252), (28, 8, 63),$
$(28, 24, 14), (28, 42, 12), (30, 7, 210), (30, 8, 60), (30, 10, 30), (30, 15, 18),$
$(30, 20, 15), (30, 40, 12), (31, 1860, 10), (32, 960, 10), (33, 12, 22), (33, 66, 11),$
$(33, 660, 10), (34, 510, 10), (35, 420, 10), (36, 8, 54), (36, 9, 36), (36, 36, 12),$

$(36,360,10),(38,285,10),(39,8,52),(39,26,13),(39,260,10),(40,240,10),$
$(42,7,126),(42,12,21),(42,14,18),(42,21,14),(42,210,10),(44,33,12)$
$(45,10,27),(45,18,15),(45,180,10),(48,7,112),(48,8,48),(48,16,16),$
$(48,32,12),(48,160,10),(50,150,10),(52,24,13),(54,135,10),(55,132,10),$
$(60,8,45),(60,12,20),(60,30,12),(60,120,10),(66,8,44),(66,11,22),$
$(66,44,11),(66,110,10),(70,7,90),(70,105,10),(75,10,25),(75,100,10),$
$(77,42,11),(78,13,18),(80,15,16),(80,96,10),(84,7,84),(84,8,42),$
$(84,28,12),(90,9,30),(90,90,10),(102,85,10),(105,84,10),(108,27,12),$
$(114,12,19),(120,8,40),(120,10,24),(120,16,15),(120,80,10),(126,18,14),$
$(130,78,10),(150,75,10),(156,8,39),(156,26,12),(168,7,72),(180,72,10),$
$(198,36,11),(210,7,70),(210,70,10),(228,8,38),(230,69,10),(252,9,28),$
$(255,68,10),(300,25,12),(330,66,10),(336,14,16),(345,10,23),(390,20,13),$
$(390,65,10),(444,8,37),(462,7,66),(480,64,10),(561,34,11),(630,63,10),$
$(660,11,20),(714,17,14),(930,62,10),(1344,7,64),(1830,61,10).$

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