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A note on fuzzy almost interior hyperideals of semihypergroups

Warud Nakkhasen, Piyaporn Khathipphathi, Sudapa Panmuang

Department of Mathematics
Faculty of Science
Mahasarakham University
Maha Sarakham 44150, Thailand

email: warud.n@msu.ac.th, 61010213108@msu.ac.th,
61010213117@msu.ac.th

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Abstract

The concept of almost interior hyperideals in semihypergroups which is a special case of almost interior Γ-hyperideals in ordered Γ-semihypergroups was introduced by Rao et al. in 2021. In this paper, we introduce the concept of fuzzy almost interior hyperideals of semihypergroups and give some properties of them. Then, we provide some connections between almost interior hyperideals and fuzzy almost interior hyperideals in semihypergroups.

1 Introduction

In 1980, Grošek and Satko [2] extended the concept of ideals in semigroups to the concept of almost ideals, characterizing the semigroups that have proper almost ideals. Later, Bogdanović [1] defined the notion of almost bi-ideals in semigroups as a generalization of bi-ideals. The concept of fuzzy subsets was first introduced by Zadeh [10] as a function from a nonempty set X to the unit interval $[0, 1]$. The fuzzy subsets are an extension of classical sets in mathematics. In 2018, Wattanatripop et al. [9] introduced the notion of

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Corresponding author: Warud Nakkhasen.
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fuzzy almost bi-ideals of semigroups and studied some relationships between almost bi-ideals and fuzzy almost bi-ideals in semigroups. Following that the concepts of almost interior ideals and weakly almost interior ideals of semigroups were introduced by Kaopusek et al. [3] in 2020. Later, Krailoet et al. [4] investigated the notions of fuzzy almost interior ideals and weakly fuzzy almost interior ideals of semigroups.

The work of Marty [5], who established the study of hyperstructures in 1934, has been examined by many mathematicians since then. Among algebraic hypertructures, the authors concentrate on semihypergroups. In 2020, Suebsung et al. [8] defined almost hyperideals in semihypergroups, which is a generalization of hyperideals, and gave some interesting their properties. Later, in 2021, Muangdoo et al. [6] introduced the concept of almost bi-hyperideals of semihypergroups and discussed some connections between almost bi-hyperideals and their fuzzification in semihypergroups. It is known that ordered Γ-semihypergroups are generalize semihypergroups. Recently, Rao et al. [7] defined the concept of almost interior Γ-hyperideals; that is, they generalized the concept of almost interior hyperideals in semihypergroups. In this paper, we introduce the notion of fuzzy almost interior hyperideals of semihypergroups. Then, we present some relationships between almost interior hyperideals and their fuzzification in semihypergroups.

2 Preliminaries

A *hyperstructure* $(H, \cdot)$ is a nonempty set H together with a mapping $\cdot : H \times H \to \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ denotes the set of all nonempty sets of H. If $A, B \in \mathcal{P}^*(H)$ and $x \in H$, then we use the notations $A \cdot B = \bigcup_{a \in A, b \in B} a \cdot b$, $A \cdot x = A \cdot \{x\}$ and $x \cdot B = \{x\} \cdot B$. A hyperstructure $(S, \cdot)$ is called a *semihypergroup* if, for every $x, y, z \in S$, $(x \cdot y) \cdot z = x \cdot (y \cdot z)$. More conveniently, we write a semihypergroup S instead of a semihypergroup $(S, \cdot)$. A nonempty subset A of a semihypergroup S is called a *subsemihypergroup* of S if $A \cdot A \subseteq A$. A subsemihypergroup I of a semihypergroup S is called an *interior hyperideal* of S if $S \cdot I \cdot S \subseteq I$.

Next, we present the definition of almost interior hyperideals of a semihypergroup S which is a special case of Definition 4 in [7].

Definition 2.1. [7] *A nonempty subset I of a semihypergroup S is called an almost interior hyperideal of S if $(x \cdot I \cdot y) \cap I \neq \emptyset$, for all $x, y \in S$.*

A *fuzzy subset* [10] of a nonempty set X is a mapping $f : X \to [0,1]$. Let f and g be any two fuzzy subsets of a nonempty set X. Then, (i) $f \subseteq g$ if and only if $f(x) \leq g(x)$ for all $x \in X$; (ii) $(f \cap g)(x) = \min\{f(x), g(x)\}$ for all $x \in X$; (iii) $(f \cup g)(x) = \max\{f(x), g(x)\}$ for all $x \in X$.

Let X be a nonempty set. For any fuzzy subset f of X, the *support* of f is defined by $supp(f) := \{x \in X \mid f(x) \neq 0\}$. Let A be any subset of X, $s \in X$ and $\alpha \in (0,1]$. The *characteristic mapping* C_A of A and the *fuzzy point* s_α of X are defined, for every $x \in X$, as follows

$$C_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad s_\alpha(x) = \begin{cases} \alpha & \text{if } x = s, \\ 0 & \text{otherwise,} \end{cases},$$

respectively.

Lemma 2.2. [6] *Let A and B be nonempty subsets of a nonempty set X and let f and g be fuzzy subsets of X. Then, the following statements hold:*

(i) $C_{A \cap B} = C_A \cap C_B$; (iii) $supp(C_A) = A$;

(ii) $A \subseteq B \Leftrightarrow C_A \subseteq C_B$; (iv) $f \subseteq g \Rightarrow supp(f) \subseteq supp(g)$.

Let f and g be fuzzy subsets of a semihypergroup S. A *product* $f \circ g$ is defined by for any $x \in S$,

$$(f \circ g)(x) = \begin{cases} \sup\limits_{x \in y \cdot z} [\min\{f(y), g(z)\}] & \text{if } \exists\, y, z \in S \text{ such that } x \in y \cdot z, \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 2.3. [6] *If A and B are subsets of a semihypergroup S, then $C_A \circ C_B = C_{A \cdot B}$.*

3 Fuzzy almost interior hyperideals

In this section, we introduce the concept of fuzzy almost interior hyperideals of semihypergroups and investigate some of their properties.

Definition 3.1. *Let f be a fuzzy subset of a semihypergroup S such that $f \neq 0$. Then, f is called a fuzzy almost interior hyperideal of S if $(s_\alpha \circ f \circ t_\beta) \cap f \neq 0$ for all fuzzy points s_α, t_β of S.*

Theorem 3.2. *Let f be a fuzzy almost interior hyperideal of a semihypergroup S. If g is a fuzzy subset of S such that $f \subseteq g$, then g is also a fuzzy almost interior hyperideal of S.*

Proof. Assume that g is a fuzzy subset of S such that $f \subseteq g$. Obviously, $g \neq 0$. Let s_α and t_β be any fuzzy points of S. Then, $0 \neq (s_\alpha \circ f \circ t_\beta) \cap f \subseteq (s_\alpha \circ g \circ t_\beta) \cap g$. Hence, g is a fuzzy almost interior hyperideal of S. □

Corollary 3.3. *The union of any two fuzzy almost interior hyperideals of S is also a fuzzy almost interior hyperideal of S.*

Example 3.4. *Consider $S = \{a, b, c\}$ together with the hyperoperation "$\cdot$" on S defined by the following table:*

$\cdot$	a	b	c
a	$\{a\}$	$\{b\}$	$\{c\}$
b	$\{b\}$	$\{a, c\}$	$\{b, c\}$
c	$\{c\}$	$\{b, c\}$	$\{a, b\}$

Then, $(S, \cdot)$ is a semihypergroup. Define two fuzzy subsets f and g of S by for every $x \in S$,

$$f(x) = \begin{cases} 0.4 & \text{if } x \in \{a, b\}, \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 0.5 & \text{if } x \in \{b, c\}, \\ 0 & \text{otherwise.} \end{cases}$$

By routine calculations, we have that f and g are fuzzy almost interior hyperideals of S. Moreover, for fuzzy points a_α and b_β of S where $\alpha, \beta \in (0, 1]$, we obtain that $f \cap g$ is not a fuzzy almost interior hyperideal of S, because $[(b_\beta \circ (f \cap g) \circ a_\alpha) \cap (f \cap g)](x) = 0$ for all $x \in S$.

From Example 3.4, we conclude that the intersection of any two fuzzy almost interior hyperideals of a semihypergroup S need not to be a fuzzy almost interior hyperideal of S.

Theorem 3.5. *Let A be a nonempty subset of a semihypergroup S. Then, A is an almost interior hyperideal of S if and only if C_A is a fuzzy almost interior hyperideal of S.*

Proof. Assume that A is an almost interior hyperideal of S. Let $x, y \in S$ and $\alpha, \beta \in (0, 1]$. Then, $(x \cdot A \cdot y) \cap A \neq \emptyset$. So, there exists $a \in S$ such that $a \in x \cdot A \cdot y$ and $a \in A$. This implies that $(x_\alpha \circ C_A \circ y_\beta)(a) \neq 0$ and $C_A(a) = 1$. We obtain that $(x_\alpha \circ C_A \circ y_\beta) \cap C_A \neq 0$. Hence, C_A is a fuzzy almost interior hyperideal of S.

Conversely, assume that C_A is a fuzzy almost interior hyperideal of S. Let $x, y \in S$ and $\alpha, \beta \in (0, 1]$. Then, $(x_\alpha \circ C_A \circ y_\beta) \cap C_A \neq 0$. Thus, there exists $a \in S$ such that $(x_\alpha \circ C_A \circ y_\beta)(a) \neq 0$ and $C_A(a) \neq 0$. It turns out that $a \in (x \cdot A \cdot y) \cap A$, which implies that $(x \cdot A \cdot y) \cap A \neq \emptyset$. Therefore, A is an almost interior hyperideal of S. □

Theorem 3.6. *Let f be a fuzzy subset of a semihypergroup S. Then, f is a fuzzy almost interior hyperideal of S if and only if $supp(f)$ is an almost interior hyperideal of S.*

Proof. Assume that f is a fuzzy almost interior hyperideal of S. Let $x, y \in S$ and $\alpha, \beta \in (0,1]$. Then, $(x_\alpha \circ f \circ y_\beta) \cap f \neq 0$. So, there exists $a \in S$ such that $(x_\alpha \circ f \circ y_\beta)(a) \neq 0$ and $f(a) \neq 0$. Also, there exists $a' \in S$ such that $a \in x \cdot a' \cdot y$, we have $0 \neq (x_\alpha \circ f \circ y_\beta)(a) = \sup\limits_{a \in x \cdot a' \cdot y} [\min\{x_\alpha(x), f(a'), y_\beta(y)\}]$. Thus, $f(a') \neq 0$ and $f(a) \neq 0$. Hence, $a, a' \in supp(f)$. It follows that $(x_\alpha \circ C_{supp(f)} \circ y_\beta)(a) \neq 0$ and $C_{supp(f)}(a) \neq 0$, which implies that $(x_\alpha \circ C_{supp(f)} \circ y_\beta) \cap C_{supp(f)} \neq 0$. We obtain that $C_{supp(f)}$ is a fuzzy almost interior hyperideal of S. By Theorem 3.5, $supp(f)$ is an almost interior hyperideal of S.

Conversely, assume that $supp(f)$ is an almost interior hyperideal of S. By Theorem 3.5, $C_{supp(f)}$ is a fuzzy almost interior hyperideal of S. Let x_α and y_β be any fuzzy points of S. Then, $(x_\alpha \circ C_{supp(f)} \circ y_\beta) \cap C_{supp(f)} \neq 0$. Thus, there exists $a \in S$ such that $(x_\alpha \circ C_{supp(f)} \circ y_\beta)(a) \neq 0$ and $C_{supp(f)}(a) \neq 0$. So, there exists $a' \in S$ such that $a \in x \cdot a' \cdot y$, we have $0 \neq (x_\alpha \circ C_{supp(f)} \circ y_\beta)(a) = \sup\limits_{a \in x \cdot a' \cdot y} \left[\min\{x_\alpha(x), C_{supp(f)}(a'), y_\beta(y)\}\right]$, this implies that $C_{supp(f)}(a') \neq 0$. It follows that $f(a) \neq 0$ and $f(a') \neq 0$. Hence, $(x_\alpha \circ f \circ y_\beta) \cap f \neq 0$. Therefore, f is a fuzzy almost interior hyperideal of S. □

A fuzzy almost interior hyperideal f of a semihyperrgroup S is called *minimal* if for any fuzzy almost interior hyperideal g of S such that $g \subseteq f$, then $supp(g) = supp(f)$.

Theorem 3.7. *Let I be a nonempty subset of a semihypergroup S. Then, I is a minimal almost interior hyperideal of S if and only if C_I is a minimal fuzzy almost interior hyperideal of S.*

Proof. Assume that I is a minimal almost interior hyperideal of S. By Theorem 3.5, C_I is a fuzzy almost interior hyperideal of S. Let g be any fuzzy almost interior hyperideal of S such that $g \subseteq C_I$. By Lemma 2.2, $supp(g) \subseteq supp(C_I) = I$. By Theorem 3.6, $supp(g)$ is an almost interior hyperideal of S. By the minimality of I, we get that $supp(g) = I = supp(C_I)$. Therefore, C_I is a minimal fuzzy almost interior hyperideal of S.

Conversely, assume that C_I is a minimal fuzzy almost interior hyperideal of S. By Theorem 3.5, I is an almost interior hyperideal of S. Let A be any almost interior hyperideal of S such that $A \subseteq I$. Again, by Theorem 3.5, C_A is a fuzzy almost interior hyperideal of S such that $C_A \subseteq C_I$. Since

C_I is minimal and Lemma 2.2, we have $A = supp(C_A) = supp(C_I) = I$. Consequently, I is a minimal almost interior hyperideal of S. $\square$

The following corollary follows from Theorems 3.5 and 3.6.

Corollary 3.8. *Let S be a semihypergroup. Then, S has no proper almost interior hyperideal if and only if for every fuzzy almost interior hyperideal f of S, $supp(f) = S$.*

Let P be an almost interior hyperideal of a semihypergroup S. Then: (i) P is called *prime* if for any almost interior hyperideals A and B of S such that $A \cdot B \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$; (ii) P is called *semiprime* if for any almost interior hyperideal A of S such that $A \cdot A \subseteq P$ implies that $A \subseteq P$; (iii) P is called *strongly prime* if for any almost interior hyperideals A and B of S such that $(A \cdot B) \cap (B \cdot A) \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 3.9. *A fuzzy almost interior hyperideal h of a semihypergroup S is called a fuzzy prime almost interior hyperideal of S if for any two fuzzy almost interior hyperideals f and g of S, $f \circ g \subseteq h$ implies that $f \subseteq h$ or $g \subseteq h$.*

Definition 3.10. *A fuzzy almost interior hyperideal h of a semihypergroup S is called a fuzzy semiprime almost interior hyperideal of S if for any fuzzy almost interior hyperideal f of S, $f \circ f \subseteq h$ implies that $f \subseteq h$.*

Definition 3.11. *A fuzzy almost interior hyperideal h of a semihypergroup S is called a fuzzy strongly prime almost interior hyperideal of S if for any two fuzzy almost interior hyperideals f and g of S, $(f \circ g) \cap (g \circ f) \subseteq h$ implies that $f \subseteq h$ or $g \subseteq h$.*

It is clear that every fuzzy strongly prime almost interior hyperideal of a semihypergroup is a fuzzy prime almost interior hyperideal, and every fuzzy prime almost interior hyperideal of a semihypergroup is a fuzzy semiprime almost interior hyperideal.

Finally, we present the relationships between strongly prime (resp. prime, semiprime) almost interior hyperideals and their fuzzifications in semihypergroups.

Theorem 3.12. *Let P be a nonempty subset of a semihypergroup S. Then, P is a strongly prime almost interior hyperideal of S if and only if C_P is a fuzzy strongly semiprime almost interior hyperideal of S.*

Proof. Assume that P is a strongly prime almost interior hyperideal of S. By Theorem 3.5, C_P is a fuzzy almost interior hyperideal of S. Let f and g be any two fuzzy almost interior hyperideals of S such that $(f \circ g) \cap (g \circ f) \subseteq C_P$. Suppose that $f \nsubseteq C_P$ and $g \nsubseteq C_P$. Thus, there exist $x, y \in S$ such that $f(x) \neq 0$ and $g(y) \neq 0$, but $C_P(x) = 0$ and $C_P(y) = 0$. It turns out that $x \notin P$ and $y \notin P$. By Theorem 3.6, $supp(f)$ and $supp(g)$ are almost interior hyperideals of S such that $x \in supp(f)$ and $y \in supp(g)$. This means that $supp(f) \nsubseteq P$ and $supp(g) \nsubseteq P$. By the hypothesis, $(supp(f) \cdot supp(g)) \cap (supp(g) \cdot supp(f)) \nsubseteq P$. So, there exists $t \in (supp(f) \cdot supp(g)) \cap (supp(g) \cdot supp(f))$, but $t \notin P$. Also, $C_P(t) = 0$ and then $[(f \circ g) \cap (g \circ f)](t) = 0$, because $(f \circ g) \cap (g \circ f) \subseteq C_P$. Since $t \in supp(f) \cdot supp(g)$ and $t \in supp(g) \cdot supp(f)$, we have that $t \in a_1 \cdot b_1$ and $t \in b_2 \cdot a_2$ for some $a_1, a_2 \in supp(f)$ and for some $b_1, b_2 \in supp(g)$. It follows that $(f \circ g)(t) = \sup\limits_{t \in a_1 \cdot b_1} [\min\{f(a_1), g(b_1)\}] \neq 0$ and $(g \circ f)(t) = \sup\limits_{t \in b_2 \cdot a_2} [\min\{g(b_2), f(a_2)\}] \neq 0$. This implies that $[(f \circ g) \cap (g \circ f)](t) \neq 0$, which is a contradiction. Hence, $f \subseteq C_P$ or $g \subseteq C_P$. Therefore, C_P is a fuzzy strongly prime almost interior hyperideal of S.

Conversely, assume that C_P is a fuzzy strongly prime almost interior hyperideal of S. By Theorem 3.5, P is an almost interior hyperideal of S. Let A and B be any two almost interior hyperideals of S such that $(A \cdot B) \cap (B \cdot A) \subseteq P$. By Lemma 2.2, Lemma 2.3 and Theorem 3.5, we get that C_A and C_B are fuzzy almost interior hyperideals of S such that $(C_A \circ C_B) \cap (C_B \circ C_A) = C_{A \cdot B} \cap C_{B \cdot A} = C_{(A \cdot B) \cap (B \cdot A)} \subseteq C_P$. By the given assumption, $C_A \subseteq C_P$ or $C_B \subseteq C_P$. That is, $A \subseteq P$ or $B \subseteq P$. Consequently, P is a strongly prime almost interior hyperideal of S. $\square$

The following theorems can be proved similar to Theorem 3.12.

Theorem 3.13. *Let P be a nonempty subset of a semihypergroup S. Then, P is a prime almost interior hyperideal of S if and only if C_P is a fuzzy prime almost interior hyperideal of S.*

Theorem 3.14. *Let P be a nonempty subset of a semihypergroup S. Then, P is a semiprime almost interior hyperideal of S if and only if C_P is a fuzzy semiprime almost interior hyperideal of S.*

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