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Almost subsemirings and fuzzifications

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Abstract

In this paper, we introduce the concepts of almost subsemirings and fuzzy almost subsemirings of semirings. These are extended from almost subsemigroups and fuzzy almost subsemigroups of semigroups,

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respectively. Moreover, we demonstrate the basic properties of almost subsemirings and their fuzzifications. Furthermore, we investigate the relationships between almost subsemirings and their fuzzifications.

1 Introduction and Preliminaries

Almost ideals of semigroups were first introduced and examined by Grosek and Satko [4]. The concept of almost ideals was used to study almost ideals in many algebraic structures. In 1965, Zadeh [8] gave the notion of fuzzy sets. Later, fuzzy sets were applied to various fields. One of those fields in which we are interested in is the study of fuzzy almost ideals of semigroups [6, 7]. Recently, almost subsemigroups and fuzzy almost subsemigroups of semigroups were defined in [3]. In addition, those concepts were extended to explore almost ternary subsemigroups and their fuzzifications in [2]. A fuzzy subset of a set S is a membership function from S into the closed interval $[0, 1]$. Let f and g be any two fuzzy subsets of S. Fuzzy subsets $f \cap g$ and $f \cup g$ of S are defined by, for all $x \in S$,

$$(f \cap g)(x) = \min\{f(x), g(x)\} \text{ and } (f \cup g)(x) = \max\{f(x), g(x)\}.$$

We say that $f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in S$. The *support* of f is defined by $supp(f) = \{x \in S \mid f(x) \neq 0\}$. The *characteristic mapping* of a subset A of S is a fuzzy subset C_A of S defined by

$$C_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise.} \end{cases}$$

In 1934, Vandiver [5] studied the structure of semirings. An algebra $(R, +, \cdot)$ is called a *semiring* if $(R, +)$ and $(R, \cdot)$ are semigroups and $\cdot$ is distributive over $+$. Ahsan et al. [1] studied Fuzzy semirings. A nonempty subset S of a semiring R is called a *subsemiring* of R if $x + y \in S$ and $xy \in S$, for all $x, y \in S$. For any two fuzzy subsets f and g of a semiring R, fuzzy subsets $f + g$ and $f \circ g$ are defined as follows:
For any $x \in R$,

$$(f + g)(x) = \begin{cases} \sup\limits_{x=a+b} \{\min\{f(a), g(b)\}\} & \text{if } x = a + b \text{ for some } a, b \in R, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$(f \circ g)(x) = \begin{cases} \sup\limits_{x=ab}\{\min\{f(a), g(b)\}\} & \text{if } x = ab \text{ for some } a, b \in R, \\ 0 & \text{otherwise.} \end{cases}$$

A fuzzy subset of a semiring R is called a *fuzzy subsemiring* of R if $f + f \subseteq f$ and $f \circ f \subseteq f$. Our objectives are to define the notions of almost subsemirings and fuzzy almost subsemirings of semirings, to investigate their basic properties, and to demonstrate the relationships between almost subsemirings of semirings and their fuzzifications.

2 Main results

2.1 Almost subsemirings

In this section, we present the new definition of an almost subsemiring and procure its basic properties.

Definition 2.1. A nonempty set A of a semiring R is called a *almost subsemiring* of R if $(A + A) \cap A \neq \emptyset$ and $A^2 \cap A \neq \emptyset$.

Let A be any subsemiring of a semiring R. We obtain $A + A \subseteq A$ and $A^2 \subseteq A$. This implies that $(A+A) \cap A \neq \emptyset$ and $A^2 \cap A \neq \emptyset$ and therefore A is an almost subsemiring of R. Hence, we can conclude that every subsemiring of a semiring R is also an almost subsemiring of R.

Example 2.1. Consider a semiring $(\mathbb{N}, +, \cdot)$ and let $A = \{2, 4\}$ and $B = \{4, 8, 16\}$. It is clear that A and B are almost subsemirings but are not subsemirings of $\mathbb{N}$. Nevertheless, $A \cap B = \{4\}$ is not an almost subsemiring of $\mathbb{N}$. We have the following facts:

(1) In general, an almost subsemiring of a semiring R need not be a subsemiring of R.

(2) In general, the intersection of almost subsemirings of a semiring R need not be an almost subsemiring of R.

Proposition 2.2. *Let a be any element of a semigroup R. Then $\{a, a+a, a^2\}$ is an almost subsemiring of R.*

Proof. It is obvious. □

Theorem 2.3. *Let A and B be any two nonempty subsets of a semiring R. If $A \subseteq B$ and A is an almost subsemiring of R, then B is also an almost subsemiring of R.*

Proof. Assume that A is an almost subsemiring of a semiring R. It follows that $(A+A) \cap A \neq \emptyset$ and $A^2 \cap A \neq \emptyset$. Since $A \subseteq B$, we have $(A+A) \cap A \subseteq (B+B) \cap B$ and $A^2 \cap A \subseteq B^2 \cap B$. This implies that $(B+B) \cap B \neq \emptyset$ and $B^2 \cap B \neq \emptyset$. □

The following corollary follows from Theorem 2.3:

Corollary 2.4. *The union of almost subsemirings of a semiring R is also an almost subsemiring of R.*

2.2 Fuzzy almost subsemirings

In this section, we introduce the concept of fuzzy almost subsemirings and provide their basic properties. Additionally, we focus on the relationships among almost subsemirings of semirings and their fuzzifications.

Definition 2.5. A fuzzy subset f of a semiring R is called a *fuzzy almost subsemiring* of S if $(f+f) \cap f \neq 0$ and $(f \circ f) \cap f \neq 0$.

Let f be any fuzzy subsemiring of a semiring R such that $f+f \neq 0$ and $f \circ f \neq 0$. Then $f+f \subseteq f$ and $f \circ f \subseteq f$ and hence $(f+f) \cap f \neq 0$ and $(f \circ f) \cap f \neq 0$. Therefore, f is a fuzzy almost subsemiring of R.

Example 2.2. Consider the semiring $(\mathbb{N}, +, \cdot)$. Let f and g be the following fuzzy subsets of $\mathbb{N}$:

$$f(x) = \begin{cases} 0.5 & \text{if } x \in \{2,4\}, \\ 0 & \text{otherwise,} \end{cases} \text{ and } g(x) = \begin{cases} 0.4 & \text{if } x \in \{4,8,16\}, \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to see that f and g are fuzzy almost subsemirings but are not fuzzy subsemirings of $\mathbb{N}$. However, $f \cap g$ is not a fuzzy almost subsemiring of $\mathbb{N}$. We get the following facts:

(1) Any fuzzy almost subsemiring of a semiring R generally need not be a fuzzy subsemiring of R.

(2) The intersection of fuzzy almost subsemirings of a semiring R generally need not be a fuzzy almost subsemiring of R.

Theorem 2.6. *Let f and g be fuzzy subsets of a semiring R such that $f \subseteq g$. If f is a fuzzy almost subsemiring of R, then g is also a fuzzy almost subsemiring of R.*

Proof. Assume that f is a fuzzy almost subsemiring of a semiring R. Then $(f+f)\cap f \neq 0$ and $(f \circ f)\cap f \neq 0$ and hence $(f+f)\cap f \subseteq (g+g)\cap g$ and $(f \circ f)\cap f \subseteq (g \circ g)\cap g$ because $f \subseteq g$. Consequently, $(g+g)\cap g \neq 0$ and $(g \circ g)\cap g \neq 0$. □

Corollary 2.7. *The union of fuzzy almost subsemirings of a semiring R is also a fuzzy almost subsemiring of R.*

Proof. This follows from Theorem 2.6. □

We now consider relationships among almost subsemirings and their fuzzifications.

Theorem 2.8. *Let A be a nonempty subset of a semiring R. Then A is an almost subsemiring of R if and only if C_A is a fuzzy almost subsemiring of R.*

Proof. Suppose that A is an almost subsemiring of a semiring R. This implies that $(A+A)\cap A \neq \emptyset$ and $A^2 \cap A \neq \emptyset$ and hence there exists $x \in (A+A)\cap A$. So $x = y+z$ for some $y, z \in A$. We obtain $(C_A+C_A)(x) = 1$ and $C_A(x) = 1$. Thus $[(C_A+C_A)\cap C_A](x) = 1 \neq 0$ which implies that $(C_A+C_A)\cap C_A \neq 0$. Similarly, we can show that $(C_A \circ C_A)\cap C_A \neq 0$. Therefore, C_A is a fuzzy almost subsemiring of R.

Conversely, assume that C_A is a fuzzy almost subsemiring of a semiring R. We have $(C_A+C_A)\cap C_A \neq 0$ and $(C_A \circ C_A)\cap C_A \neq 0$. Then there exists $x \in R$ such that $[(C_A+C_A)\cap C_A](x) \neq 0$. This implies that $(C_A+C_A)(x) \neq 0$ and $C_A(x) \neq 0$. It follows that $(C_A+C_A)(x) = 1$ and $C_A(x) = 1$. Hence, $x \in A+A$ and $x \in A$. Eventually, $(A+A)\cap A \neq \emptyset$. Similarly, $A^2 \cap A \neq \emptyset$. We can conclude that A is an almost subsemiring of R. □

Theorem 2.9. *Let f be a fuzzy subset of a semiring R. Then f is a fuzzy almost subsemiring of R if and only if $supp(f)$ is an almost subsemiring of R.*

Proof. Assume that f is a fuzzy almost subsemiring of R. Thus $(f+f)\cap f \neq 0$ and $(f \circ f)\cap f \neq 0$. So there exists $x \in R$ such that $[(f+f)\cap f](x) \neq 0$. This implies that $f(x) \neq 0$ and $(f+f)(x) \neq 0$. We obtain $x = y+z$, for some $y, z \in R$ such that $f(y) \neq 0$ and $f(z) \neq 0$. This implies that

$y, z \in supp(f)$. Thus $(C_{supp(f)} + C_{supp(f)})(x) \neq 0$ and $C_{supp(f)}(x) \neq 0$. Hence $[(C_{supp(f)} + C_{supp(f)}) \cap C_{supp(f)}](x) \neq 0$. In the same way, we have $[(C_{supp(f)} \circ C_{supp(f)}) \cap C_{supp(f)}](x) \neq 0$. Therefore, $C_{supp(f)}$ is a fuzzy almost subsemiring of R and so $supp(f)$ is an almost subsemiring of R by Theorem 2.8.

Conversely, suppose that $supp(f)$ is an almost subsemiring of R. Then $C_{supp(f)}$ is a fuzzy almost subsemiring of R by Theorem 2.8. Hence $[(C_{supp(f)} + C_{supp(f)}) \cap C_{supp(f)}] \neq 0$ and $[(C_{supp(f)} \circ C_{supp(f)}) \cap C_{supp(f)}] \neq 0$. Then there exists $x \in S$ such that $[(C_{supp(f)} + C_{supp(f)}) \cap C_{supp(f)}](x) \neq 0$ and hence $(C_{supp(f)} + C_{supp(f)})(x) \neq 0$ and $C_{supp(f)}(x) \neq 0$. Therefore, there exist $y, z \in supp(f)$ such that $x = y + z$. It follows that $f(y) \neq 0$ and $f(z) \neq 0$. Hence $(f + f)(x) \neq 0$, and so $[(f + f) \cap f](x) \neq 0$. Thus $(f + f) \cap f \neq 0$. Similarly, we can show that $[(f \circ f) \cap f](x) \neq 0$. Consequently, f is a fuzzy almost subsemiring of R. □

3 Conclusions

In this paper, we have introduced the concepts of almost subsemirings and fuzzy almost subsemirings of semirings. The union of almost subsemirings (fuzzy almost subsemirings) is also an almost subsemiring [a fuzzy almost subsemiring]. However, the intersection of almost subsemirings [fuzzy almost subsemirings] generally need not be an almost subsemiring [a fuzzy almost subsemiring]. From our study, we have found that an almost subsemiring and its characteristic mapping are equivalent. Moreover, a fuzzy almost subsemiring and its support are also equivalent.

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