International Journal of Mathematics and
Computer Science, **18**(2023), no. 1, 61–65

Almost contra-(Λ, sp)-continuity and $s(\Lambda, sp)$-closed sets

Jeeranunt Khampakdee, Chawalit Boonpok

Mathematics and Applied Mathematics Research Unit
Department of Mathematics
Faculty of Science
Mahasarakham University
Maha Sarakham, 44150, Thailand

email: jeeranunt.k@msu.ac.th, chawalit.b@msu.ac.th

(Received September 1, 2022, Accepted October 2, 2022)

Abstract

The aim of this paper is to investigate some characterizations of upper and lower almost contra-(Λ, sp)-continuous multifunctions by utilizing the concept of $s(\Lambda, sp)$-closed sets.

1 Introduction

The concept of contra-continuity was introduced by Dontchev [6]. In 1999, Dontchev and Noiri [5] considered a slightly weaker form of contra-continuity called contra-semicontinuity and investigated the class of strongly S-closed spaces. In 2002, Jafari and Noiri [10] introduced and investigated a new generalization of contra-continuity called contra-precontinuity. In 2004, Ekici [9] introduced the concept of almost contra-precontinuity as a new generalization of regular set-connectedness, contra-precontinuity, almost s-continuity and perfectly continuity. Moreover, basic properties and preservation theorems of almost contra-precontinuity and the relationships between almost contra-precontinuity and P-regular graphs were obtained in [9]. The first initiation of the concept of contra-continuous multifunctions has been done by

Key words and phrases: lower almost contra-(Λ, sp)-continuous multifunction, upper almost contra-(Λ, sp)-continuous multifunction.
AMS (MOS) Subject Classifications: 54C08, 54C60.
ISSN 1814-0432, 2023, http://ijmcs.future-in-tech.net

Ekici et al. [8]. In 2009, Ekici et al. [7] introduced and studied a new generalization of contra-continuous multifunctions called almost contra-continuous multifunctions. In this paper, we investigate several characterizations of upper and lower almost contra-(Λ, sp)-continuous multifunctions.

2 Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *β-open* [1] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$. The complement of a β-open set is called *β-closed*. The family of all β-open sets of a topological space (X, τ) is denoted by $\beta(X, \tau)$. A subset $\Lambda_{sp}(A)$ [11] is defined as follows:
$\Lambda_{sp}(A) = \cap\{U \mid A \subseteq U, U \in \beta(X, \tau)\}$. A subset A of a topological space (X, τ) is called a *Λ_{sp}-set* [11] if $A = \Lambda_{sp}(A)$. A subset A of a topological space (X, τ) is called *(Λ, sp)-closed* [4] if $A = T \cap C$, where T is a Λ_{sp}-set and C is a β-closed set. The complement of a (Λ, sp)-closed set is called *(Λ, sp)-open*. The family of all (Λ, sp)-open sets in a topological space (X, τ) is denoted by $\Lambda_{sp}O(X, \tau)$. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, sp)-cluster point* [4] of A if $A \cap U \neq \emptyset$ for every (Λ, sp)-open set U of X containing x. The set of all (Λ, sp)-cluster points of A is called the *(Λ, sp)-closure* [4] of A and is denoted by $A^{(\Lambda, sp)}$. The union of all (Λ, sp)-open sets contained in A is called the *(Λ, sp)-interior* [4] of A and is denoted by $A_{(\Lambda, sp)}$. A subset A of a topological space (X, τ) is said to be *$s(\Lambda, sp)$-open* (resp. *$r(\Lambda, sp)$-open*) if $A \subseteq [A_{(\Lambda, sp)}]^{(\Lambda, sp)}$ (resp. $A = [A^{(\Lambda, sp)}]_{(\Lambda, sp)}$) [4]. The complement of a $s(\Lambda, sp)$-open (resp. $r(\Lambda, sp)$-open) set is said to be *$s(\Lambda, sp)$-closed* (resp. *$r(\Lambda, sp)$-closed*). The family of all $s(\Lambda, sp)$-open (resp. $r(\Lambda, sp)$-open) sets in a topological space (X, τ) is denoted by $s\Lambda_{sp}O(X, \tau)$ (resp. $r\Lambda_{sp}O(X, \tau)$). Let A be a subset of a topological space (X, τ). The intersection of all $s(\Lambda, sp)$-closed sets containing A is called the *$s(\Lambda, sp)$-closure* [12] of A and is denoted by $A^{s(\Lambda, sp)}$.

Throughout this paper, the spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces and $F : X \to Y$ (resp. $f : X \to Y$) presents a multivalued (resp. single valued) function. For a multifunction $F : X \to Y$, following [2] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively; that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3 Almost contra-(Λ, sp)-continuity and $s(\Lambda, sp)$-closed sets

In this section, we investigate some characterizations of upper and lower almost contra-(Λ, sp)-continuous multifunctions.

Definition 3.1. [3] *A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be:*

(i) lower almost contra-(Λ, sp)-continuous at $x \in X$ if, for each $r(\Lambda, sp)$-closed set K of Y with $x \in F^{-}(K)$, there exists a (Λ, sp)-open set U of X containing x such that $U \subseteq F^{-}(K)$;

(ii) upper almost contra-(Λ, sp)-continuous at $x \in X$ if, for each $r(\Lambda, sp)$-closed set K of Y with $x \in F^{+}(K)$, there exists a (Λ, sp)-open set U of X containing x such that $U \subseteq F^{+}(K)$;

(iii) lower (upper) almost contra-(Λ, sp)-continuous if F has this property at each point of X.

Theorem 3.2. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is upper almost contra-(Λ, sp)-continuous;

(2) $[F^{-}(K_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^{-}(K)$ for every $s(\Lambda, sp)$-closed set K of Y;

(3) $[F^{-}([B^{s(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^{-}(B^{s(\Lambda,sp)})$ for every subset B of Y;

(4) $F^{+}(B_{s(\Lambda,sp)}) \subseteq [F^{+}([B_{s(\Lambda,sp)}]^{(\Lambda,sp)})]_{(\Lambda,sp)}$ for every subset B of Y.

Proof. (1) $\Rightarrow$ (2): Let K be any $s(\Lambda, sp)$-closed set of Y. Then, $Y - K$ is $s(\Lambda, sp)$-open in Y. By Theorem 1 of [3], we have

$$\begin{aligned} X - F^{-}(K) = F^{+}(Y - K) &\subseteq [F^{+}([Y - K]^{(\Lambda,sp)})]_{(\Lambda,sp)} \\ &= [F^{+}(Y - K_{(\Lambda,sp)})]_{(\Lambda,sp)} \\ &= [X - F^{-}(K_{(\Lambda,sp)})]_{(\Lambda,sp)} \\ &= X - [F^{-}(K_{(\Lambda,sp)})]^{(\Lambda,sp)} \end{aligned}$$

and hence $[F^{-}(K_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^{-}(K)$.

(2) $\Rightarrow$ (3): Let B be any subset of Y. Then, $B^{s(\Lambda,sp)}$ is $s(\Lambda, sp)$-closed in Y, by (2), $[F^{-}([B^{s(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^{-}(B^{s(\Lambda,sp)})$.

(3) ⇒ (4): Let B be any subset of Y. By (3), we have

$$\begin{aligned} X - F^+(B_{s(\Lambda,sp)}) &= F^-([Y - B]^{s(\Lambda,sp)}) \\ &\supseteq [F^-([[Y - B]^{s(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \\ &= [F^-([Y - B_{s(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \\ &= [F^-(Y - [B_{s(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \\ &= [X - F^+([B_{s(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \\ &= X - [F^+([B_{s(\Lambda,sp)}]^{(\Lambda,sp)})]_{(\Lambda,sp)} \end{aligned}$$

and hence $F^+(B_{s(\Lambda,sp)}) \subseteq [F^+([B_{s(\Lambda,sp)}]^{(\Lambda,sp)})]_{(\Lambda,sp)}$.

(4) ⇒ (1): Let V be any $s(\Lambda, sp)$-open set of Y. By (4), we have $F^+(V) = F^+(V_{s(\Lambda,sp)}) \subseteq [F^+([V_{s(\Lambda,sp)}]^{(\Lambda,sp)})]_{(\Lambda,sp)} = [F^+(V^{(\Lambda,sp)})]_{(\Lambda,sp)}$ and by Theorem 1 of [3], F is upper almost contra-(Λ, sp)-continuous. □

Theorem 3.3. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is lower almost contra-(Λ, sp)-continuous;

(2) $[F^+(K_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^+(K)$ for every $s(\Lambda, sp)$-closed set K of Y;

(3) $[F^+([B^{s(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^+(B^{s(\Lambda,sp)})$ for every subset B of Y;

(4) $F^-(B_{s(\Lambda,sp)}) \subseteq [F^-([B_{s(\Lambda,sp)}]^{(\Lambda,sp)})]_{(\Lambda,sp)}$ for every subset B of Y.

Proof. The proof is similar to that of Theorem 3.2. □

Definition 3.4. [3] *A function $f : (X, \tau) \to (Y, \sigma)$ is called almost contra-(Λ, sp)-continuous if, for each $x \in X$ and each $r(\Lambda, sp)$-closed set K of Y containing $f(x)$, there exists a (Λ, sp)-open set U of X containing x such that $f(U) \subseteq K$.*

Corollary 3.5. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is almost contra-(Λ, sp)-continuous;

(2) $[f^{-1}(K_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(K)$ for every $s(\Lambda, sp)$-closed set K of Y;

(3) $[f^{-1}([B^{s(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(B^{s(\Lambda,sp)})$ for every subset B of Y;

(4) $f^{-1}(B_{s(\Lambda,sp)}) \subseteq [f^{-1}([B_{s(\Lambda,sp)}]^{(\Lambda,sp)})]_{(\Lambda,sp)}$ for every subset B of Y.

Acknowledgment. This research project was financially supported by Mahasarakham University.

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb, R. A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Assiut Univ., **12,** (1983), 77–90.

[2] C. Berge, Espaces topologiques fonctions multivoques, Dunod, Paris, 1959.

[3] C. Boonpok, J. Khampakdee, Upper and lower almost contra-(Λ, sp)-continuity (accepted).

[4] C. Boonpok, (Λ, sp)-closed sets and related topics in topological spaces, WSEAS Tran. Math., **19,** (2020), 312–322.

[5] J. Dontchev, T. Noiri, Contra-semicontinuous functions, Math. Pannonica, **10,** no. 2, (1999), 159–168.

[6] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, Int. J. Math. Math. Sci., **19,** no. 2, (1996), 303–310.

[7] E. Ekici, S. Jafari, V. Popa, On almost contra-continuous multifunctions, Lobachevskii J. Math., **30,** no. 2, (2009), 124–131.

[8] E. Ekici, S. Jafari, T. Noiri, On upper and lower contra-continuous multifunctions, Analele Sti. Univ. Al I. Cuza Mat., **54,** no. 1, (2008), 75–85.

[9] E. Ekici, Almost contra-precontinuous functions, Bull. Malays. Math. Sci. Soc., **27,** (2004), 53–65.

[10] S. Jafari, T. Noiri, On contra-precontinuous functions, Bull. Malays. Math. Sci. Soc., **25,** (2002), 115–128.

[11] T. Noiri, E. Hatir, Λ_{sp}-sets and some weak separation axioms, Acta Math. Hungar., **103,** no. 3, (2004), 225–232.

[12] C. Viriyapong, C. Boonpok, Weak quasi (Λ, sp)-continuity for multifunctions, Int. J. Math. Comput. Sci., **17,** no. 3, (2022), 1201–1209.