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Properties of locally (Λ, p)-closed sets

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Abstract

In this paper, we deal with the notion of locally (Λ, p)-closed sets. Moreover, we investigate some properties of those sets. Furthermore, we characterize (Λ, p)-submaximal spaces by utilizing locally (Λ, p)-closed sets.

1 Introduction

The concept of locally closed sets was introduced by Bourbaki [3]. Ganster and Reilly [5] introduced and studied three different notions of generalized continuity; namely, LC-irresoluteness, LC-continuity and sub-LC-continuity. All three notions were defined by using the concept of a locally closed set. Arenas et al. [1] introduced and investigated the notion of λ-closed sets as a generalization of locally closed sets. Modak and Noiri [8] introduced and studied the concept of λ-locally closed sets which is a generalization of locally closed sets. Mashhour et al. [7] introduced and investigated the concept of preopen sets and preclosed sets. Ganster et al. [4] introduced the notions of a pre-Λ-set and a pre-V-set in topological spaces. In particular, the fundamental properties of pre-Λ-sets and pre-V-sets were investigated in

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[4]. Boonpok and Viriyapong [2] introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets which were defined by utilizing the notions of Λ_p-sets and preclosed sets. In this paper, we introduce the concept of locally (Λ, p)-closed sets. Moreover, we discuss some of their properties. Furthermore, we characterize (Λ, p)-submaximal spaces by utilizing locally (Λ, p)-closed sets.

2 Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [7] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. Let A be a subset of a topological space (X, τ). A subset $\Lambda_p(A)$ [4] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p*-set* [4] if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)*-closed* [2] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)*-open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)*-cluster point* [2] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)*-closure* [2] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets contained in A is called the (Λ, p)*-interior* [2] of A and is denoted by $A_{(\Lambda,p)}$.

Lemma 2.1. [2] *For subsets A, B of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq A^{(\Lambda,p)}$ and $[A^{(\Lambda,p)}]^{(\Lambda,p)} = A^{(\Lambda,p)}$.

(2) If $A \subseteq B$, then $A^{(\Lambda,p)} \subseteq B^{(\Lambda,p)}$.

(3) $A^{(\Lambda,p)} = \cap\{F | A \subseteq F$ and F is (Λ, p)-closed$\}$.

(4) $A^{(\Lambda,p)}$ is (Λ, p)-closed.

(5) A is (Λ, p)-closed if and only if $A = A^{(\Lambda,p)}$.

Lemma 2.2. [2] *Let A and B be subsets of a topological space (X, τ). For the (Λ, p)-interior, the following properties hold:*

(1) $A_{(\Lambda,p)} \subseteq A$ *and* $[A_{(\Lambda,p)}]_{(\Lambda,p)} = A_{(\Lambda,p)}$.

(2) If $A \subseteq B$*, then* $A_{(\Lambda,p)} \subseteq B_{(\Lambda,p)}$.

(3) $A_{(\Lambda,p)}$ *is* (Λ, p)*-open;*

(4) A *is* (Λ, p)*-open if and only if* $A_{(\Lambda,p)} = A$.

(5) $[X - A]^{(\Lambda,p)} = X - A_{(\Lambda,p)}$.

3 Properties of locally (Λ, p)-closed sets

In this section, we introduce the notion of locally (Λ, p)-closed sets. Moreover, we discuss some properties of locally (Λ, p)-closed sets. Furthermore, we characterize (Λ, p)-submaximal spaces by utilizing locally (Λ, p)-closed sets.

Definition 3.1. *A subset A of a topological space (X, τ) is said to be locally (Λ, p)-closed if $A = U \cap F$, where $U \in \Lambda_pO(X, \tau)$ and F is (Λ, p)-closed.*

Theorem 3.2. *For a subset A of a topological space (X, τ), the following properties are equivalent:*

(1) A *is locally* (Λ, p)*-closed;*

(2) $A = U \cap A^{(\Lambda,p)}$ *for some* $U \in \Lambda_pO(X, \tau)$*;*

(3) $A^{(\Lambda,p)} - A$ *is* (Λ, p)*-closed;*

(4) $A \cup (X - A^{(\Lambda,p)})$ *is* (Λ, p)*-open;*

(5) $A \subseteq [A \cup (X - A^{(\Lambda,p)})]_{(\Lambda,p)}$.

Proof. (1) $\Rightarrow$ (2): Let $A = U \cap F$, where $U \in \Lambda_pO(X, \tau)$ and F is (Λ, p)-closed. Since $A \subseteq F$, we have $A^{(\Lambda,p)} \subseteq F^{(\Lambda,p)} = F$. Since $A \subseteq U$, $A \subseteq U \cap A^{(\Lambda,p)} \subseteq U \cap F = A$. Thus, $A = U \cap A^{(\Lambda,p)}$.

(2) $\Rightarrow$ (3): Suppose that $A = U \cap A^{(\Lambda,p)}$ for some $U \in \Lambda_pO(X, \tau)$. Then, we have $A^{(\Lambda,p)} - A = (X - [U \cap A^{(\Lambda,p)}]) \cap A^{(\Lambda,p)} = (X - U) \cap A^{(\Lambda,p)}$. Thus, $A^{(\Lambda,p)} - A$ is (Λ, p)-closed.

(3) $\Rightarrow$ (4): Since $X - (A^{(\Lambda,p)} - A) = (X - A^{(\Lambda,p)}) \cup A$ and by (3), $A \cup (X - A^{(\Lambda,p)})$ is (Λ, p)-open.

(4) $\Rightarrow$ (5): By (4), $A \subseteq A \cup (X - A^{(\Lambda,p)}) = [A \cup (X - A^{(\Lambda,p)})]_{(\Lambda,p)}$.

(5) $\Rightarrow$ (1): We put $U = [A \cup (X - A^{(\Lambda,p)})]_{(\Lambda,p)}$. Then, U is (Λ, p)-open and $A = A \cap U \subseteq U \cap A^{(\Lambda,p)} \subseteq [A \cup (X - A^{(\Lambda,p)})] \cap A^{(\Lambda,p)} = A \cap A^{(\Lambda,p)} = A$.

Thus, $A = U \cap A^{(\Lambda,p)}$, where $U \in \Lambda_pO(X,\tau)$ and $A^{(\Lambda,p)}$ is (Λ,p)-closed. This shows that A is locally (Λ,p)-closed. □

Definition 3.3. *A subset A of a topological space (X,τ) is said to be:*

(i) (Λ,p)*-dense if* $A^{(\Lambda,p)} = X$*;*

(ii) (Λ,p)*-codense if its complement is* (Λ,p)*-dense.*

Definition 3.4. *A topological space (X,τ) is said to be (Λ,p)-submaximal if for each (Λ,p)-dense subset of X is (Λ,p)-open.*

Theorem 3.5. *For a topological space (X,τ), the following properties are equivalent:*

(1) (X,τ) *is* (Λ,p)*-submaximal;*

(2) every subset of X is a locally (Λ,p)-closed set;

(3) every subset of X is the union of a (Λ,p)-open set and a (Λ,p)-closed set;

(4) every (Λ,p)-dense set of X is the intersection of a (Λ,p)-closed set and a (Λ,p)-open set;

(5) every (Λ,p)-codense set of X is the union of a (Λ,p)-open set and a (Λ,p)-closed set.

Proof. (1) $\Rightarrow$ (2): Suppose that (X,τ) is (Λ,p)-submaximal. Let A be any subset of X. Then, we have $[X-(A^{(\Lambda,p)}-A)]^{(\Lambda,p)} = [A\cup(X-A^{(\Lambda,p)})]^{(\Lambda,p)} = X$. Thus, $X-(A^{(\Lambda,p)}-A)$ is (Λ,p)-dense and hence $X-(A^{(\Lambda,p)}-A)$ is (Λ,p)-open. Thus, $X-(A^{(\Lambda,p)}-A) = A\cup(X-A^{(\Lambda,p)})$ is (Λ,p)-open. This shows that $A = [A\cup(X-A^{(\Lambda,p)})]\cap A^{(\Lambda,p)}$ is locally (Λ,p)-closed.

(2) $\Leftrightarrow$ (3): Suppose that every subset of X is a locally (Λ,p)-closed set. Let A be any subset of X. By (2), we have $X-A = U\cap F$, where U is (Λ,p)-open and F is (Λ,p)-closed. Thus, $A = (X-U)\cup(X-F)$, where $X-U$ is (Λ,p)-closed and $X-F$ is (Λ,p)-open. The converse is similar.

(2) $\Rightarrow$ (4) and (4) $\Leftrightarrow$ (5) are obvious.

(4) $\Rightarrow$ (1): Let A be a (Λ,p)-dense set. By (4), there exist a (Λ,p)-open set U and a (Λ,p)-closed set F such that $A = U\cap F$. Since $A\subseteq F$ and A is (Λ,p)-dense, $X\subseteq F$. Thus, $F = X$ and hence $A = U$ is (Λ,p)-open. This shows that (X,τ) is (Λ,p)-submaximal. □

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