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Almost (Λ, sp)-continuity and $p(\Lambda, sp)$-open sets

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Abstract

The purpose of the present paper is to investigate some characterizations of upper and lower almost (Λ, sp)-continuous multifunctions.

1 Introduction

Topology as a field of mathematics is concerned with all questions directly or indirectly related to continuity. Continuity is an important concept for the study and investigation in topological spaces. This concept has been extended to the setting of multifunctions and has been generalized by weaker forms of open sets such as semi-open sets, preopen sets, α-open sets and β-open sets. In 1968, M. Singal and A. Singal [9] introduced and studied the notion of almost continuous functions. In 1982, Popa [8] introduced the concepts of upper and lower almost continuous multifunctions. Several characterizations of upper and lower almost continuous multifunctions were presented in [7] and other articles. In 2010, Noiri and Popa [5] introduced and studied the notions of upper and lower almost m-continuous multifunctions as multifunctions from a set satisfying some minimal conditions into a topological space. In [4], the present author introduced and investigated the

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concepts of upper and lower (Λ, sp)-continuous multifunctions. In this paper, we investigate some characterizations of upper and lower almost (Λ, sp)-continuous multifunctions.

2 Preliminaries

Throughout this paper, unless explicitly stated, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed. Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *β-open* [1] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$. The complement of a β-open set is called *β-closed*. The family of all β-open sets of a topological space (X, τ) is denoted by $\beta(X, \tau)$. Let A be a subset of a topological space (X, τ). A subset $\Lambda_{sp}(A)$ [6] is defined as follows:
$\Lambda_{sp}(A) = \cap\{U \mid A \subseteq U, U \in \beta(X, \tau)\}$. A subset A of a topological space (X, τ) is called a *Λ_{sp}-set* [6] if $A = \Lambda_{sp}(A)$. A subset A of a topological space (X, τ) is called *(Λ, sp)-closed* [4] if $A = T \cap C$, where T is a Λ_{sp}-set and C is a β-closed set. The complement of a (Λ, sp)-closed set is called *(Λ, sp)-open*. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, sp)-cluster point* [4] of A if $A \cap U \neq \emptyset$ for every (Λ, sp)-open set U of X containing x. The set of all (Λ, sp)-cluster points of A is called the *(Λ, sp)-closure* [4] of A and is denoted by $A^{(\Lambda,sp)}$. The union of all (Λ, sp)-open sets contained in A is called the *(Λ, sp)-interior* [4] of A and is denoted by $A_{(\Lambda,sp)}$. A subset A of a topological space (X, τ) is said to be *$p(\Lambda, sp)$-open* (resp. *$r(\Lambda, sp)$-open*) if $A \subseteq [A^{(\Lambda,sp)}]_{(\Lambda,sp)}$ (resp. $A = [A^{(\Lambda,sp)}]_{(\Lambda,sp)}$) [4]. The complement of a $p(\Lambda, sp)$-open (resp. $r(\Lambda, sp)$-open) set is said to be *$p(\Lambda, sp)$-closed* (resp. *$r(\Lambda, sp)$-closed*).

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, following [2] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3 Characterizations

In this section, we investigate some characterizations of upper and lower almost (Λ, sp)-continuous multifunctions.

Definition 3.1. [3] *A multifunction* $F : (X, \tau) \to (Y, \sigma)$ *is said to be:*

(i) upper almost (Λ, sp)*-continuous at a point* $x \in X$ *if, for each* (Λ, sp)*-open set* V *of* Y *containing* $F(x)$*, there exists a* (Λ, sp)*-open set* U *of* X *containing* x *such that* $F(U) \subseteq [V^{(\Lambda,sp)}]_{(\Lambda,sp)}$*;*

(ii) lower almost (Λ, sp)*-continuous at a point* $x \in X$ *if, for each* (Λ, sp)*-open set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$*, there exists a* (Λ, sp)*-open set* U *of* X *containing* x *such that* $F(z) \cap [V^{(\Lambda,sp)}]_{(\Lambda,sp)} \neq \emptyset$ *for each* $z \in U$*.*

(iii) upper (lower) almost (Λ, sp)*-continuous if* F *has this property at each point of* X*.*

Theorem 3.2. *For a multifunction* $F : (X, \tau) \to (Y, \sigma)$*, the following properties are equivalent:*

(1) F *is upper almost* (Λ, sp)*-continuous;*

(2) $[F^-([[V^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V^{(\Lambda,sp)})$ *for every* $p(\Lambda, sp)$*-open set* V *of* Y*;*

(3) $[F^-([V_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V^{(\Lambda,sp)})$ *for every* $p(\Lambda, sp)$*-open set* V *of* Y*;*

(4) $F^+(V) \subseteq [F^+([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}$ *for every* $p(\Lambda, sp)$*-open set* V *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let V be any $p(\Lambda, sp)$-open set of Y. Then, $V^{(\Lambda,sp)}$ is (Λ, sp)-closed in Y and by Theorem 3 of [3], $[F^-([[V^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V^{(\Lambda,sp)})$.

(2) $\Rightarrow$ (3): Let V be any $p(\Lambda, sp)$-open set of Y. By (2), we have $[F^-([V_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq [F^-([[V^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V^{(\Lambda,sp)})$.

(3) $\Rightarrow$ (4): Let V be any $p(\Lambda, sp)$-open set of Y. By (3),

$$\begin{aligned} X - [F^+([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)} &= [X - F^+([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \\ &= [F^-([[Y - V^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \\ &\subseteq F^-([Y - V^{(\Lambda,sp)}]^{(\Lambda,sp)}) \\ &\subseteq X - F^+(V) \end{aligned}$$

and hence $F^+(V) \subseteq [F^+([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}$.

(4) $\Rightarrow$ (1): Let V be any $r(\Lambda, sp)$-open set of Y. Then, V is $p(\Lambda, sp)$-open, by (4), $F^+(V) \subseteq [F^+([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)} = [F^+(V)]_{(\Lambda,sp)}$. Thus, $F^+(V)$ is (Λ, sp)-open in X. From Theorem 3 of [3], it follows that F is upper almost (Λ, sp)-continuous. □

Theorem 3.3. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is lower almost (Λ, sp)-continuous;

(2) $[F^+([[V^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^+(V^{(\Lambda,sp)})$ for every $p(\Lambda, sp)$-open set V of Y;

(3) $[F^+([V_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^+(V^{(\Lambda,sp)})$ for every $p(\Lambda, sp)$-open set V of Y;

(4) $F^-(V) \subseteq [F^-([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}$ for every $p(\Lambda, sp)$-open set V of Y.

Proof. The proof is similar to that of Theorem 3.2. □

Definition 3.4. [3] *A function $f : (X, \tau) \to (Y, \sigma)$ is called almost (Λ, sp)-continuous at a point $x \in X$ if, for each (Λ, sp)-open set V of Y containing $f(x)$, there exists a (Λ, sp)-open set U of X containing x such that $f(U) \subseteq [V^{(\Lambda,sp)}]_{(\Lambda,sp)}$. A function $f : (X, \tau) \to (Y, \sigma)$ is called almost (Λ, sp)-continuous if f has this property at each point of X.*

Corollary 3.5. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is almost (Λ, sp)-continuous;

(2) $[f^{-1}([[V^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ for every $p(\Lambda, sp)$-open set V of Y;

(3) $[f^{-1}([V_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ for every $p(\Lambda, sp)$-open set V of Y;

(4) $f^{-1}(V) \subseteq [f^{-1}([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]_{(\Lambda,sp)}$ for every $p(\Lambda, sp)$-open set V of Y.

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