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On generalized (Λ, p)-closed sets

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Abstract

In this paper, we deal with with the concept of generalized (Λ, p)-closed sets. Moreover, we investigate some properties of (Λ, p)-closed sets.

1 Introduction

In 1970, Levine [4] introduced the notion of generalized closed sets in topological spaces. In 1980, Dunham and Levine [2] investigated further properties of generalized closed sets where the concept of generalized closed sets has been modified and investigated by using weaker forms of open sets such as semi-open sets, α-open sets, preopen sets, β-open sets and δ-open sets. In 1982, Mashhour et al. [5] introduced and investigated the concept of preopen sets and preclosed sets. In 2002, Ganster et al. [3] introduced the notions of pre-Λ-sets and pre-V-sets in topological spaces and investigated the fundamental properties of pre-Λ-sets and pre-V-sets. In [1], the authors introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets which were defined by utilizing the notions of Λ_p-sets and preclosed sets. In this paper, we introduce the concept of generalized (Λ, p)-closed sets. Moreover, we discuss some properties of generalized (Λ, p)-closed sets.

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2 Preliminaries

Throughout this paper, unless explicitly stated, a space (X, τ) (or simply X) always mean a topological space on which no separation axioms are assumed. Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [5] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [3] is defined as follows:
$\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [3] if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [1] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_pO(X, \tau)$ (resp. $\Lambda_pC(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [1] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets contained in A is called the (Λ, p)-*interior* [1] of A and is denoted by $A_{(\Lambda,p)}$.

Lemma 2.1. [1] *For subsets A, B of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq A^{(\Lambda,p)}$ and $[A^{(\Lambda,p)}]^{(\Lambda,p)} = A^{(\Lambda,p)}$.

(2) If $A \subseteq B$, then $A^{(\Lambda,p)} \subseteq B^{(\Lambda,p)}$.

(3) $A^{(\Lambda,p)} = \cap\{F | A \subseteq F$ and F is (Λ, p)-closed$\}$.

(4) $A^{(\Lambda,p)}$ is (Λ, p)-closed.

(5) A is (Λ, p)-closed if and only if $A = A^{(\Lambda,p)}$.

3 Generalized (Λ, p)-closed sets

We begin this section by introducing the notion of generalized (Λ, p)-closed sets.

Definition 3.1. *A subset A of a topological space (X, τ) is said to be generalized (Λ, p)-closed (briefly g-(Λ, p)-closed) if $A^{(\Lambda,p)} \subseteq U$ whenever $A \subseteq U$ and U is (Λ, p)-open.*

Theorem 3.2. *A subset A of a topological space (X, τ) is g-(Λ, p)-closed if and only if $A^{(\Lambda,p)} - A$ contains no nonempty (Λ, p)-closed set.*

Proof. Let F be a (Λ, p)-closed subset of $A^{(\Lambda,p)} - A$. Now, $A \subseteq X - F$ and since A is g-(Λ, p)-closed, we have $A^{(\Lambda,p)} \subseteq X - F$ and $F \subseteq X - A^{(\Lambda,p)}$. Thus, $F \subseteq A^{(\Lambda,p)} \cap (X - A^{(\Lambda,p)}) = \emptyset$ and F is empty.

Conversely, suppose that $A \subseteq U$ and U is (Λ, p)-open. If $A^{(\Lambda,p)} \nsubseteq U$, then $A^{(\Lambda,p)} \cap (X - U)$ is a nonempty (Λ, p)-closed subset of $A^{(\Lambda,p)} - A$. □

Proposition 3.3. *Let A, B be subsets of a topological space (X, τ). If A is g-(Λ, p)-closed and $A \subseteq B \subseteq A^{(\Lambda,p)}$, then B is g-(Λ, p)-closed.*

Proof. Let $B \subseteq U$ and U be (Λ, p)-open. Since A is g-(Λ, p)-closed and $A \subseteq U$, $A^{(\Lambda,p)} \subseteq U$. Since $A \subseteq B \subseteq A^{(\Lambda,p)}$, $B^{(\Lambda,p)} = A^{(\Lambda,p)}$ and hence $B^{(\Lambda,p)} \subseteq U$. Thus, B is g-(Λ, p)-closed. □

Theorem 3.4. *A subset A of a topological space (X, τ) is g-(Λ, p)-open if and only if $F \subseteq A_{(\Lambda,p)}$ whenever $F \subseteq A$ and F is (Λ, p)-closed.*

Proof. Suppose that A is g-(Λ, p)-open. Let $F \subseteq A$ and F be (Λ, p)-closed. Then, $X - A \subseteq X - F$. Since $X - F$ is (Λ, p)-open and $X - A$ is g-(Λ, p)-closed, $X - A_{(\Lambda,p)} = [X - A]^{(\Lambda,p)} \subseteq X - F$ and hence $F \subseteq A_{(\Lambda,p)}$.

Conversely, let $X - A \subseteq U$ and $U \in \Lambda_p O(X, \tau)$. Then, $X - U \subseteq A$ and $X - U$ is (Λ, p)-closed. By the hypothesis, we have $X - U \subseteq A_{(\Lambda,p)}$ and hence $[X - A]^{(\Lambda,p)} = X - A_{(\Lambda,p)} \subseteq U$. Thus, $X - A$ is g-(Λ, p)-closed and A is g-(Λ, p)-open. □

Corollary 3.5. *Let A and B be subsets of a topological space (X, τ). If A is g-(Λ, p)-open and $A_{(\Lambda,p)} \subseteq B \subseteq A$, then B is g-(Λ, p)-open.*

Proof. This follows from Proposition 3.3. □

Lemma 3.6. *Let A be a subset of a topological space (X, τ) and $G \in \Lambda_p O(X, \tau)$. If $A \cap G = \emptyset$, then $A^{(\Lambda,p)} \cap G = \emptyset$.*

Theorem 3.7. *For a subset A of a topological space (X, τ), the following properties are equivalent:*

(1) A is g-(Λ, p)-closed;

(2) $A^{(\Lambda,p)} - A$ *contains no nonempty* (Λ, p)*-closed set;*

(3) $A^{(\Lambda,p)} - A$ *is g-*(Λ, p)*-open.*

Proof. (1) ⇒ (2): This follows from Theorem 3.2.

(2) ⇒ (3): Let $F \subseteq A^{(\Lambda,p)} - A$ and F be (Λ, p)-closed. By (2), we have $F = \emptyset$ and $F \subseteq [A^{(\Lambda,p)} - A]_{(\Lambda,p)}$. It follows from Theorem 3.4 that $A^{(\Lambda,p)} - A$ is g-(Λ, p)-open.

(3) ⇒ (1): Let $A \subseteq U$ and $U \in \Lambda_p O(X, \tau)$. Thus, $A^{(\Lambda,p)} - U \subseteq A^{(\Lambda,p)} - A$. Since $A^{(\Lambda,p)} - A$ is g-(Λ, p)-open and $A^{(\Lambda,p)} - U$ is (Λ, p)-closed, by Theorem 3.4, $A^{(\Lambda,p)} - U \subseteq [A^{(\Lambda,p)} - A]_{(\Lambda,p)} = \emptyset$. Thus, $A^{(\Lambda,p)} \subseteq U$ and hence A is g-(Λ, p)-closed. Now, the proof of $[A^{(\Lambda,p)} - A]_{(\Lambda,p)} = \emptyset$ is given as follows. Suppose that $[A^{(\Lambda,p)} - A]_{(\Lambda,p)} \neq \emptyset$. Let $x \in [A^{(\Lambda,p)} - A]_{(\Lambda,p)}$. Then, there exists $G \in \Lambda_p O(X, \tau)$ such that $x \in G \subseteq A^{(\Lambda,p)} - A$. Since $G \subseteq X - A$, we have $G \cap A = \emptyset$ and by Lemma 3.6, $G \cap A^{(\Lambda,p)} = \emptyset$. Thus, $G \subseteq X - A^{(\Lambda,p)}$ and hence $G \subseteq (X - A^{(\Lambda,p)}) \cap A^{(\Lambda,p)} = \emptyset$. This is a contradiction. □

Proposition 3.8. *A subset* A *of a topological space* (X, τ) *is g-*(Λ, p)*-closed if and only if* $F \cap A^{(\Lambda,p)} = \emptyset$ *whenever* $F \cap A = \emptyset$ *and* F *is* (Λ, p)*-closed.*

Proof. Suppose that A is g-(Λ, p)-closed. Let $F \cap A = \emptyset$ and F be (Λ, p)-closed. Then, $A \subseteq X - F$. Since A is g-(Λ, p)-closed and $X - F$ is (Λ, p)-open, $A^{(\Lambda,p)} \subseteq X - F$. Thus, $F \cap A^{(\Lambda,p)} = \emptyset$.

Conversely, let $A \subseteq U$ and U be (Λ, p)-open. Then, $(X - U) \cap A = \emptyset$ and $X - U$ is (Λ, p)-closed. By the hypothesis, $(X - U) \cap A^{(\Lambda,p)} = \emptyset$ and hence $A^{(\Lambda,p)} \subseteq U$. Thus, A is g-(Λ, p)-closed. □

Theorem 3.9. *A subset* A *of a topological space* (X, τ) *is g-*(Λ, p)*-closed if and only if* $A \cap \{x\}^{(\Lambda,p)} \neq \emptyset$ *for each* $x \in A^{(\Lambda,p)}$.

Proof. Suppose that A is g-(Λ, p)-closed. Let $A \cap \{x\}^{(\Lambda,p)} = \emptyset$ for some $x \in A^{(\Lambda,p)}$. Then, we have $A \subseteq X - \{x\}^{(\Lambda,p)}$. Since A is g-(Λ, p)-closed and $X - \{x\}^{(\Lambda,p)}$ is (Λ, p)-open, $A^{(\Lambda,p)} \subseteq X - \{x\}^{(\Lambda,p)} \subseteq X - \{x\}$. This contradicts that $x \in A^{(\Lambda,p)}$.

Conversely, suppose that A is not g-(Λ, p)-closed. Then, $\emptyset \neq A^{(\Lambda,p)} - U$ for some $U \in \Lambda_p O(X, \tau)$ containing A. There exists $x \in A^{(\Lambda,p)} - U$. Since $x \notin U$, by Lemma 3.6, $U \cap \{x\}^{(\Lambda,p)} = \emptyset$ and hence $A \cap \{x\}^{(\Lambda,p)} \subseteq U \cap \{x\}^{(\Lambda,p)} = \emptyset$. This shows that $A \cap \{x\}^{(\Lambda,p)} = \emptyset$ for some $x \in A^{(\Lambda,p)}$. □

Theorem 3.10. *A subset* A *of a topological space* (X, τ) *is g-*(Λ, p)*-open if and only if* $U = X$ *whenever* U *is* (Λ, p)*-open and* $(X - A) \cup A_{(\Lambda,p)} \subseteq U$.

Proof. Let U be (Λ, p)-open and $(X - A) \cup A_{(\Lambda,p)} \subseteq U$. Then, $X - U \subseteq (X - A)^{(\Lambda,p)} - (X - A)$. Since $X - A$ is g-(Λ, p)-closed and $X - U$ is (Λ, p)-closed, by Theorem 3.2, $X - U = \emptyset$ and hence $X = U$.

Conversely, suppose that $F \subseteq A$ and F is (Λ, p)-closed. Then, we have $(X - A) \cup A_{(\Lambda,p)} \subseteq (X - F) \cup A_{(\Lambda,p)}$ and $(X - F) \cup A_{(\Lambda,p)}$ is (Λ, p)-open. By the hypothesis, $X = (X - F) \cup A_{(\Lambda,p)}$ and hence $F = F \cap [(X - F) \cup A_{(\Lambda,p)}] = F \cap A_{(\Lambda,p)} \subseteq A_{(\Lambda,p)}$. It follows from Theorem 3.4 that A is g-(Λ, p)-open. □

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