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Model Tumor Response to Cancer Treatment Using Fuzzy Partial SH-Transform: An Analytic Study

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Abstract

In this paper, we show that fuzzy partial Integra-differential equations have an important role in the fields of science and medicine. SHA transformation has been suggested as a new method for solving fuzzy partial differential equations. We discover derivatives of fuzzy partial derivatives formulas for the first and second orders and then we apply the results to find solutions of fuzzy partial differential equations. Finally, we demonstrate the proposed method's capability in solving a model tumor response to cancer treatment.

1 Introduction

The use of fuzzy partial differential equations as a modeling tool for naturally occurring propagation events under uncertainty is a straightforward method. Problems involving the independent variable of time [5]. The last several decades have seen an increase in the application of fuzzy differential equations (FDEs) in both science and engineering. Fuzzy sets were introduced in [1] by Zadeh who outlined the fundamental concepts and operations. As a result of these studies, the concepts of fuzzy derivatives and fuzzy integration were examined in [2]. PDEs are a type of mathematical equations that can be found in a wide

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range of fields including physics, engineering, chemistry, and biology [4], among others. Mathematicians use partial differential equations (PDEs) when they have to deal with several variables and their underlying relationships. Ordinary differential equations fall short of their potential when applied to real-world problems [6]. The reason for this is that observing an event often requires dealing with a number of different variables at the same time. For example, when modeling a wire's heat transfer, we must consider both distance and time simultaneously. Several researchers have solved partial differential equations analytically and numerically [3].

Definition 1.1. *Let $\Im = \Im(\sigma, \alpha)$ be a continuous fuzzy-valued function and let t be a real parameter. Then a fuzzy SHA-transform of a function is defined as follows:*
$SHA_\alpha(\sigma, \alpha) = SHA_\alpha\left[\Im(\sigma, \alpha)\right] = \varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\Im(\sigma, \alpha)d\alpha$
$= \lim_{\tau\to\infty} \varepsilon \int_0^\tau e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\Im(\sigma, \alpha)d\alpha,\ SHA_\alpha(\sigma, \alpha)$
$= \left[\lim_{\tau\to\infty} \varepsilon \int_0^\tau e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\underline{\Im}(\sigma, \alpha)d\alpha, \lim_{\tau\to\infty} \varepsilon \int_0^\tau e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\overline{\Im}(\sigma, \alpha)d\alpha\right]$

whenever the limits exist. The ϑ-cut representation of $SHA_\alpha(\sigma, \varepsilon)$ is given as:
$SHA_\alpha(\sigma, \alpha; \vartheta) = SHA_\alpha\left[\Im(\sigma, \alpha; \vartheta)\right] = \left[\gamma\left(\underline{\Im}(\sigma, \alpha)\right), \gamma\left(\overline{\Im}\left(\sigma, \alpha\right)\right)\right]$

Theorem 1.2. *Assume that $\Im : (0, \infty) \times (0, \infty) \longrightarrow E$ s a continuous fuzzy-valued function, $\varepsilon e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha} \cdot \Im\left(\sigma, \alpha\right)$ is improper fuzzy Riemann-integrable on $[0, \infty]$. Then*

1. $SHA_\alpha\left[\frac{\partial}{\partial\sigma}\Im(\sigma, \alpha)\right] = \frac{\partial}{\partial\sigma}SHA_\alpha\left[\Im(\sigma, \alpha)\right] = \frac{\partial}{\partial\sigma}SHA_\alpha\Im(\sigma, \varepsilon)$,
2. $SHA_\alpha\left[\frac{\partial^2}{\partial\sigma^2}\Im(\sigma, \alpha)\right] = \frac{\partial^2}{\partial\sigma^2}SHA_\alpha\left[\Im(\sigma, \alpha)\right] = \frac{\partial^2}{\partial\sigma^2}SHA_\alpha\Im(\sigma, \varepsilon)$,

Proof.

1. From the definition of a Fuzzy Partial SHA-transform:
$SHA_\alpha\left[\frac{\partial}{\partial\sigma}\Im(\sigma, \alpha)\right] = \varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\frac{\partial}{\partial\sigma}\Im(\sigma, \alpha)d\alpha$
$= \left[\varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\frac{\partial}{\partial\sigma}\underline{\Im}(\sigma, \alpha; \vartheta)d\alpha, \varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\frac{\partial}{\partial\sigma}\overline{\Im}(\sigma, \alpha; \vartheta)d\alpha\right]$
$= \frac{\partial}{\partial\sigma}\left[\varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\underline{\Im}(\sigma, \alpha; \vartheta)d\alpha, \varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\overline{\Im}(\sigma, \alpha; \vartheta)d\alpha\right]$
2. Since $SHA_\alpha\left[\frac{\partial^2}{\partial\sigma^2}\Im(\sigma, \alpha)\right] = \varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\frac{\partial^2}{\partial\sigma^2}\Im(\sigma, \alpha)d\alpha$
$= \frac{\partial^2}{\partial\sigma^2}\left[\varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\underline{\Im}(\sigma, \alpha; \vartheta)d\alpha, \varepsilon \int_0^\infty e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}\overline{\Im}(\sigma, \alpha; \vartheta)d\alpha\right]$
$= \frac{\partial^2}{\partial\sigma^2}SHA_\alpha(\sigma, \varepsilon)$

Theorem 1.3. *Let $\Im(\sigma, \alpha)$ be a differentiable function with respect to t, ε is a real positive function and $-(i\sqrt[2n]{\varepsilon} + \varepsilon$ is complex positive function. Then*

1. $SHA_\alpha\left[\frac{\partial}{\partial\sigma}\Im(\sigma, \alpha)\right] = -(i\sqrt[2n]{\varepsilon} + \varepsilon)SHA_\alpha\left[\Im(\sigma, \alpha)\right] - \varepsilon\Im(\sigma, 0)$,
2. $SHA_\alpha\left[\frac{\partial^2}{\partial\sigma^2}\Im(\sigma, \alpha)\right] = -(i\sqrt[2n]{\varepsilon}+\varepsilon)^2 SHA_\alpha\left[\Im(\sigma, \alpha)\right] - \varepsilon - (i\sqrt[2n]{\varepsilon}+\varepsilon)\Im(\sigma, 0) - \varepsilon\Im_\alpha(\sigma, 0)$,

Proof.

1. Since $SHA_\alpha\left[\frac{\partial}{\partial\sigma}\Im(\sigma,\alpha)\right] = \varepsilon\int_0^\infty \frac{\partial}{\partial\sigma}\Im(\sigma,\alpha)e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}d\alpha$, integrating by parts: $\varepsilon\int_0^\infty \frac{\partial}{\partial\sigma}\Im(\sigma,\alpha)e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}d\alpha = (i\sqrt[2n]{\varepsilon}+\varepsilon)SHA_\alpha\left[\Im(\sigma,\alpha)\right] - \varepsilon\Im(\sigma,0)$, thus $SHA_\alpha\left[\Im(\sigma,\alpha)\right] - \varepsilon\Im(\sigma,0)$,
2. Since $SHA_\alpha\left[\frac{\partial^2}{\partial\sigma^2}\Im(\sigma,\alpha)\right] = \varepsilon\int_0^\infty \frac{\partial^2}{\partial\sigma^2}\Im(\sigma,\alpha)e^{-(i\sqrt[2n]{\varepsilon}+\varepsilon)\alpha}d\alpha$, integrating by parts twice: $SHA_\alpha\left[\frac{\partial^2}{\partial\sigma^2}\Im(\sigma,\alpha)\right] = -(i\sqrt[2n]{\varepsilon}+\varepsilon)^2 SHA_\alpha\left[\Im(\sigma,\alpha)\right] - \varepsilon - (i\sqrt[2n]{\varepsilon}+\varepsilon) - \varepsilon\Im_\alpha(\sigma,0)$

2 Formulas of Fuzzy Partial Derivatives about First and Second

Theorem 2.1. *Let* $\Im : (0,\infty)\times(0,\infty) \to \mathrm{E}$ *be continuous fuzzy-valued function and* $\Im_\alpha$ *the partial derivative of* $\Im$ *with respect to* α. *Suppose that* $\varepsilon e^{(-i\sqrt[2n]{\varepsilon+\varepsilon})\delta}$ $\Im(\sigma,\alpha)$ *and* $\varepsilon e^{(-i\sqrt[2n]{\varepsilon+\varepsilon})\delta}$ $\Im_\alpha(\sigma,\alpha)$ *are improper fuzzy Riemann-integrable on* $[0,\infty]$. *Then*

1. *if* $\Im$ *is the first form differentiable function with respect to* α
   $SHA_\alpha\left[\Im_\alpha(\sigma,\alpha)\right] = (i\sqrt[2n]{\varepsilon+\varepsilon})SHA_\alpha\left[\Im(\sigma,\alpha)\right] \ominus \varepsilon\Im(\sigma,\alpha)$
2. *if* $\Im$ *is the second form differentiable function with respect to* α
   $SHA_\alpha\left[\Im_\alpha(\sigma,\alpha)\right] = -\varepsilon\Im(\sigma,\alpha) \ominus (i\sqrt[2n]{\varepsilon+\varepsilon})SHA_\alpha\left[\Im(\sigma,\alpha)\right]$

Proof.
Since $\Im_\alpha(\sigma,\alpha)u_t(\varpi,t)$ is continuous fuzzy-valued function, there are two cases as follows:

Case 1

Since $\Im$ is the first form differentiable function, for any arbitrary $\vartheta \in [0,1]$, from Therom 2.1 :

$$SHA_\alpha\left[\Im_\alpha(\sigma,\alpha)\right] = \gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right], \gamma_\alpha\left[\overline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] \tag{1}$$

$$\begin{aligned}\gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] &= \left(i\sqrt[2n]{\varepsilon+\varepsilon}\right)\gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] - \varepsilon\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\\ \gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] &= \left(i\sqrt[2n]{\varepsilon+\varepsilon}\right)\gamma_\alpha\left[\overline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] - \varepsilon\overline{\Im}_\alpha(\sigma,\alpha;\vartheta)\end{aligned} \tag{2}$$

Substituting (2) int0 (1), we get
$SHA_\alpha\left[\Im_\alpha(\sigma,\alpha)\right] = \left(i\sqrt[2n]{\varepsilon+\varepsilon}\right)\gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] - \varepsilon\underline{\Im}_\alpha(\sigma,\alpha;\vartheta), \gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] = \left(i\sqrt[2n]{\varepsilon+\varepsilon}\right)\gamma_\alpha\left[\overline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] - \varepsilon\overline{\Im}_\alpha(\sigma,\alpha;\vartheta)$
By Theorem 1,

$$SHA_\alpha\left[\Im_\alpha(\sigma,\alpha)\right] = (i\sqrt[2n]{\varepsilon+\varepsilon})SHA_\alpha\left[\Im(\sigma,\alpha)\right] \ominus \varepsilon\Im(\sigma,0)$$

Case 2

Since is $\Im$ the second form differentiable function, for any arbitrary $\vartheta \in [0,1]$ From Theorem 2.2:

$$SHA_\alpha\left[\Im_\alpha(\sigma,\alpha)\right]=\gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right],\gamma_\alpha\left[\overline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right] \tag{3}$$

From Theorem 2.1:

$$\begin{aligned}\gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right]&=\left(i\sqrt[2n]{\varepsilon+\varepsilon}\right)\gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right]-\varepsilon\underline{\Im}(\sigma,\alpha;\vartheta)\\ \gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right]&=\left(i\sqrt[2n]{\varepsilon+\varepsilon}\right)\gamma_\alpha\left[\overline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right]-\varepsilon\overline{\Im}(\sigma,\alpha;\vartheta)\end{aligned} \tag{4}$$

Substituting (4) into (3), we get
$SHA_\alpha\left[\Im_\alpha(\sigma,\alpha)\right]=\left(i\sqrt[2n]{\varepsilon+\varepsilon}\right)\gamma_\alpha\left[\overline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right]-\varepsilon\overline{\Im}(\sigma,\alpha;\vartheta),\left(i\sqrt[2n]{\varepsilon+\varepsilon}\right)\gamma_\alpha\left[\underline{\Im}_\alpha(\sigma,\alpha;\vartheta)\right]-\varepsilon\underline{\Im}(\sigma,\alpha;\vartheta)$
By Theorem 1,

$$SHA_\alpha\left[\Im_\alpha(\sigma,\alpha)\right]=-\varepsilon\Im(\sigma,\alpha)\ominus(-i\sqrt[2n]{\varepsilon+\varepsilon})SHA_\alpha\left[\Im(\sigma,\alpha)\right]$$

3 The Proposed Method

The object enhancement algorithm Fuzzy partial SHA-transform Filter is used to blur images in a noisy environment. The processes are as follows:

1. Select a fuzzy partial SHA-transform type in the first step.
2. Use fuzzy partial SHA-transformation to indefinitely repeat the component, iteratively, up to the level of decomposition, which yields the following results : as well as some other approximations coefficients that represent the I's spectral content spatial information is encoded as a component, detail coefficients.
3. Use the fuzzy partial SHA- transform technique to transform image up to the n=1 and n=0 and n=-1 level of the panchromatic image.

4 Experimental Results

A sample of color images is displayed when the system was run on a database of 20 blurred images of any size and format.
In addition to speed, the fuzzy partial SHA-transform filter is the ideal option because it provides additional features as it is orthogonal and rotates continuously.

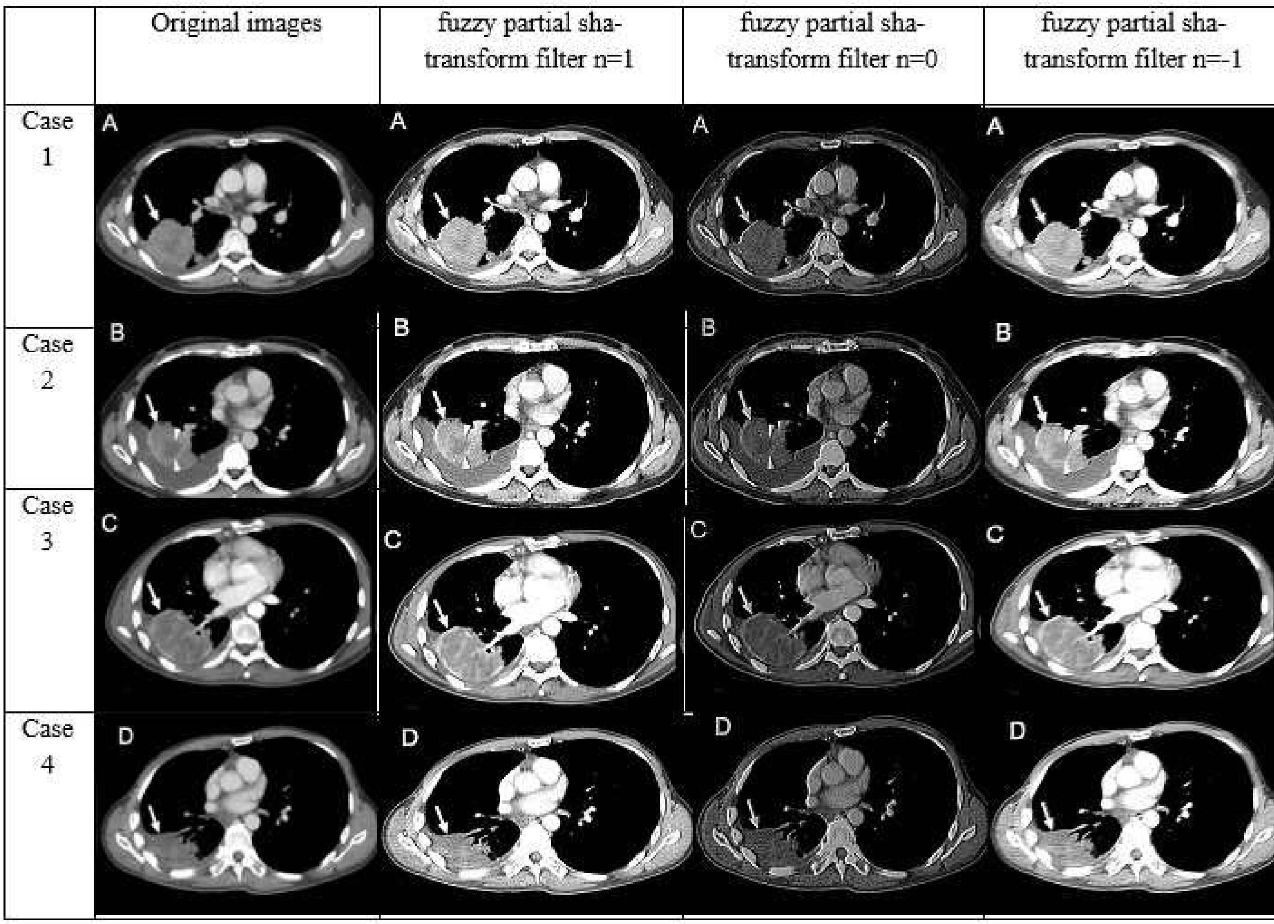

Figure 1: Enhancement objects and regions in the blurry images

5 Conclusion

This study implies that non-small cell lung cancer (NSCLC) patients undergoing targeted therapy who exhibit a high degree of tumor necrosis can expect a positive early response. A 53-year-old man was given Afatinib after being diagnosed with NSCLC (EGFR exon 20 insertion mutation positive). An enhancing tumor (arrow) measuring 64 mm in diameter is shown on a pre-treatment contrast-enhanced (CE) axial CT scan. B, C Three months (B) (tumor diameter of 67 mm) and eight months (C) (tumor diameter of 74 mm) following targeted therapy showed pseudo-progression with increasing tumor growth (arrow). At the same time, we saw focal tumor necrosis (B, arrowheads) transform into diffuse tumor necrosis (C). D A new method known as (fuzzy partial SHA -transform), where the data set consists of 20 photos, was used to categorize the shrinkage of the tumor mass (50 mm in diameter) 15 months following therapy. When compared to the other transformations, which do not supply such solutions, it yields superior results in a shorter amount of time and with fewer calculations because it always picks the best option for the proposed conversion.

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