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The Weighted Maximum Product of Spacings for Extreme Value Distributions

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Abstract

The Maximum Product of Spacing (MPS) is an alternative parameter estimation method for the Maximum Likelihood Estimator (MLE), since the MLE may not exist in some circumstances. However, the MPS estimator still has some weaknesses despite being an alternative method. This is due to the variations in distance between data points, particularly with extreme data points, as the MPS is based on the calculation of spacings in a data set. It is also possible that any slight difference in the estimation of the parameters may have a substantial impact on the fitted values of the extreme value distribution. As a consequence, it is very important to estimate the parameters of the extreme value distribution as accurately as possible. Therefore, the power of the mean function could be introduced and considered as

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a weight function, leading to the weighted MPS. This paper shows the improvement of the MPS method. The Weighted Maximum Product of Spacing (WMPS) is a method that gives weights to the maximization of the logarithm of the spacings in the data set. This is done to reduce the root mean square error of the models and improve the goodness of fit of the extreme value distributions.

1 Introduction

An extreme value distribution (EVD) is a limiting model for the maximums or minimums of a data set [1]. There are three types of EVD model: Type I (Gumbel), Type II (Fréchet), and Type III (Weibull). The Generalized Extreme Value distribution (GEVD) is a three-parameter distribution that unites the thee types extreme value distributions. The CDF for the three-parameter is as follows:

$$F(x;\mu,\sigma,\alpha) = \exp\left\{-\left[1+\alpha\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\alpha}\right\} \tag{1.1}$$

whereby μ = location parameter, σ = scale parameter, and α = shape parameter. The σ and $1+\alpha(x-\mu)/\sigma > 0$, where μ and α can take any real value. The three types of EVD can be obtained through GEVD based on the value of alpha where $\alpha = 0$ is the Type I EVD (Gumbel distribution), $\alpha > 0$ is the Type II EVD (Fréchet distribution), and $\alpha < 0$ is theType II EVD (Weibull distribution) [6].

Extreme value distributions are widely used models in a wide range of applications. The Maximum likelihood estimation (MLE) is a popular estimation method. It has been thoroughly investigated and widely used in various areas with extreme value distributions due to its nice asymptotic behavior [2, 3]. Nevertheless, the MLE may not exist when the shape parameter α is less than -0.5, since the PDF of the extreme value distributions goes to ∞ and is also an inconsistent method [4]. Furthermore, MLE is not accurate if the sample size of the extreme observations is small. As a result, some alternative estimation methods have been proposed which provide better estimation results when compared to the MLE method [3].

The Maximum Product of Spacing (MPS) is proposed by Cheng [5]. It is now becoming one of the most widely used estimation methods [6, 7, 8]. However, the MPS estimators tend to underestimate some parameters in extreme value distributions due to the variations in distance between data

points. Furthermore, the MPS estimators provide a much less accurate estimation when the shape parameter is greater than a specific value [9]. This is because MPS estimators are based on the calculation of spacings between data points [3, 10].

The MPS method is considered one of the fittest methods for extreme value distribution (EVD) and has more or less better goodness of fit compared to the MLE method [6, 7, 8, 11, 12]. Abdulali [6] provided a comprehensive comparison of various estimation methods for extreme value distributions and concluded his study by highly recommending that the MPS might be taken into consideration for future studies. Moreover, there have been many approaches for weighted parameter estimation methods, such as modifying from the MLE to the weighted MLE [2, 13, 14, 15], the least-squares method to weighted least-squares [16, 17, 18], and the method of moments to the weighted method of moments [19, 20, 21], all of which yield better results.

In this article, the weight function is well explained in Section 2. Moreover, the slopes of the CDF curve fitting in the power weight function are considered. It also shows how to measure the slopes of the CDF curve using the right machine learning method. The Weighted Maximum Product of Spacing (WMPS) is covered in Section 3 and then a simulation study is carried out in Section 4, which can evaluate the performance of the WMPS. Lastly, a summary of the proposed method's results based on the simulation study is given in Section 5.

2 Weight function

Fitting nonlinear models is quite a challenging task and is often complicated not only by heteroscedasticity but also by inconstant variability in a data set. The weighting approach has been utilized and supported for some estimation methods. There are many types of weight functions. Nevertheless, finding an appropriate weight function that predominantly works well for all types of extreme distributions is also challenging in itself. The weight function should have all the theoretical properties that are intact for any choice of tuning parameters. According to [15], the choice of the weight functions has no effect on the estimator's significant theoretical properties as an impact function on the parameters for consistency and asymptotic efficiency when the right model is adequately defined.

In extreme datasets, modeling the variances is very crucial. The power of the mean function could be used as a weight function for each combination

of predictor variable values. Taking into consideration that some conditions are attached to the power of the mean function, which are all automatically satisfied by the construction of this weight function in the case of being used with the MPS scenario.

The weight function has been chosen to be the modeling variance that will be utilized in the next section. This weight function was chosen for two reasons. Firstly, regardless of the sample size, there is always a connection between the weight function and the data set via the slopes of the data points. Secondly, it also allocates more weights to data close to short or medium-distance positions in the data set ranked in ascending order. It was implemented into the weighted least squares model, which was addressed by [22, 23], where they used the power function model as a weight function. The weight function is defined as follows:

$$\sigma_i^2 \approx a * \mu_i^b \tag{2.2}$$

By taking the log for both sides of the model. A simple linear regression is obtained as the following :

$$\log(\sigma_i^2) = \log(a) + b * \log(\mu_i) \tag{2.3}$$

This power function model is convenient, especially when there is only one predictor variable. The weight function is based only on the slope (b) from the fit to the weight data because the weights only need to be proportional to the replicate variances. As a result, we can ignore the estimate of (a) in the power function since it is only a proportionality constant (in the original units of the model). In this situation, the general model given above can be simplified to the power function of the variances from each set of responses in the data. Therefore, the finding weights used can be written as:

$$w_i = \frac{1}{(|X_{(i)}|)^{b_i}} \tag{2.4}$$

where X_i is an ascending data set representing the curve of the cumulative distribution function (CDF), and b_i is the slops of the CDF curve for $i = 1, 2, ..., n$.

2.1 Properties of the weight function

The weight function has the following properties.

1. w_i is a nonnegative function on $(0, 1]$ with the property $|X_{(i)}|$ for $i = 1, .., n$.

2. The trend of the CDF curve is always upward until it reaches a probability value of 1. In other words, the CDF is a non-decreasing function. Thus, the slopes of tangent lines are non-negative values, and the value slopes b are always ≥ 0.

3. The value of x_i=1 indicates that the slope is $\rightarrow 0$.

4. If w_i=1 for $i = 1, .., n$ for all calculations of spacings, then WMPS = MPS.

To determine a possible weight function having the above-mentioned properties, we proceed with scaling the weights by dividing the max values. Then w_i, which will be mapped between 0 and 1 for $i = 1, .., n$.

2.2 The slopes of the CDF curve

The value of slope b_i for the CDF curve can be found via multiple methods. However, one of the most crucial methods for estimating the value of the slopes is local regression, which could be taken into consideration in this study. One of the main advantages of the local regression method is that it can provide a vector of estimated slopes of b_i for each data point $X_{(i)}$ for $i = 1, .., n$.

2.3 Local regression

Local regression or local polynomial regression, also known as moving regression, is a form of regression analysis in which it models the relationship between outcomes and predictors by fitting different linear or quadratic functions to different segments or intervals of data set. The overall curve obtained by combining the individually fitted curves for the different data segments shows the general shape of the CDF plot

Local regression is also considered a non-parametric method, and there are many non-parametric regression techniques, such as:

- **LOWESS** (locally weighted scatterplot smoothing) proposed in 1979 by Cleveland [24].

- **LOESS** (locally estimated scatterplot smoothing) proposed in 1988 by Cleveland and Susan J. Devlin [25]

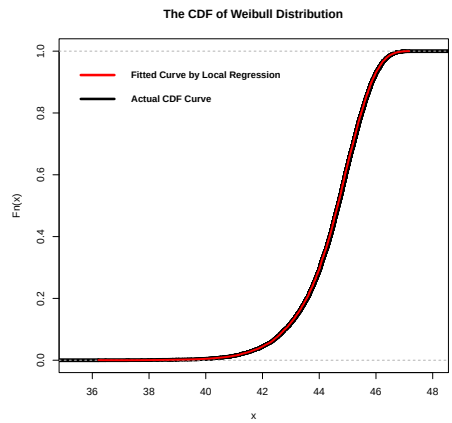

Figure 1: The fitted curve by local regression method to CDF curve of EVD.

Both methods could be implemented to find the fitted CDF curve. LOESS is arguably one of the most flexible and powerful smoothing functions that attempts to capture general patterns in response to relationships while reducing the noise between data points. It also makes minimal assumptions about the relationships among variables. Moreover, it has some other features that can support not only multidimensional predictors but also multiple dependent variables. We assume a model of the following form:

$$y_i = f(x_i) + \epsilon_i$$

where $f(x_i)$ is an unknown function, and ϵ_i is an error term, representing random errors in the observations or variability from sources not included in the x_i. In this study, $f(x_i)$ is approximated locally by the Polynomial function e.g. $f(x_i) = \beta_0 + \beta_1 x$. The degree of the locally fitted polynomial is 1. The coefficient estimates $\beta = (\beta_0, \beta_1)'$ are chosen to minimize

$$\beta = \arg\min_{\beta \in \mathbb{R}} \sum_{i=1}^{n} w_i\Big(\frac{x_i - x}{s}\Big) * [Y_i - (\beta_0 + \beta_1(x_i - x))]^2 \tag{2.5}$$

where s is a fixed parameter known as the span. The value of the slopes in the β_1 vector could be extracted from the locally fitted approximation by locally weighted least squares.

3 Weighted maximum product of spacing method

Let $\mathbf{x} = (x_1, x_2, x_3, .., x_n)$ be a dataset with size n, and $\mathbf{X} = (X_1, X_2, X_3, .., X_n)$ be the ascending order of $\mathbf{x}$ dataset. Let $\mathbf{w} = (w_1, w_2, w_3, .., w_n)$ be the weight vector of the ordered dataset. We define the spacings as the gaps between the values of the distribution function at adjacent ordered points, as follows:

$$D_i(\mu, \sigma, \alpha) = F(X_{(i)};\, \mu, \sigma, \alpha) - F(X_{(i-1)};\, \mu, \sigma, \alpha), \quad i = 1, \ldots, n+1. \tag{3.6}$$

where $\mu =$ location parameter, $\sigma =$ scale parameter, and $\alpha =$ shape parameter of EVD.

$$F(X_{0:n}|\mu, \sigma, \alpha) = 0, \quad F(X_{n+1:n}|\mu, \sigma, \alpha) = 1. \tag{3.7}$$

It is evident that the MPS is based on maximizing the logarithm of the sample spacings. Meanwhile, the composition of the WMPS method is the weight function along with the MPS method. But it must be taken into account that the weight function may be equal to zero in some cases, where the logarithm of zero will be undefined. Therefore, in order to avoid those obstacles, the weight function could only be the power of the logarithm of sample spacings, where it does not lead to malfunctions in calculating WMPS. The $\hat{\mu}_{\text{wmps}}$, $\hat{\sigma}_{\text{wmps}}$, and $\hat{\alpha}_{\text{wmps}}$ estimators are then regarded as values that maximize the logarithm of the sample spacings geometric:

$$H(\hat{\mu}, \hat{\sigma}, \hat{\alpha}) = \underset{\mu,\sigma,\alpha\in\Theta}{\arg\max}\, S_n^{w_X}(\mu, \sigma, \alpha) \tag{3.8}$$

where w_i is set to be the weight function for each different distance movement between data points. Thus the WMPS is as the following :

$$S_n^{w_X}(\mu, \sigma, \alpha) = \ln \sqrt[n+1]{D_1^{w_1} D_2^{w_2} \cdots D_n^{w_n} D_{n+1}^{1}} = \frac{1}{n+1} \sum_{i=1}^{n+1} \ln {D_i}^{w_i}(\mu, \sigma, \alpha). \tag{3.9}$$

where the weight function $w_i = \dfrac{1}{|X_{(i)}|^{b_i}}$, and b_i is the slopes of CDF curve.

From (3.8) and (3.9), the $\hat{\mu}_{\text{wmps}}$, $\hat{\sigma}_{\text{wmps}}$ and $\hat{\alpha}_{\text{wmps}}$ estimators could be achieved by solving the nonlinear equations as follows:

$$\frac{\partial H(\mu, \sigma, \alpha; x)}{\partial \mu} = \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{1}{|X_{(i)}|^{b_i} * D_i(\mu, \sigma, \alpha)} \Delta = 0 \tag{3.10}$$

$$\frac{\partial H(\mu, \sigma, \alpha; x)}{\partial \alpha} = \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{1}{|X_{(i)}|^{b_i} * D_i(\mu, \sigma, \alpha)} \Delta = 0 \tag{3.11}$$

$$\frac{\partial H(\mu,\sigma,\alpha;x)}{\partial\sigma} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{|X_{(i)}|^{b_i} * D_i(\mu,\sigma,\alpha)}\Delta = 0 \tag{3.12}$$

where

$$\Delta = \delta(X_i|\mu,\sigma,\alpha) - \delta(X_{i-1}|\mu,\sigma,\alpha)$$

whereby δ = the derivative of the cumulative function of the extreme distribution with respect to the estimated parameter.

4 Simulation study

Simulation studies have been conducted to investigate the performance of the WMPS approach and compare it to the MPS and MLE methods with different sizes of datasets by using R software. The sample sizes for the simulation studies are widely discussed by many researchers [12, 26, 27, 28]. Nevertheless, two sample sizes have been considered, with $N = 100$ for the small size of the dataset and $N = 10,000$ for the large size of the dataset. Data sets were generated for the Gumbel Distribution (Type I), Frechet Distribution (Type II), and Weibull Distribution (Type III). The EVD for two and three parameters has different values for parameters for both sample sizes.

The performance of three estimation methods is not only examined by comparing the root mean square errors (RMSE) but also by Akaikes Information Criterion (AIC), the Bayesian Information Criterion (BIC), the Anderson-Darling (A) test, and the Cramr-von Mises (W) test. For this purpose, we generate the datasets following the extreme value distributions for the parameters of interest by MLE, MPS, and WMPS methods respectively. The simulation results for all EVD of MLE, MPS, and WMPS estimation methods are presented in the Tables 1 & 2.

Table 1: Estimation Parameters with Size $N = 100$.

$N = 100$ EVD	Estimated Parameters By MLE		
	location	scale	shape
	μ=5	σ=7	-
Gumbel (Type I)	5.397040	6.566514	-
	-	σ=9	α=3
2-Fréchet (Type II)	-	8.212115	3.430645
	μ=0.5	σ=7	α=3
3-Fréchet (Type II)	1.8691787	5.6668813	2.3766289

	-	σ=1	α=5
2-Weibull (Type III)	-	0.9991391	5.4236505
	μ=0.5	σ=3	α=2
3-Weibull (Type III)	0.7222131	4.610721	1.6663683
	μ=2	σ=3	α=0.1
GEV-Distribution (GEVD)	2.09498253	2.82466755	0.01972448
$N = 100$	Estimated Parameters By MPS		
EVD	location	scale	shape
	μ=5	σ=7	-
Gumbel (Type I)	5.360579	6.875822	-
	-	σ=9	α=3
2-Fréchet (Type II)	-	8.192851	3.286586
	μ=0.5	σ=7	α=3
3-Fréchet (Type II)	1.637546	5.898064	2.364839
	-	σ=1	α=5
2-Weibull (Type III)	-	0.9999133	5.2107155
	μ=0.5	σ=3	α=2
3-Weibull (Type III)	0.6393905	2.6509675	1.6868107
	μ=2	σ=3	α=0.1
GEV-Distribution (GEVD)	2.05428524	2.93026428	0.03637715
$N = 100$	Estimated Parameters By WMPS		
EVD	location	scale	shape
	μ=5	σ=7	-
Gumbel (Type I)	5.323002	7.076661	-
	-	σ=9	α=3
2-Fréchet (Type II)	-	8.340154	2.998637
	μ=0.5	σ=7	σ=3
3-Fréchet (Type II)	0.8798338	6.9989168	2.3772184
	-	σ=1	α=5
2-Weibull (Type III)	-	0.9695959	5.0571887
	μ=0.5	σ=3	α=2
3-Weibull (Type III)	0.5657216	2.7859650	1.6994437
	μ=2	σ=3	α=0.1
GEV-Distribution (GEVD)	1.84873221	2.95518108	0.09832042

Table 2: Estimation Parameters with Size $N = 10000$.

$N = 10000$	Estimated Parameters By MLE		
EVD	location	scale	shape
	μ=5	σ=9	-
Gumbel (Type I)	4.879680	8.884994	-
	-	σ=2	α=5
2-Fréchet (Type II)	-	2.003168	5.027983
	μ=5	σ=30	α=10
3-Fréchet (Type II)	7.215161	27.756319	9.379527

	-	σ=2	α=10
2-Weibull (Type III)	-	1.998252	9.921407
	μ=1	σ=3	α=30
3-Weibull (Type III)	1.042261	2.956307	29.553351
	μ=3	σ=10	α=0.01
GEV-Distribution (GEVD)	2.882597044	9.987487261	2.882597044
$N = 10000$	Estimated Parameters By MPS		
EVD	location	scale	shape
	μ=5	σ=9	-
Gumbel (Type I)	4.879442	8.893716	-
	-	σ=2	α=5
2-Fréchet (Type II)	-	2.003119	5.022712
	μ=5	σ=30	α=10
3-Fréchet (Type II)	5.033644	29.949967	10.088762
	-	σ=2	α=10
2-Weibull (Type III)	-	1.998807	9.972420
	μ=1	σ=3	σ=30
3-Weibull (Type III)	0.9421131	3.0565143	30.5340238
	μ=3	σ=10	α=0.01
GEV-Distribution (GEVD)	2.889919710	9.997768139	0.007874064
$N = 10000$	Estimated Parameters By WMPS		
EVD	location	scale	shape
	μ=5	σ=9	-
Gumbel (Type I)	4.767553	8.962183	-
	-	σ=2	α=5
2-Fréchet (Type II)	-	2.000857	4.985115
	μ=5	σ=30	α=10
3-Fréchet (Type II)	5.003881	30.168918	10.150674
	-	σ=2	α=10
2-Weibull (Type III)	-	1.991324	9.972603
	μ=1	σ=3	α=30
3-Weibull (Type III)	0.9361494	3.0627187	30.6059502
	μ=3	σ=10	α=0.01
GEV-Distribution (GEVD)	2.77371618	9.99747554	0.010625624

The Goodness of Fit tests and RMSE are presented in the Tables 3 & 4

Table 3: Test of Goodness of Fit and RMSE with $N = 100$.

Model	Loglikelihood	AIC	BIC	A	W	RMSE
Gumbel-MLE	-348.2736	700.5472	705.7575	0.32889164	0.04669522	0.01364266
Gumbel-MPS	-348.4592	700.9183	706.1287	0.38220000	0.06388000	0.00393335
Gumbel-WMPS	-290.7687	585.5374	590.7478	0.21024976	0.00613402	0.00224123
Fréchet-MLE	-260.8655	525.7310	530.9414	0.20690485	0.03225485	0.12178530
Fréchet-MPS	-260.7909	525.5818	530.7921	0.22635000	0.03960200	0.10036620

Model	Loglikelihood	AIC	BIC	A	W	RMSE
Fréchet-WMPS	-208.6178	421.2355	426.4459	0.11046590	0.01496952	0.04495697
Fréchet-3-MLE	-269.4808	544.9617	552.7772	0.46504696	0.06037190	0.04144314
Fréchet-3-MPS	-269.0696	545.3930	553.2085	0.57593000	0.08572800	0.04056556
Fréchet-3-WMPS	-212.6682	431.3363	439.1519	0.22931250	0.02321757	0.01290916
Weibull- MLE	20.83141	-37.66282	-32.45248	0.35971945	0.05701530	0.56960140
Weibull- MPS	20.68785	-37.37570	-32.16536	0.35326000	0.05958800	0.28879040
Weibull-WMPS	13.42555	-22.85109	-17.64075	0.20662500	0.01513184	0.14584870
Weibull-3-MLE	-169.7593	345.5185	353.334	0.24691200	0.02495165	0.66834730
Weibull-3-MPS	-170.0708	346.1416	353.957	0.27475650	0.03825779	0.06877046
Weibull-3-WMPS	-132.9855	271.971	279.7865	0.16268320	0.00324056	0.01760126
GEVD-MLE	-262.7859	531.5718	539.3873	0.16836210	0.02923236	5.36825800
GEVD-MPS	-262.9716	531.9432	539.7587	0.19379000	0.03265700	0.03571904
GEVD-WMPS	-164.6143	335.2286	343.0441	0.01001955	0.01065642	0.00453723

Table 4: Test of Goodness of Fit and RMSE with $N = 10000$.

Model	Loglikelihood	AIC	BIC	A	W	RMSE
Gumbel-MLE	-37613.06	75230.11	75244.53	0.990236425	0.18171877	0.1798249
Gumbel-MPS	-37613.06	75230.11	75244.53	0.976310000	0.18025000	0.1659857
Gumbel-WMPS	-29384.49	58772.98	58778.19	0.502175000	0.04156094	0.0537062
Fréchet-MLE	-7738.047	15480.09	15494.51	0.457414686	0.06457617	0.1279719
Fréchet-MPS	-7738.054	15480.11	15494.53	0.453240000	0.06574000	0.0958593
Fréchet-WMPS	-5041.672	10087.34	10110.97	0.392828100	0.00629087	0.0125343
Fréchet-3-MLE	-27239.66	54485.31	54506.95	0.218008015	0.03156076	0.0519615
Fréchet-3-MPS	-27240.19	54486.38	54508.01	0.609510000	0.09497700	0.0478979
Fréchet-3-WMPS	-20334.42	40674.83	40696.46	0.052088640	0.00224780	0.0279240
Weibull- MLE	817.2641	-1630.528	-1616.107	.284359629	0.04400983	0.4289099
Weibull- MPS	817.0455	-1630.091	-1615.670	0.429460000	0.06665700	0.1314943
Weibull-WMPS	366.9993	-729.9986	-715.5779	0.189703300	0.03315033	0.0564669
Weibull-3-MLE	7446.599	-14887.20	-14865.57	0.21797140	0.02119137	0.1776381
Weibull-3-MPS	7446.574	-14887.15	-14865.52	0.217310000	0.02015000	0.1397980
Weibull-3-WMPS	6705.000	-13404.00	-13382.37	0.05942381	0.00888378	0.1019555
GEVD-MLE	-38834.95	77675.91	77697.54	0.248724651	0.03924340	0.0023059
GEVD-MPS	-38834.98	77675.96	77697.59	0.244730000	0.03670700	0.0011012
GEVD-WMPS	-30452.88	60911.76	60933.39	0.154138340	0.01260231	0.0001908

Tables 1 and 2 show that estimates made with the MLE and MPS methods are very different from those made with the WMPS methods. The primary difference was observed in the evaluation of all EVD, particularly with three

parameters. In other words, the WMPS and MPS variations for all EVD ranged from 0.36% for the location parameter μ, 1.1% for the scale parameter σ, and 0.29% for the shape parameter α in Table 1. Meanwhile, the WMPS and MPS variations for all EVD ranged from 0.24% for the location parameter μ, 0.37% for the scale parameter σ, and 0.19% for the shape parameter α in Table 2.

Moreover, the estimates from the MLE and MPS methods aren't that significantly different. The Estimates derived from the WMPS Goodness of Fit tests were consistently significantly lower than those derived from other methods, regardless of sample size or the parameter values. The MLE provided lower accuracy in estimating almost all of the EVD parameters. Nonetheless, it was still able to provide a reliable estimate for GEVD.

Meanwhile, Tables 3 and 4 show the Root Mean Square Error (RMSE) for each parameter estimate technique for all probability distributions studied at two sample sizes. The WMPS technique offers the lowest RMSE estimates for all EVD in both sample sizes. However, in certain distributions, the difference in RMSE values between MPS and MLE estimate techniques is considered almost non-existent. The RMSE values for WMPS estimations, on the other hand, are much lower for practically all extreme value distributions. Despite the MPS method having a lower RMSE than the MLE approach, the MPS and MLE methods have nearly identical goodness of fit values for almost all extreme value distributions. WMPS, on the other hand, shows substantially lower scores for the goodness of fit tests, indicating that it is a better fit for the EVD simulated datasets. Again, it has been shown that the WMPS method is the best estimation method for figuring out how well the EVD fits. The tables above can be transformed from numerical outcomes into graphical results for more clarity. Thus, here are the CDF plots of all the EVDs that were calculated using MLE, MPS, and WMPS methods:

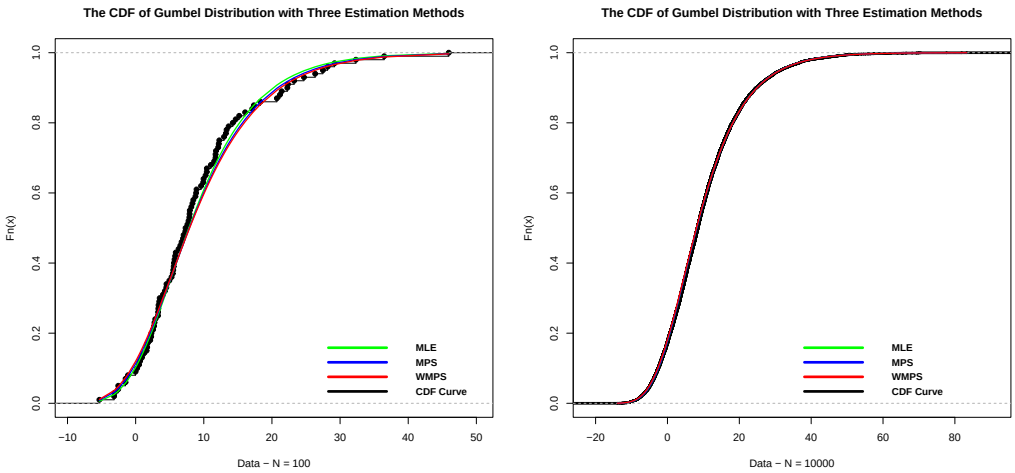

Figure 2: The plot of CDF for Gumbel(μ,σ) with three methods.

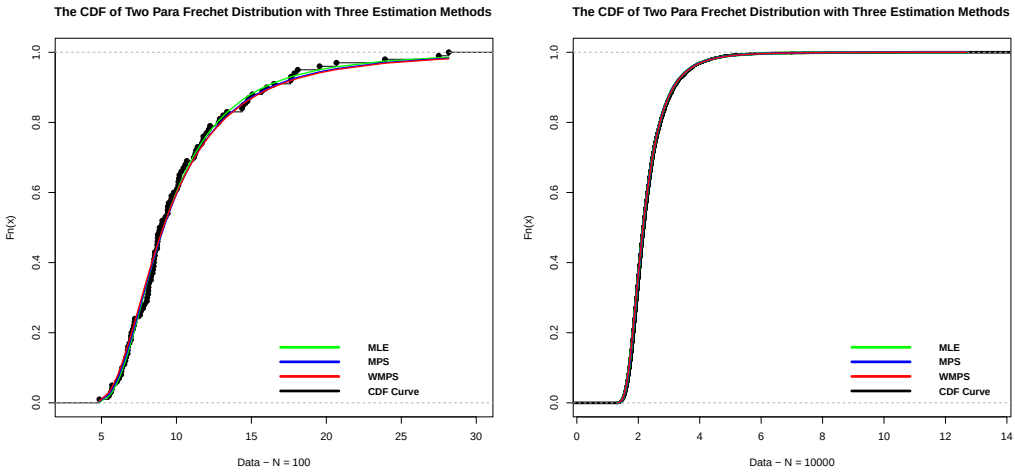

Figure 3: The plot of CDF for Frechet (σ,α) with three methods.

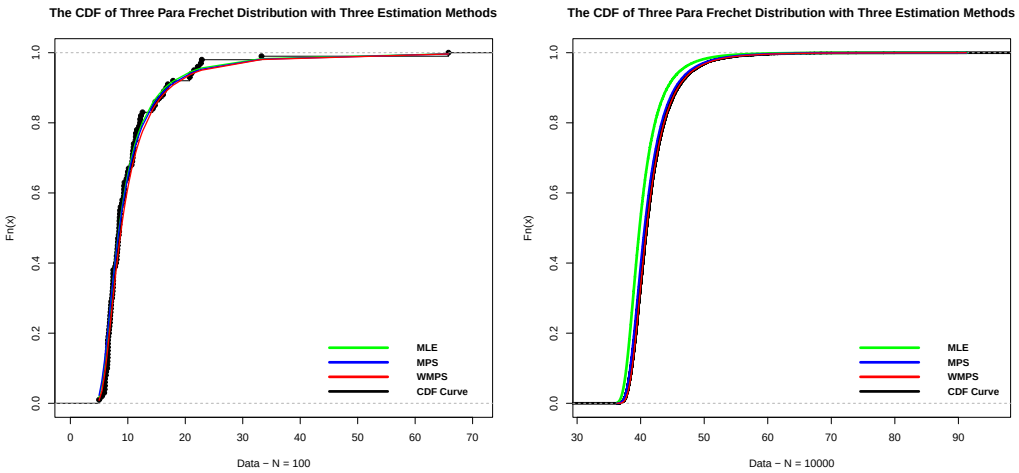

Figure 4: The plot of CDF for Frechet (μ,σ,α) with three methods.

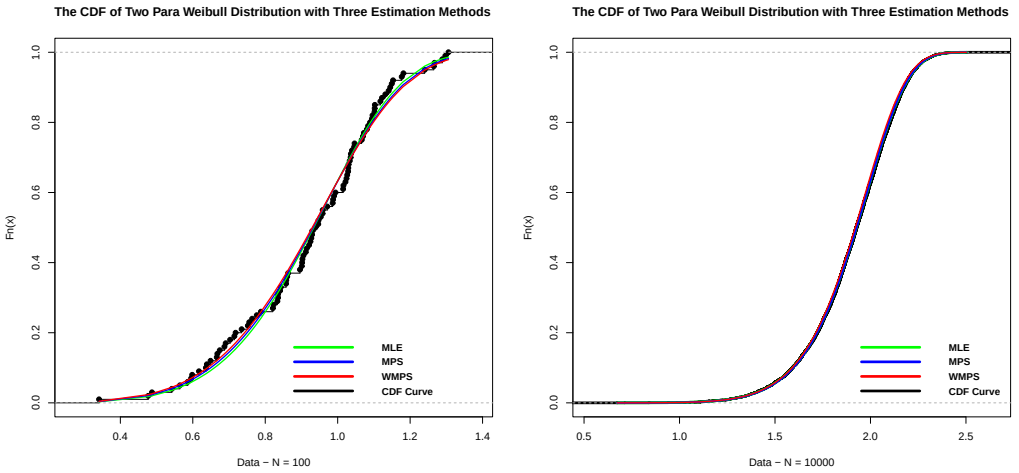

Figure 5: The plot of CDF for Weibull(σ,α) with three methods.

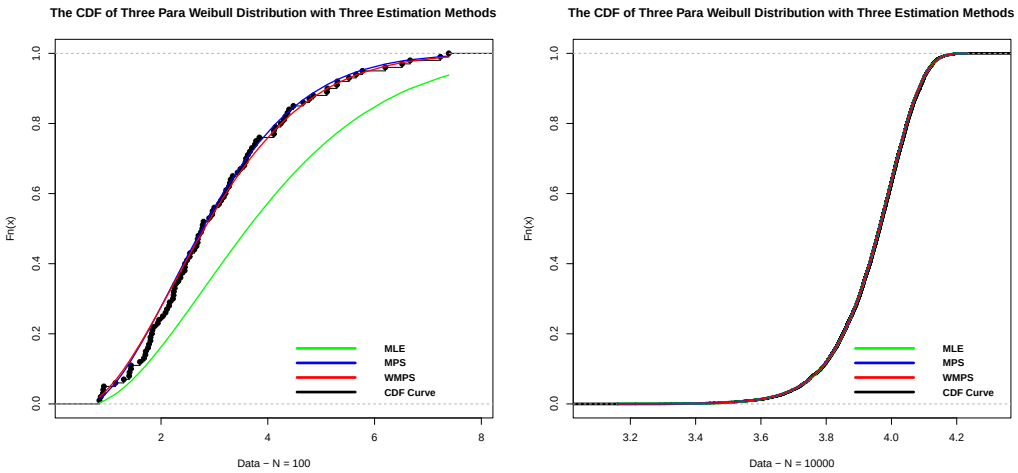

Figure 6: The plot of CDF for Weibull(μ,σ,α) with three methods.

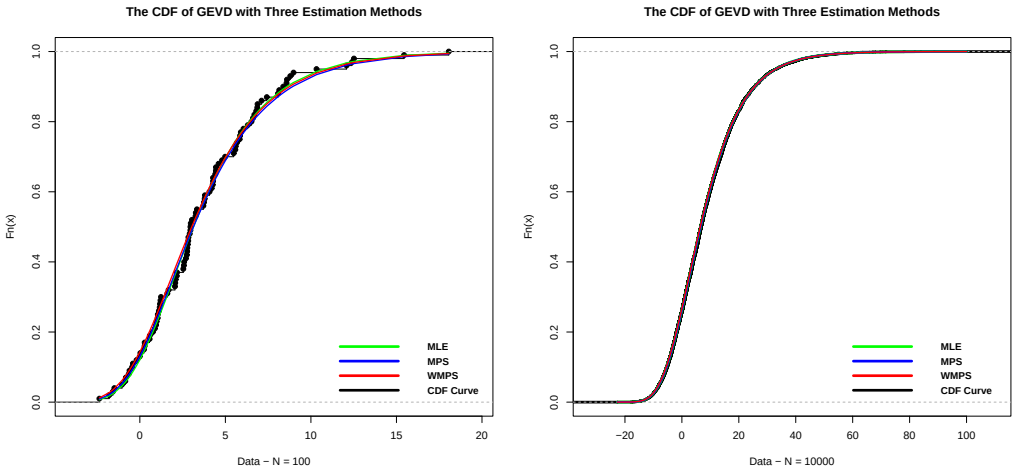

Figure 7: The plot of CDF for GEVD with three methods.

From Figures 2–7, which show the fitted models for all EVD at $N = 100$ and $N = 10,000$, indicating that the WMPS estimation method fitted the data well for all EVD. In contrast, in some distributions, the MLE and MPS fitted the data to EVD with less accuracy. As illustrated in Figure 6, the MLE consistently provides a poor fit for the three parameters of Weibull Distribution. This outcome is consistent with the goodness-of-fit test results for all EVD, ... shown in Tables 3 and 4. The WMPS estimator is the more accurate method compared to the MLE and MPS methods, with much lower values of goodness-of-fit tests for all extreme distributions.

5 Conclusions

In this article, the WMPS approach was introduced for estimation of parameters of the extreme value distributions. The weight function played a vital role in many estimation methods; not only for more accurate estimation but also for more stable efficiency and consistency. Various weight functions have been implemented in different estimation methods. Thus, the proposed weight function is thought to improve the estimation of parameters; namely, location, size, and shape. The weight function was also introduced to model the variance which required finding the appropriate slopes of the CDF curve. The local regression method was used to find the local slope for the data points in the subsets of the data set. Finally, a simulation study was conducted to provide a comprehensive view of the performance and validate the estimation methods using various extreme value distributions. Despite the fact that the MPS method was better than the MLE method in terms of RMSE, the MLE method was at least better than the MPS method in terms of the goodness of fit tests for almost all extreme value distributions. Nevertheless, the WMPS method was superior to other methods based on RMSE and goodness of fit tests.

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