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Characterizations of (Λ, p)-$T_{\frac{1}{2}}$-spaces

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Abstract

In this paper, we deal with the concept of (Λ, p)-$T_{\frac{1}{2}}$-spaces. Moreover, we investigate some characterizations of (Λ, p)-$T_{\frac{1}{2}}$-spaces.

1 Introduction

In 1970, Levine [6] introduced the concept of generalized closed sets in topological spaces and defined a class of topological spaces called $T_{\frac{1}{2}}$-spaces; a topological space (X, τ) is $T_{\frac{1}{2}}$ if every generalized closed set is closed. Dunham [4] showed that a topological space (X, τ) is $T_{\frac{1}{2}}$ if and only if each singleton of X is open or closed. Moreover, Arenas et al. [1] proved that a topological space (X, τ) is $T_{\frac{1}{2}}$ if and only if every subset of X is λ-closed. In 2012, Dungthaisong et al. [3] introduced and studied the concept of pairwise μ-$T_{\frac{1}{2}}$ spaces. Torton et al. [8] introduced and investigated the notions of $\mu_{(m,n)}$-T_1 spaces, $\mu_{(m,n)}$-T_3 spaces and $\mu_{(m,n)}$-T_4 spaces. Ganster et al. [5] studied the notions of pre-Λ-sets and pre-V-sets. In [2], the present authors introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets which are

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defined by utilizing the notions of Λ_p-sets and preclosed sets. In this paper, we introduce the concept of (Λ,p)-$T_{\frac{1}{2}}$-spaces. Moreover, we discuss some characterizations of (Λ,p)-$T_{\frac{1}{2}}$-spaces.

2 Preliminaries

For a subset A of a topological space (X,τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X,τ) is said to be *preopen* [7] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X,τ) is denoted by $PO(X,\tau)$. A subset $\Lambda_p(A)$ [5] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X,\tau)\}$. A subset A of a topological space (X,τ) is called a Λ_p-*set* [2] (pre-Λ-set [5]) if $A = \Lambda_p(A)$. A subset A of a topological space (X,τ) is called (Λ,p)-*closed* [2] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ,p)-closed set is called (Λ,p)-*open*. The family of all (Λ,p)-open (resp. (Λ,p)-closed) sets in a topological space (X,τ) is denoted by $\Lambda_pO(X,\tau)$ (resp. $\Lambda_pC(X,\tau)$). Let A be a subset of a topological space (X,τ). A point $x \in X$ is called a (Λ,p)-*cluster point* [2] of A if $A \cap U \neq \emptyset$ for every (Λ,p)-open set U of X containing x. The set of all (Λ,p)-cluster points of A is called the (Λ,p)-*closure* [2] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ,p)-open sets contained in A is called the (Λ,p)-*interior* [2] of A and is denoted by $A_{(\Lambda,p)}$.

Lemma 2.1. [2] *For subsets A, B of a topological space (X,τ), the following properties hold:*

(1) $A \subseteq A^{(\Lambda,p)}$ and $[A^{(\Lambda,p)}]^{(\Lambda,p)} = A^{(\Lambda,p)}$.

(2) If $A \subseteq B$, then $A^{(\Lambda,p)} \subseteq B^{(\Lambda,p)}$.

(3) $A^{(\Lambda,p)} = \cap\{F \mid A \subseteq F$ and F is (Λ,p)-closed$\}$.

(4) $A^{(\Lambda,p)}$ is (Λ,p)-closed.

(5) A is (Λ,p)-closed if and only if $A = A^{(\Lambda,p)}$.

Lemma 2.2. [2] *Let A and B be subsets of a topological space (X,τ). For the (Λ,p)-interior, the following properties hold:*

(1) $A_{(\Lambda,p)} \subseteq A$ and $[A_{(\Lambda,p)}]_{(\Lambda,p)} = A_{(\Lambda,p)}$.

(2) If $A \subseteq B$, then $A_{(\Lambda,p)} \subseteq B_{(\Lambda,p)}$.

(3) $A_{(\Lambda,p)}$ *is* (Λ, p)*-open.*

(4) A *is* (Λ, p)*-open if and only if* $A_{(\Lambda,p)} = A$.

(5) $[X - A]^{(\Lambda,p)} = X - A_{(\Lambda,p)}$.

3 Characterizations of (Λ, p)-$T_{\frac{1}{2}}$-spaces

In this section, we introduce the notion of (Λ, p)-$T_{\frac{1}{2}}$-spaces. Moreover, we discuss some characterizations of (Λ, p)-$T_{\frac{1}{2}}$-spaces.

Definition 3.1. [2] *Let* A *be a subset of a topological space* (X, τ)*. A subset* $\Lambda_{(\Lambda,p)}(A)$ *is defined as follows:* $\Lambda_{(\Lambda,p)}(A) = \cap\{U \in \Lambda_p O(X, \tau) \mid A \subseteq U\}$.

Lemma 3.2. [2] *For subsets* A, B *of a topological space* (X, τ)*, the following properties hold:*

(1) $A \subseteq \Lambda_{(\Lambda,p)}(A)$.

(2) If $A \subseteq B$*, then* $\Lambda_{(\Lambda,p)}(A) \subseteq \Lambda_{(\Lambda,p)}(B)$.

(3) $\Lambda_{(\Lambda,p)}[\Lambda_{(\Lambda,p)}(A)] = \Lambda_{(\Lambda,p)}(A)$.

(4) If A *is* (Λ, p)*-open, then* $\Lambda_{(\Lambda,p)}(A) = A$.

Definition 3.3. *A subset* A *of a topological space* (X, τ) *is said to be:*

(i) a $\Lambda_{(\Lambda,p)}$*-set if* $A = \Lambda_{(\Lambda,p)}(A)$*;*

(ii) a $\circledast\Lambda_{(\Lambda,p)}$*-set if* $\Lambda_{(\Lambda,p)}(A) \subseteq F$ *whenever* $A \subseteq F$ *and* F *is a* (Λ, p)*-closed set.*

Recall that a subset A of a topological space (X, τ) is said to be *generalized* (Λ, p)*-closed* [9] (briefly g-(Λ, p)-closed) if $A^{(\Lambda,p)} \subseteq U$ whenever $A \subseteq U$ and $U \in \Lambda_p O(X, \tau)$.

Definition 3.4. *A topological space* (X, τ) *is called* (Λ, p)*-*$T_{\frac{1}{2}}$ *if every g-*(Λ, p)*-closed set of* X *is* (Λ, p)*-closed.*

Lemma 3.5. *For a topological space* (X, τ)*, the following properties hold:*

(1) for each $x \in X$*, the singleton* $\{x\}$ *is* (Λ, p)*-closed or* $X - \{x\}$ *is g-*(Λ, p)*-closed;*

(2) for each $x \in X$, the singleton $\{x\}$ is (Λ, p)-open or $X - \{x\}$ is a $\circledast\Lambda_{(\Lambda,p)}$-set.

Proof. (1) Let $x \in X$ and the singleton $\{x\}$ be not (Λ, p)-closed. Then, we have $X - \{x\}$ is not (Λ, p)-open and X is the only (Λ, p)-open set which contains $X - \{x\}$ and hence $X - \{x\}$ is g-(Λ, p)-closed.

(2) Let $x \in X$ and the singleton $\{x\}$ be not (Λ, p)-open. Then, we have $X - \{x\}$ is not (Λ, p)-closed and the only (Λ, p)-closed set which contains $X - \{x\}$ is X and hence $X - \{x\}$ is a $\circledast\Lambda_{(\Lambda,p)}$-set. □

Theorem 3.6. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, p)-$T_{\frac{1}{2}}$;

(2) for each $x \in X$, the singleton $\{x\}$ is (Λ, p)-open or (Λ, p)-closed;

(3) every $\circledast\Lambda_{(\Lambda,p)}$-set is a $\Lambda_{(\Lambda,p)}$-set.

Proof. (1) ⇒ (2): By Lemma 3.5, for each $x \in X$, the singleton $\{x\}$ is (Λ, p)-closed or $X - \{x\}$ is g-(Λ, p)-closed. Since (X, τ) is a (Λ, p)-$T_{\frac{1}{2}}$-space, $X - \{x\}$ is (Λ, p)-closed and hence $\{x\}$ is (Λ, p)-open in the latter case. Therefore, the singleton $\{x\}$ is (Λ, p)-open or (Λ, p)-closed.

(2) ⇒ (3): Suppose that there exists a $\circledast\Lambda_{(\Lambda,p)}$-set A which is not a $\Lambda_{(\Lambda,p)}$-set. There exists $x \in \Lambda_{(\Lambda,p)}(A)$ such that $x \notin A$. In case the singleton $\{x\}$ is (Λ, p)-open, $A \subseteq X - \{x\}$ and $X - \{x\}$ is (Λ, p)-closed. Since A is a $\circledast\Lambda_{(\Lambda,p)}$-set, $\Lambda_{(\Lambda,p)}(A) \subseteq X - \{x\}$. This is a contradiction. In case the singleton $\{x\}$ is (Λ, p)-closed, $A \subseteq X - \{x\}$ and $X - \{x\}$ is (Λ, p)-open. By Lemma 3.2, $\Lambda_{(\Lambda,p)}(A) \subseteq \Lambda_{(\Lambda,p)}(X - \{x\}) = X - \{x\}$. This is a contradiction. Thus, every $\circledast\Lambda_{(\Lambda,p)}$-set is a $\Lambda_{(\Lambda,p)}$-set.

(3) ⇒ (1): Suppose that (X, τ) is not a (Λ, p)-$T_{\frac{1}{2}}$-space. Then, there exists a g-(Λ, p)-closed set A which is not (Λ, p)-closed. Since A is not (Λ, p)-closed, there exists $x \in A^{(\Lambda,p)}$ such that $x \notin A$. By Lemma 3.5, the singleton $\{x\}$ is (Λ, p)-open or $X - \{x\}$ is a $\circledast\Lambda_{(\Lambda,p)}$-set. (a) In case $\{x\}$ is (Λ, p)-open, since $x \in A^{(\Lambda,p)}$, $\{x\} \cap A \neq \emptyset$ and $x \in A$. This is a contradiction. (b) In case $X - \{x\}$ is a $\circledast\Lambda_{(\Lambda,p)}$-set, if $\{x\}$ is not (Λ, p)-closed, $X - \{x\}$ is not (Λ, p)-open and $\Lambda_{(\Lambda,p)}[X - \{x\}] = X$. Hence, $X - \{x\}$ is not a $\circledast\Lambda_{(\Lambda,p)}$-set. This contradicts (3). If $\{x\}$ is (Λ, p)-closed, $A \subseteq X - \{x\} \in \Lambda_p O(X, \tau)$ and A is g-(Λ, p)-closed. Thus, we have $A^{(\Lambda,p)} \subseteq X - \{x\}$. This contradicts that $x \in A^{(\Lambda,p)}$. Therefore, (X, τ) is (Λ, p)-$T_{\frac{1}{2}}$. □

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