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Properties of (Λ, p)-normal spaces

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Abstract

In this paper, we deal with the concept of (Λ, p)-normal spaces. In particular, we investigate some properties of (Λ, p)-normal spaces.

1 Introduction

It is well known that various types of separation axioms play a significant role in the theory of classical point set topology. In the literature, separation axioms have been studied by many mathematicians. Viglino [10] introduced the notion of seminormal spaces. Singal and Arya [8] introduced the class of almost normal spaces and proved that a space is normal if and only if it is both a seminormal space and an almost normal space. Paul and Bhattacharyya [7] introduced and studied the notion of p-normal spaces. Maheshwari and Prasad [5] introduced and investigated the concept of s-normal spaces. Ekici [3] introduced a new class of spaces, called γ-normal spaces and investigated the relationships among s-normal spaces, p-normal spaces and γ-normal spaces. Ekici and Noiri [2] introduced and studied the notions of δp-normal spaces, almost δp-normal spaces and mildly δp-normal spaces.

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Torton et al. [9] introduced and investigated the concepts of $\mu_{(m,n)}$-regular spaces and $\mu_{(m,n)}$-normal spaces. Mashhour et al. [6] introduced and studied the concepts of preopen and preclosed sets. Ganster et al. [4] introduced the notions of pre-Λ-sets and pre-V-sets in topological spaces and investigated the fundamental properties of pre-Λ-sets and pre-V-sets. In [1], the present authors introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets which are defined by utilizing the notions of Λ_p-sets and preclosed sets. In this paper, we introduce the concept of (Λ, p)-normal spaces. Moreover, we discuss some properties of (Λ, p)-normal spaces.

2 Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [6] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [4] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [1] (*pre-*Λ-*set* [4]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [1] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [1] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets contained in A is called the (Λ, p)-*interior* [1] of A and is denoted by $A_{(\Lambda,p)}$.

Lemma 2.1. [1] *For subsets A, B of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq A^{(\Lambda,p)}$ *and* $[A^{(\Lambda,p)}]^{(\Lambda,p)} = A^{(\Lambda,p)}$.

(2) If $A \subseteq B$, *then* $A^{(\Lambda,p)} \subseteq B^{(\Lambda,p)}$.

(3) $A^{(\Lambda,p)} = \cap\{F | A \subseteq F$ *and* F *is* (Λ, p)-*closed*$\}$.

(4) $A^{(\Lambda,p)}$ *is* (Λ, p)-*closed.*

(5) A is (Λ, p)-closed if and only if $A = A^{(\Lambda,p)}$.

Lemma 2.2. [1] *Let A and B be subsets of a topological space (X, τ). For the (Λ, p)-interior, the following properties hold:*

(1) $A_{(\Lambda,p)} \subseteq A$ and $[A_{(\Lambda,p)}]_{(\Lambda,p)} = A_{(\Lambda,p)}$.

(2) If $A \subseteq B$, then $A_{(\Lambda,p)} \subseteq B_{(\Lambda,p)}$.

(3) $A_{(\Lambda,p)}$ is (Λ, p)-open.

(4) A is (Λ, p)-open if and only if $A_{(\Lambda,p)} = A$.

(5) $[X - A]^{(\Lambda,p)} = X - A_{(\Lambda,p)}$.

3 Properties of (Λ, p)-normal spaces

In this section, we introduce the concept of (Λ, p)-normal spaces. Moreover, we discuss several properties of (Λ, p)-normal spaces.

Recall that a subset A of a topological space (X, τ) is said to be *generalized (Λ, p)-closed* [11] (briefly g-(Λ, p)-closed) if $A^{(\Lambda,p)} \subseteq U$ whenever $A \subseteq U$ and $U \in \Lambda_p O(X, \tau)$. The complement of a generalized (Λ, p)-closed set is said to be *generalized (Λ, p)-open* (briefly g-(Λ, p)-open).

Lemma 3.1. [11] *A subset A of a topological space (X, τ) is g-(Λ, p)-open if and only if $F \subseteq A_{(\Lambda,p)}$ whenever $F \subseteq A$ and F is (Λ, p)-closed.*

Definition 3.2. *A topological space (X, τ) is said to be (Λ, p)-normal if, for any pair of disjoint (Λ, p)-closed sets F_1 and F_2, there exist disjoint (Λ, p)-open sets U_1 and U_2 such that $F_1 \subseteq U_1$ and $F_2 \subseteq U_2$.*

Theorem 3.3. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, p)-normal;

(2) for any pair of disjoint (Λ, p)-closed sets F_1 and F_2, there exist disjoint g-(Λ, p)-open sets U_1 and U_2 such that $F_1 \subseteq U_1$ and $F_2 \subseteq U_2$;

(3) for each (Λ, p)-closed set F and each (Λ, p)-open set G containing F, there exists a g-(Λ, p)-open set U such that $F \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G$;

(4) *for each* (Λ, p)*-closed set* F *and each g-*(Λ, p)*-open set* G *containing* F*, there exists a* (Λ, p)*-open set* U *such that* $F \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G_{(\Lambda,p)}$*;*

(5) *for each* (Λ, p)*-closed set* F *and each g-*(Λ, p)*-open set* G *containing* F*, there exists a g-*(Λ, p)*-open set* U *such that* $F \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G_{(\Lambda,p)}$*;*

(6) *for each g-*(Λ, p)*-closed set* F *and each* (Λ, p)*-open set* G *containing* F*, there exists a* (Λ, p)*-open set* U *such that* $F^{(\Lambda,p)} \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G$*;*

(7) *for each g-*(Λ, p)*-closed set* F *and each* (Λ, p)*-open set* G *containing* F*, there exists a g-*(Λ, p)*-open set* U *such that* $F^{(\Lambda,p)} \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G$*.*

Proof. (1) ⇒ (2): The proof is obvious.

(2) ⇒ (3): Let F be a (Λ, p)-closed set and G be a (Λ, p)-open set containing F. Then, F and $X-G$ are two disjoint (Λ, p)-closed sets. By (2), there exist disjoint g-(Λ, p)-open sets U and V such that $F \subseteq U$ and $X-G \subseteq V$. Since V is g-(Λ, p)-open and $X-G$ is (Λ, p)-closed, by Lemma 3.1, $X-G \subseteq V_{(\Lambda,p)}$. Since $U \cap V = \emptyset$, we have $U^{(\Lambda,p)} \subseteq [X-V]^{(\Lambda,p)} = X - V_{(\Lambda,p)} \subseteq G$. Thus, $F \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G$.

(3) ⇒ (1): Let F_1 and F_2 be any two disjoint (Λ, p)-closed sets. Then, we have $X - F_2$ is a (Λ, p)-open set containing F_1. Thus, by (3), there exists a g-(Λ, p)-open set U such that $F_1 \subseteq U \subseteq U^{(\Lambda,p)} \subseteq X - F_2$ and hence $F_2 \subseteq X - U^{(\Lambda,p)}$. Since F_1 is (Λ, p)-closed and U is g-(Λ, p)-open, by Lemma 3.1, we have $F_1 \subseteq U_{(\Lambda,p)}$. This shows that (X, τ) is (Λ, p)-normal.

(6) ⇒ (7) ⇒ (3): This is obvious.

(3) ⇒ (5): Let F be a (Λ, p)-closed set and G be a g-(Λ, p)-open set containing F. Since G is g-(Λ, p)-open and F is (Λ, p)-closed, by Lemma 3.1, $F \subseteq G_{(\Lambda,p)}$. Thus by (3), there exists a g-(Λ, p)-open set U such that $F \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G_{(\Lambda,p)}$.

(5) ⇒ (6): Let F be a g-(Λ, p)-closed set and G be a (Λ, p)-open set containing F. Then, we have $F^{(\Lambda,p)} \subseteq G$. Since G is g-(Λ, p)-open, by (5), there exists a g-(Λ, p)-open set U such that $F^{(\Lambda,p)} \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G$. Since U is g-(Λ, p)-open and $F^{(\Lambda,p)}$ is (Λ, p)-closed, by Lemma 3.1, $F^{(\Lambda,p)} \subseteq U_{(\Lambda,p)}$. Put $V = U_{(\Lambda,p)}$. Then, we have V is (Λ, p)-open and $F^{(\Lambda,p)} \subseteq V \subseteq V^{(\Lambda,p)} = [U_{(\Lambda,p)}]^{(\Lambda,p)} \subseteq U^{(\Lambda,p)} \subseteq G$.

(4) ⇒ (5) ⇒ (2): This is obvious.

(6) ⇒ (4): Let F be a (Λ, p)-closed set and G be a g-(Λ, p)-open set containing F. By Lemma 3.1, $F \subseteq G_{(\Lambda,p)}$. Since F is g-(Λ, p)-closed and $G_{(\Lambda,p)}$ is (Λ, p)-open, by (6), there exists a (Λ, p)-open set U such that $F = F^{(\Lambda,p)} \subseteq U \subseteq U^{(\Lambda,p)} \subseteq G_{(\Lambda,p)}$. □

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