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Almost (Λ, p)-continuous functions

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Abstract

We introduce and study a new class of functions, called almost (Λ, p)-continuous functions. Moreover, we investigate some characterizations and several properties concerning almost (Λ, p)-continuous functions.

1 Introduction

In 1966, Husain [3] introduced the concept of almost continuous functions and investigated some of their characterizations. In 1968, Singal and Singal [6] have also introduced the concept, similarly called almost-continuous functions, which is in fact different somehow from that of Husain. In 1982, Mashhour et al. [4] introduced and investigated the concepts of preopen sets and precontinuous functions. In 1986, Noiri [5] showed that precontinuity is equivalent to almost continuity and obtained several characterizations of almost-continuity and almost continuity. In 2002, Ganster et al. [2] introduced the notions of pre-Λ-sets and pre-V-sets in topological spaces and investigated the fundamental properties of pre-Λ-sets and pre-V-sets. In [1], the authors introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets

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which are defined by utilizing the notions of Λ_p-sets and preclosed sets. Moreover, several characterizations of weakly (Λ, p)-continuous functions were investigated in [1]. The purpose of the present paper is to introduce the concept of almost (Λ, p)-continuous functions. In particular, we discuss several characterizations of almost (Λ, p)-continuous functions.

2 Preliminaries

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean a topological space on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [4] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [2] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [1] (*pre*-Λ-*set* [2]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [1] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [1] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets contained in A is called the (Λ, p)-*interior* [1] of A and is denoted by $A_{(\Lambda,p)}$. A subset A of a topological space (X, τ) is said to be $s(\Lambda, p)$-*open* (resp. $p(\Lambda, p)$-*open*, $\beta(\Lambda, p)$-*open*, $r(\Lambda, p)$-*open*) [1] if $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$ (resp. $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A \subseteq [[A^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}$, $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$). The complement of a $s(\Lambda, p)$-open (resp. $p(\Lambda, p)$-open, $\beta(\Lambda, p)$-open, $r(\Lambda, p)$-open) set is called $s(\Lambda, p)$-*closed* (resp. $p(\Lambda, p)$-*closed*, $\beta(\Lambda, p)$-*closed*, $r(\Lambda, p)$-*closed*).

3 Characterizations of almost (Λ, p)-continuous functions

We begin this section by introducing the concept of almost (Λ, p)-continuous functions.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost (Λ, p)-continuous at a point $x \in X$ if, for each (Λ, p)-open set V containing $f(x)$, there exists a (Λ, p)-open set U containing x such that $f(U) \subseteq [V^{(\Lambda,p)}]_{(\Lambda,p)}$. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost (Λ, p)-continuous if f has this property at each point $x \in X$.*

Theorem 3.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is almost (Λ, p)-continuous at $x \in X$;

(2) $x \in [f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}$ for every (Λ, p)-open set V containing $f(x)$;

(3) $x \in [f^{-1}(V)]_{(\Lambda,p)}$ for every $r(\Lambda, p)$-open set V containing $f(x)$;

(4) for every $r(\Lambda, p)$-open set V containing $f(x)$, there exists a (Λ, p)-open set U containing x such that $f(U) \subseteq V$.

Proof. (1) $\Rightarrow$ (2): Let V be any (Λ, p)-open set of Y containing $f(x)$. Then, there exists a (Λ, p)-open set U of X containing x such that $f(U) \subseteq [V^{(\Lambda,p)}]_{(\Lambda,p)}$. Thus, $x \in U \subseteq f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})$. Since U is (Λ, p)-open, $x \in [f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}$.

(2) $\Rightarrow$ (3): Let V be any $r(\Lambda, p)$-open set of Y containing $f(x)$. Since $V = [V^{(\Lambda,p)}]_{(\Lambda,p)}$, by (2), $x \in [f^{-1}(V)]_{(\Lambda,p)}$

(3) $\Rightarrow$ (4): Let V be any $r(\Lambda, p)$-open set of Y containing $f(x)$. By (3), there exists a (Λ, p)-open set U containing x such that $U \subseteq f^{-1}(V)$ and hence $f(U) \subseteq V$.

(4) $\Rightarrow$ (1): Let V be any (Λ, p)-open set of Y containing $f(x)$. Then, $f(x) \in V \subseteq [V^{(\Lambda,p)}]_{(\Lambda,p)}$. Since $[V^{(\Lambda,p)}]_{(\Lambda,p)}$ is $r(\Lambda, p)$-open, by (4), there exists a (Λ, p)-open set U containing x such that $f(U) \subseteq [V^{(\Lambda,p)}]_{(\Lambda,p)}$. This shows that f is almost (Λ, p)-continuous. □

Theorem 3.3. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is almost (Λ, p)-continuous;

(2) $f^{-1}(V) \subseteq [f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}$ for every (Λ, p)-open set V of Y;

(3) $[f^{-1}([F_{(\Lambda,p)}]^{(\Lambda,p)})]^{(\Lambda,p)} \subseteq f^{-1}(F)$ for every (Λ, p)-closed set F of Y;

(4) $[f^{-1}([[B^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{(\Lambda,p)} \subseteq f^{-1}(B^{(\Lambda,p)})$ for every subset B of Y;

(5) $f^{-1}(B_{(\Lambda,p)}) \subseteq [f^{-1}([[B_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}$ *for every subset* B *of* Y*;*

(6) $f^{-1}(V)$ *is* (Λ,p)*-open in* X *for every* $r(\Lambda,p)$*-open set* V *of* Y*;*

(7) $f^{-1}(F)$ *is* (Λ,p)*-closed in* X *for every* $r(\Lambda,p)$*-closed set* F *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let V be any (Λ,p)-open set of Y and $x \in f^{-1}(V)$. There exists a (Λ,p)-open set U of X containing x such that $f(U) \subseteq [V^{(\Lambda,p)}]_{(\Lambda,p)}$. Thus, $x \in [f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}$ and hence $f^{-1}(V) \subseteq [f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}$.

(2) $\Rightarrow$ (3): Let F be any (Λ,p)-closed set of Y. Then, $F_{(\Lambda,p)}$ is (Λ,p)-open in Y, by (2), we have $f^{-1}(Y-F) \subseteq [f^{-1}([[Y-F]^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)} = [f^{-1}(Y-[F_{(\Lambda,p)}]^{(\Lambda,p)})]_{(\Lambda,p)} = X - [f^{-1}([F_{(\Lambda,p)}]^{(\Lambda,p)})]^{(\Lambda,p)}$ and hence

$$[f^{-1}([F_{(\Lambda,p)}]^{(\Lambda,p)})]^{(\Lambda,p)} \subseteq f^{-1}(F).$$

(3) $\Rightarrow$ (4): Let B be any subset of Y. Since $B^{(\Lambda,p)}$ is (Λ,p)-closed, by (3), $[f^{-1}([[B^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{(\Lambda,p)} \subseteq f^{-1}(B^{(\Lambda,p)})$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4),

$$\begin{aligned} f^{-1}(B_{(\Lambda,p)}) &= X - f^{-1}([Y-B]^{(\Lambda,p)}) \\ &\subseteq X - [f^{-1}([[Y-B]^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{(\Lambda,p)} \\ &= [f^{-1}([[B_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}. \end{aligned}$$

(5) $\Rightarrow$ (6): Let V be any $r(\Lambda,p)$-open set of Y. Since $[[V_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)} = V$, by (5), $f^{-1}(V) \subseteq [f^{-1}(V)]_{(\Lambda,p)}$. Thus, $f^{-1}(V)$ is (Λ,p)-open in X.

(6) $\Rightarrow$ (7): The proof is obvious.

(7) $\Rightarrow$ (1): Let V be any $r(\Lambda,p)$-open set of Y containing $f(x)$. By (7), $X - f^{-1}(V) = f^{-1}(Y-V) = [f^{-1}(Y-V)]^{(\Lambda,p)} = X - [f^{-1}(V)]_{(\Lambda,p)}$. Since $x \in f^{-1}(V) = [f^{-1}(V)]_{(\Lambda,p)}$, there exists a (Λ,p)-open set U of X containing x such that $U \subseteq f^{-1}(V)$; hence $f(U) \subseteq V$. Thus, by Theorem 3.2, f is almost (Λ,p)-continuous. □

Theorem 3.4. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is almost* (Λ,p)*-continuous;*

(2) $[f^{-1}(U)]^{(\Lambda,p)} \subseteq f^{-1}(U^{(\Lambda,p)})$ *for every* $\beta(\Lambda,p)$*-open set* U *of* Y*;*

(3) $[f^{-1}(U)]^{(\Lambda,p)} \subseteq f^{-1}(U^{(\Lambda,p)})$ *for every* $s(\Lambda,p)$*-open set* U *of* Y*;*

(4) $f^{-1}(U) \subseteq [f^{-1}([U^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}$ *for every* $p(\Lambda,p)$*-open set* U *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let U be any $\beta(\Lambda, p)$-open set of Y. Since $U^{(\Lambda,p)}$ is $r(\Lambda, p)$-closed, by Theorem 3.3, $[f^{-1}(U^{(\Lambda,p)})]^{(\Lambda,p)} = f^{-1}(U^{(\Lambda,p)})$. Thus,

$$[f^{-1}(U)]^{(\Lambda,p)} \subseteq [f^{-1}(U^{(\Lambda,p)})]^{(\Lambda,p)} = f^{-1}(U^{(\Lambda,p)}).$$

(2) $\Rightarrow$ (3): The proof is obvious.

(3) $\Rightarrow$ (1): Let F be any $r(\Lambda, p)$-closed set of Y. Then, since F is $s(\Lambda, p)$-open, $[f^{-1}(F)]^{(\Lambda,p)} \subseteq f^{-1}(F^{(\Lambda,p)}) = f^{-1}(F)$. Thus, by Theorem 3.3, f is almost (Λ, p)-continuous.

(1) $\Rightarrow$ (4): Let U be any $p(\Lambda, p)$-open set of Y. Then, $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$ and hence $[U^{(\Lambda,p)}]_{(\Lambda,p)}$ is $r(\Lambda, p)$-open. By Theorem 3.3, $f^{-1}([U^{(\Lambda,p)}]_{(\Lambda,p)}) = [f^{-1}([U^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}$. Thus,

$$f^{-1}(U) \subseteq f^{-1}([U^{(\Lambda,p)}]_{(\Lambda,p)}) = [f^{-1}([U^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)}.$$

(4) $\Rightarrow$ (1): Let U be any $r(\Lambda, p)$-open set of Y. Then, U is $p(\Lambda, p)$-open and $f^{-1}(U) \subseteq [f^{-1}([U^{(\Lambda,p)}]_{(\Lambda,p)})]_{(\Lambda,p)} = [f^{-1}(U)]_{(\Lambda,p)}$. Thus, by Theorem 3.3, f is almost (Λ, p)-continuous. □

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