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Characterizations of (Λ, p)-hyperconnected spaces

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Abstract

In this paper, we deal with with the concept of (Λ, p)-hyperconnected spaces. Moreover, we investigate several characterizations of (Λ, p)-hyperconnected spaces.

1 Introduction

In 1970, Steen and Seebach [8] introduced the concept of hyperconnected spaces. Several concepts which are equivalent to hyperconnectedness were defined and investigated in the literature. Levine [3] called a topological space (X, τ) a D-space if every nonempty open set of X is dense in X and showed that (X, τ) is a D-space if and only if it is hyperconnected. Pipitone and Russo [6] defined a topological space (X, τ) to be semi-connected if X is not the union of two disjoint nonempty semi-open sets of X and showed that (X, τ) is semi-connected if and only if it is a D-space. Noiri [5] investigated some properties of hyperconnected spaces by using semi-preopen sets and almost feebly continuous functions. In 1982, Mashhour et al. [4] introduced

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and investigated the concepts of preopen sets and preclosed sets. In 2002, Ganster et al. [2] introduced the notions of a pre-Λ-set and a pre-V-set in topological spaces and investigated the fundamental properties of pre-Λ-sets and pre-V-sets. Quite recently, Boonpok and Viriyapong [1] introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets which are defined by utilizing the notions of Λ_p-sets and preclosed sets. In this paper, we introduce the concept of (Λ, p)-hyperconnected spaces. Moreover, some properties of (Λ, p)-hyperconnected spaces are discussed.

2 Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [4] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [2] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a *Λ_p-set* [1] (*pre-Λ-set* [2]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called *(Λ, p)-closed* if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called *(Λ, p)-open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_pO(X, \tau)$ (resp. $\Lambda_pC(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, p)-cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the *(Λ, p)-closure* [1] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets contained in A is called the *(Λ, p)-interior* [1] of A and is denoted by $A_{(\Lambda,p)}$.

Lemma 2.1. [1] *For subsets A, B of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq A^{(\Lambda,p)}$ and $[A^{(\Lambda,p)}]^{(\Lambda,p)} = A^{(\Lambda,p)}$.

(2) If $A \subseteq B$, then $A^{(\Lambda,p)} \subseteq B^{(\Lambda,p)}$.

(3) $A^{(\Lambda,p)} = \cap\{F | A \subseteq F$ and F is (Λ, p)-closed$\}$.

(4) $A^{(\Lambda,p)}$ is (Λ, p)-closed.

(5) A is (Λ, p)-closed if and only if $A = A^{(\Lambda,p)}$.

Lemma 2.2. [1] *Let A and B be subsets of a topological space (X, τ). For the (Λ, p)-interior, the following properties hold:*

(1) $A_{(\Lambda,p)} \subseteq A$ *and* $[A_{(\Lambda,p)}]_{(\Lambda,p)} = A_{(\Lambda,p)}$.

(2) If $A \subseteq B$, *then* $A_{(\Lambda,p)} \subseteq B_{(\Lambda,p)}$.

(3) $A_{(\Lambda,p)}$ *is* (Λ, p)*-open.*

(4) A *is* (Λ, p)*-open if and only if* $A_{(\Lambda,p)} = A$.

(5) $[X - A]^{(\Lambda,p)} = X - A_{(\Lambda,p)}$.

A subset A of a topological space (X, τ) is said to be $s(\Lambda, p)$*-open* (resp. $\beta(\Lambda, p)$*-open*) [1] if $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$ (resp. $A \subseteq [[A^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}$).

3 Characterizations of (Λ, p)-hyperconnected spaces

In this section, we introduce the notion of (Λ, p)-hyperconnected spaces. Moreover, several characterizations of (Λ, p)-hyperconnected spaces are discussed.

Recall that a subset A of a topological space (X, τ) is said to be (Λ, p)*-dense* [7] if $A^{(\Lambda,p)} = X$.

Definition 3.1. *A topological space (X, τ) is called (Λ, p)-hyperconnected if U is (Λ, p)-dense for every nonempty (Λ, p)-open set U of X.*

Definition 3.2. *A subset N of a topological space (X, τ) is said to be (Λ, p)-nowhere dense if $[N^{(\Lambda,p)}]_{(\Lambda,p)} = \emptyset$.*

Lemma 3.3. *A subset A of a topological space (X, τ) is $s(\Lambda, p)$-open if and only if there exists a (Λ, p)-open set U such that $U \subseteq A \subseteq U^{(\Lambda,p)}$.*

Proof. Suppose that A is $s(\Lambda, p)$-open. Then, we have $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$. Put $U = A_{(\Lambda,p)}$. Then, U is a $s(\Lambda, p)$-open set such that $U \subseteq A \subseteq U^{(\Lambda,p)}$.

Conversely, suppose that there exists a (Λ, p)-open set U such that $U \subseteq A \subseteq U^{(\Lambda,p)}$. Then, $U \subseteq A_{(\Lambda,p)}$ and hence $U^{(\Lambda,p)} \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$. Since $A \subseteq U^{(\Lambda,p)}$, we have $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$. This shows that A is $s(\Lambda, p)$-open. □

Theorem 3.4. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) *is* (Λ, p)*-hyperconnected.*

(2) A *is* (Λ, p)*-dense or* (Λ, p)*-nowhere dense for every subset* A *of* X*.*

(3) $U \cap V \neq \emptyset$ *for every nonempty* (Λ, p)*-open sets* U *and* V *of* X*.*

(4) $U \cap V \neq \emptyset$ *for every nonempty* $s(\Lambda, p)$*-open sets* U *and* V *of* X*.*

Proof. (1) $\Rightarrow$ (2): Suppose that A is not (Λ, p)-nowhere dense. Then, $[A^{(\Lambda,p)}]_{(\Lambda,p)} \neq \emptyset$. Since (X, τ) is (Λ, p)-hyperconnected, $[[A^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)} = X$ and hence $X \subseteq A^{(\Lambda,p)}$. This shows that $A^{(\Lambda,p)} = X$. Thus, A is (Λ, p)-dense.

(2) $\Rightarrow$ (3): Suppose that $U \cap V = \emptyset$ for some nonempty (Λ, p)-open sets U and V of X. Then, we have $U^{(\Lambda,p)} \cap V = \emptyset$ and hence U is not (Λ, p)-dense. Since U is (Λ, p)-open, $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$. Thus, U is not (Λ, p)-nowhere dense.

(3) $\Rightarrow$ (4): Suppose that $U \cap V = \emptyset$ for some nonempty $s(\Lambda, p)$-open sets U and V of X. By Lemma 3.3, there exist $G, W \in \Lambda_p O(X, \tau)$ such that $G \subseteq U \subseteq G^{(\Lambda,p)}$ and $W \subseteq V \subseteq W^{(\Lambda,p)}$. Since U and V are nonempty, G and W are nonempty. Moreover, we have $G \cap W \subseteq U \cap V = \emptyset$.

(4) $\Rightarrow$ (1): Suppose that (X, τ) is not (Λ, p)-hyperconnected. There exists a nonempty (Λ, p)-open set G of X such that $G^{(\Lambda,p)} \neq X$. Thus, $X - G^{(\Lambda,p)} \neq \emptyset$ and hence $[X - G^{(\Lambda,p)}] \cap G = \emptyset$. This is a contradiction. □

Theorem 3.5. *For a topological space* (X, τ)*, the following properties are equivalent:*

(1) (X, τ) *is* (Λ, p)*-hyperconnected.*

(2) V *is* (Λ, p)*-dense for every nonempty* $\beta(\Lambda, p)$*-open set* V *of* X*.*

(3) $V \cup [V^{(\Lambda,p)}]_{(\Lambda,p)} = X$ *for every nonempty* $\beta(\Lambda, p)$*-open set* V *of* X*.*

Proof. (1) $\Rightarrow$ (2): Suppose that (X, τ) is (Λ, p)-hyperconnected. Let V be a nonempty $\beta(\Lambda, p)$-open set. It follows that $[V^{(\Lambda,p)}]_{(\Lambda,p)} \neq \emptyset$ and hence $X = [V^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)} = V^{(\Lambda,p)}$. Thus, V is (Λ, p)-dense.

(2) $\Rightarrow$ (3): Let V be a nonempty $\beta(\Lambda, p)$-open set. By (2), $V \cup [V^{(\Lambda,p)}]_{(\Lambda,p)} = V \cup X_{(\Lambda,p)} = X$.

(3) $\Rightarrow$ (1): Let V be a nonempty (Λ, p)-open set. It follows (3) that $V^{(\Lambda,p)} \supseteq V \cup [V^{(\Lambda,p)}]_{(\Lambda,p)} = X$ and hence $V^{(\Lambda,p)} = X$. This shows that (X, τ) is (Λ, p)-hyperconnected. □

Theorem 3.6. *For a topological space* (X, τ)*, the following properties are equivalent:*

(1) (X, τ) is (Λ, p)-hyperconnected.

(2) V is (Λ, p)-dense for every nonempty $s(\Lambda, p)$-open set V of X.

(3) $V \cup [V^{(\Lambda,p)}]_{(\Lambda,p)} = X$ for every nonempty $s(\Lambda, p)$-open set V of X.

Proof. The proof follows from Theorem 3.5. □

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References

[1] C. Boonpok, C. Viriyapong, On (Λ, p)-closed sets and the related notions in topological spaces, Eur. J. Pure Appl. Math., **15,** no. 2, (2022), 415–436.

[2] M. Ganster, S. Jafari, T. Noiri, On pre-Λ-sets and pre-V-sets, Acta Math. Hungar., **95,** (2002), 337–343.

[3] N. Levine, Dense topologies, Amer. Math. Monthly, **75,** (1968), 847–852.

[4] A. S. Mashhour, M. E. Abd El-Monsef, S. N. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, **53,** (1982), 47–53.

[5] T. Noiri, Properties of hyperconnected spaces, Acta Math. Hungar., **66,** (1995), 147–154.

[6] V. Pipitone, G. Russo, Spazi semiconnessi e apazi semiaperti, Rend. Circ. Mat. Palermo, **24,** (1975), 273–285.

[7] N. Srisarakham, C. Boonpok, Properties of locally (Λ, p)-closed sets, Int. J. Math. Comput. Sci., **18,** no. 1, (2023), 73–77.

[8] L. A. Steen, J. R. Seebach Jr., Counterexamples in Topology, Holt, Rinehart and Winston, New York, 1970.