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Some properties of (Λ, p)-regular spaces

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Abstract

We introduce the concept of (Λ, p)-regular spaces. In particular, we investigate some characterizations of (Λ, p)-regular spaces.

1 Introduction

Sinnal and Arya [7] defined a new separation axiom called almost regularity which is weaker than regularity. In 1982, Mashhour et al. [5] introduced and investigated the concepts of preopen sets and preclosed sets in topological spaces. In 1983, El-Deeb et al. [2] introduced and studied the notion of p-regular spaces by using preopen sets. In 1990, Malghan and Navalagi [4] introduced and investigated the concept of almost p-regular spaces as a generalization of p-regularity. In 1998, Noiri [6] defined a new class of sets called rgp-closed sets and investigated some characterizations of almost p-regular spaces by utilizing rgp-closed sets. In 2012, Torton et al. [8] introduced and studied the notions of $\mu_{(m,n)}$-regular spaces and $\mu_{(m,n)}$-normal spaces. Ganster et al. [3] introduced the notions of pre-Λ-sets and pre-V-sets in topological spaces and investigated the fundamental properties of pre-Λ-sets

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and pre-V-sets. Quite recently, Boonpok and Viriyapong [1] introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets which are defined by utilizing the notions of Λ_p-sets and preclosed sets. In this paper, we introduce the concept of (Λ, p)-regular spaces. Moreover, some properties of (Λ, p)-regular spaces are discussed.

2 Preliminaries

For a subset A of a topological space (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [5] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [3] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [1] (*pre*-Λ-*set* [3]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [1] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [1] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets contained in A is called the (Λ, p)-*interior* [1] of A and is denoted by $A_{(\Lambda,p)}$.

Lemma 2.1. [1] *For subsets A, B of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq A^{(\Lambda,p)}$ *and* $[A^{(\Lambda,p)}]^{(\Lambda,p)} = A^{(\Lambda,p)}$.

(2) If $A \subseteq B$, *then* $A^{(\Lambda,p)} \subseteq B^{(\Lambda,p)}$.

(3) $A^{(\Lambda,p)} = \cap\{F | A \subseteq F \text{ and } F \text{ is } (\Lambda, p)\text{-closed}\}$.

(4) $A^{(\Lambda,p)}$ *is* (Λ, p)*-closed.*

(5) A *is* (Λ, p)*-closed if and only if* $A = A^{(\Lambda,p)}$.

Lemma 2.2. [1] *Let A and B be subsets of a topological space (X, τ). For the (Λ, p)-interior, the following properties hold:*

(1) $A_{(\Lambda,p)} \subseteq A$ *and* $[A_{(\Lambda,p)}]_{(\Lambda,p)} = A_{(\Lambda,p)}$.

(2) *If* $A \subseteq B$*, then* $A_{(\Lambda,p)} \subseteq B_{(\Lambda,p)}$.

(3) $A_{(\Lambda,p)}$ *is* (Λ, p)*-open.*

(4) A *is* (Λ, p)*-open if and only if* $A_{(\Lambda,p)} = A$.

(5) $[X - A]^{(\Lambda,p)} = X - A_{(\Lambda,p)}$.

3 Some properties of (Λ, p)-regular spaces

In this section, we introduce the concept of (Λ, p)-regular spaces. Moreover, some properties of (Λ, p)-regular spaces are discussed.

Definition 3.1. [9] *A subset* A *of a topological space* (X, τ) *is said to be generalized* (Λ, p)*-closed (briefly g-*(Λ, p)*-closed) if* $A^{(\Lambda,p)} \subseteq U$ *whenever* $A \subseteq U$ *and* $U \in \Lambda_p O(X, \tau)$*. The complement of a generalized* (Λ, p)*-closed set is said to be generalized* (Λ, p)*-open (briefly g-*(Λ, p)*-open).*

Lemma 3.2. [9] *A subset* A *of a topological space* (X, τ) *is g-*(Λ, p)*-open if and only if* $F \subseteq A_{(\Lambda,p)}$ *whenever* $F \subseteq A$ *and* F *is* (Λ, p)*-closed.*

Definition 3.3. *A topological space* (X, τ) *is said to be* (Λ, p)*-regular if, for each* (Λ, p)*-closed set* F *and each point* $x \in X - F$*, there exist disjoint* (Λ, p)*-open sets* U *and* V *such that* $x \in U$ *and* $F \subseteq V$.

Theorem 3.4. *For a topological space* (X, τ)*, the following properties are equivalent:*

(1) (X, τ) *is* (Λ, p)*-regular.*

(2) *For each* $x \in X$ *and each* $U \in \Lambda_p O(X, \tau)$ *with* $x \in U$*, there exists* $V \in \Lambda_p O(X, \tau)$ *such that* $x \in V \subseteq V^{(\Lambda,p)} \subseteq U$.

(3) *For each* (Λ, p)*-closed set* F *of* X*,* $\cap\{V^{(\Lambda,p)} \mid F \subseteq V \in \Lambda_p O(X, \tau)\} = F$.

(4) *For each subset* A *of* X *and each* $U \in \Lambda_p O(X, \tau)$ *with* $A \cap U \neq \emptyset$*, there exists* $V \in \Lambda_p O(X, \tau)$ *such that* $A \cap V \neq \emptyset$ *and* $V^{(\Lambda,p)} \subseteq U$.

(5) *For each nonempty subset* A *of* X *and each* (Λ, p)*-closed set* F *of* X *with* $A \cap F = \emptyset$*, there exist* $V, W \in \Lambda_p O(X, \tau)$ *such that* $A \cap V \neq \emptyset$*,* $F \subseteq W$ *and* $V \cap W = \emptyset$.

(6) For each (Λ,p)-closed set F of X and $x \notin F$, there exist $U \in \Lambda_pO(X,\tau)$ and a g-(Λ,p)-open set V such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$.

(7) For each subset A of X and each (Λ,p)-closed set F with $A \cap F = \emptyset$, there exist $U \in \Lambda_pO(X,\tau)$ and a g-(Λ,p)-open set V such that $A \cap U \neq \emptyset$, $F \subseteq V$ and $U \cap V = \emptyset$.

Proof. (1) $\Rightarrow$ (2): Let G be a (Λ,p)-open set and $x \notin X - G$. Then, there exist disjoint (Λ,p)-open sets U and V such that $X - G \subseteq U$ and $x \in V$. Thus, $V \subseteq X - U$ and hence $x \in V \subseteq V^{(\Lambda,p)} \subseteq X - U \subseteq G$.

(2) $\Rightarrow$ (3): Let F be a (Λ,p)-closed set and $x \in X - F$. By (2), there exists $U \in \Lambda_pO(X,\tau)$ such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq X - F$. Therefore, $F \subseteq X - U^{(\Lambda,p)} = V \in \Lambda_pO(X,\tau)$ and $U \cap V = \emptyset$. Thus, $x \notin V^{(\Lambda,p)}$ and hence $F \supseteq \cap\{V^{(\Lambda,p)} \mid F \subseteq V \in \Lambda_pO(X,\tau)\}$.

(3) $\Rightarrow$ (4): Let A be a subset of X and $U \in \Lambda_pO(X,\tau)$ such that $A \cap U \neq \emptyset$. Let $x \in A \cap U$. Then, we have $x \notin X - U$. Thus, by (3), there exists $W \in \Lambda_pO(X,\tau)$ such that $X - U \subseteq W$ and $x \notin W^{(\Lambda,p)}$. Put $V = X - W^{(\Lambda,p)}$. Then, V is a (Λ,p)-open set containing x and $A \cap V \neq \emptyset$. Thus, $V \subseteq X - W$ and so $V^{(\Lambda,p)} \subseteq X - W \subseteq U$.

(4) $\Rightarrow$ (5): Let A be a nonempty subset of X and F be a (Λ,p)-closed set such that $A \cap F = \emptyset$. Then, $X - F \in \Lambda_pO(X,\tau)$ and $A \cap (X - F) \neq \emptyset$. By (4), there exists $V \in \Lambda_pO(X,\tau)$ such that $A \cap V \neq \emptyset$ and $V^{(\Lambda,p)} \subseteq X - F$. If we put $W = X - V^{(\Lambda,p)}$, then $F \subseteq W$ and $W \cap V = \emptyset$.

(5) $\Rightarrow$ (1): Let F be a (Λ,p)-closed set not containing x. Then, $F \cap \{x\} = \emptyset$. Thus, by (5), there exist $V, W \in \Lambda_pO(X,\tau)$ such that $x \in V, F \subseteq W$ and $V \cap W = \emptyset$.

(1) $\Rightarrow$ (6): The proof is obvious.

(6) $\Rightarrow$ (7): Let A be a subset of X and F be a (Λ,p)-closed set such that $A \cap F = \emptyset$. Then, for each $x \in A$, $x \notin F$ and by (6), there exist $U \in \Lambda_pO(X,\tau)$ and a g-(Λ,p)-open set V such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$. Thus, $A \cap U \neq \emptyset$, $F \subseteq V$ and $U \cap V = \emptyset$.

(7) $\Rightarrow$ (1): Let F be a (Λ,p)-closed set and $x \notin F$. Since $\{x\} \cap F = \emptyset$, by (7) there exist $U \in \Lambda_pO(X,\tau)$ and a g-(Λ,p)-open set W such that $x \in U$, $F \subseteq W$ and $U \cap W = \emptyset$. Since W is g-(Λ,p)-open, by Lemma 3.2, $F \subseteq W_{(\Lambda,p)} = V \in \Lambda_pO(X,\tau)$ and hence $U \cap V = \emptyset$. □

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