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Optimal Path on a Multigraph Network Involving Edge Weights of Multiple Parameters

Nahlia Rakhmawati[1,2], Agus Widodo[1], Noor Hidayat[1], Abdul Rouf Alghofari[1]

[1]Department of Mathematics
Brawijaya University
Malang, Indonesia

[2]Mathematics Education
STKIP PGRI Jombang.
Jombang, East Java, Indonesia

email: rakhmanahlia.stkipjb@gmail.com, prof.agus_widodo@yahoo.com

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Abstract

In this paper, we discuss the concept of using multiple parameters to find the optimal path in the multigraph. Multiple parameters indicate the number of parameters used in compiling a network. To be compiled into an edge weight of a multigraph involves converting the parameters into the same quantity without losing meaning. Here, the interval-valued intuitionistic fuzzy approach is used to perform the conversion. We will obtain the optimal multigraph path using a multiple-parameter transportation problem.

1 Introduction

The shortest path problem is always related to the parameters used to construct the network system. In the form of a graph, as mentioned in many

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studies [1, 2, 3], the shortest path obtained is often an honest answer that may not be a solution to the real problem. It is always up to the decision maker since it only involves one parameter to figure out the network as a simple graph. The multiple edges are often neglected in solving graph problems to facilitate the calculation process and parameter fitingt. However, as mentioned in [4, 5], a multigraph with multiple edges is the most common condition in existing network systems. IT Required a conversion that does not affect the meaning of each parameter so that we can still apply the solution in actual conditions. Using many parameters in preparing a network system is difficult because the parameters used have their meanings and units. In this paper, the conversion is used with an interval-valued intuitionistic fuzzy number approach.

Intuitionistic fuzzy was first introduced by Attanassov in 1986 and has been widely developed in graph theory [6, 7, 8, 9, 10]. The shortest path involving fuzzy intuitionistic has been widely developed since 2005 in [11, 12, 13, 14, 15, 16]. The use of interval-valued fuzzy intuitionistic in graphs to solve the shortest path problem has been widely discussed in [17, 18]. Interval-valued intuitionistic fuzzy is used to overcome the ambiguity and uncertainty of the data used effectively. Since 2013, Biswas [19, 20, 21] developed a multigraph on intuitionistic fuzzy to solve the communication and transportation problem. Using the intuitionistic fuzzy multigraph, we will obtain an optimal path to solve the transportation problem with three parameters: road length, travel time, and travel costs. All the parameters will be one edge weight we wish to solve as minimally as possible.

In this research, we first discuss the shortest path in a multigraph with multiple parameters in a regular graph. Here, we will talk about the main problem of why we need to use multiple parameters to solve the optimal path in a multigraph. Then we go to the intuitionistic fuzzy multigraph and how to compile it. Finally, we provide some applications in transportation problems that contain multigraph and multiple parameters.

2 Shortest Path in Multigraph with Multi Parameters

In the transportation network, we often find a problem in optimizing the shortest path that minimizes travel cost, travel time, or road length (Rarely do we find a solution that fits all the three-parameter simultaneously). It always becomes an available answer that belongs to the decision-maker to make. For example, here we will discuss the shortest path between cities 1 and 9 which has a network system, as shown in Figure 1.

Figure 1 shows that it was not a simple graph since it has double edges in some vertices. If three parameters are used to find the shortest path with quantity, as mentioned in Table 1 below, then the graph will become 3 different graphs that each parameter will solve.

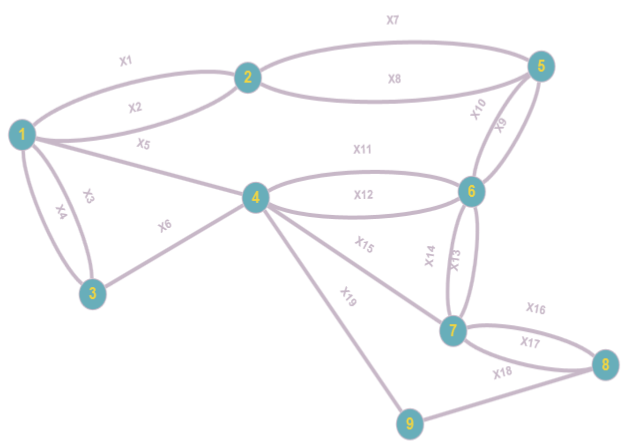


Figure 1: Multigraph of Transportation Network.

The usual algorithm (Djikstra Algorithm) will find the shortest path between any vertices before searching to find the shortest one in this transportation network. With parameter road lengths and the evaluation of its shortest length, the graph of the transportation network becomes Figure 2.

There are 7 double edges in the transportation network in Figure 1. Since the algorithm does not fit with multigraph, we do the elimination before the iteration. The elimination is done by choosing the minimum length between two vertices. For example, $X2$ was chosen because it has a minimum value of road lengths, so $X4$, $X8$, $X9$, $X12$, $X14$ and $X17$. Then the multigraph of the transportation network becomes a simple graph and can be solved by an algorithm. The shortest path between cities 1 and 9 is $X2 - X8 - X9 - X14 - X17 - X18$, with a minimum value of 227 km. This solution only fits with the parameter road length to be used.

If we change the parameter into the other two, we will have a different graph, as shown in Figure 3(a). From all those figures, we can see that different parameters will make different graphs and shortest paths. The objective function among all three parameters was the same: minimizing the value of road length, time travel, and travel cost. Nevertheless, the

Table 1: The parameter of transportation network.

Variable	Road Length (km)	Travel Time $(minutes)$	Travel Cost (Rp)
$X1$	28.81	25	24,000.00
$X2$	25.80	19	48,000.00
$X3$	75.98	65	65,000.00
$X4$	48.49	73	50,000.00
$X5$	117.65	177	122,000.00
$X6$	70.18	105	73,000.00
$X7$	55.94	48	48,000.00
$X8$	36.27	27	65,000.00
$X9$	29.56	25	25,000.00
$X10$	45.00	34	42,000.00
$X11$	66.62	57	57,000.00
$X12$	52.06	39	72,000.00
$X13$	42.69	37	36,000.00
$X14$	34.15	66	48,000.00
$X15$	44.15	66	46,000.00
$X16$	48.33	42	41,000.00
$X17$	43.70	33	62,000.00
$X18$	56.87	49	49,000.00
$X19$	132.86	114	113,000.00

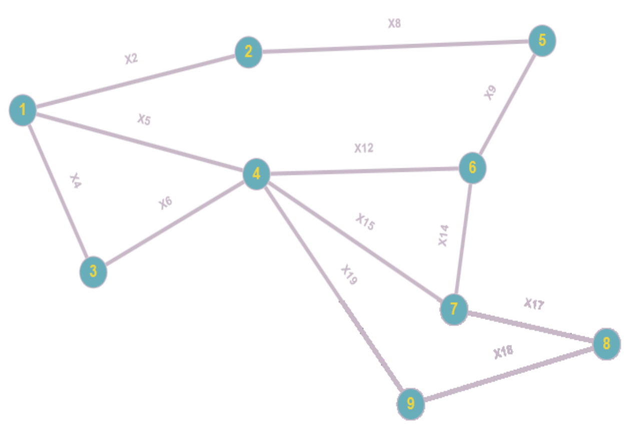
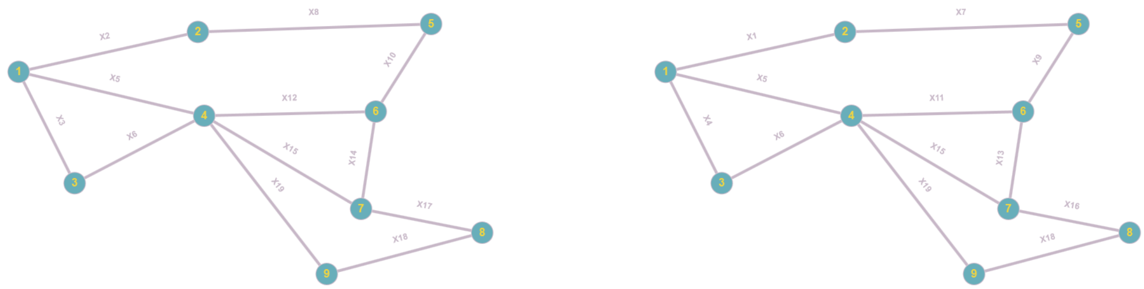


Figure 2: Transportation network based on the road Length.

shortest path differed depending on the parameter. The shortest path and the minimum value are shown in Table 2.



(a) Transportation network based on the time travel. (b) Transportation network based on the travel cost.

Figure 3: Transportation network.

Table 2: The Difference shortest path based on the parameter

Parameter	The Shortest Path	The Length of The Path	The Value of Shortest Path
Road Length	$X2 - X8 - X9 - X14 - X17 - X18$	6	227 km
Time Travel	$X2 - X8 - X10 - X14 - X17 - X18$	6	179 $minutes$
Travel Cost	$X1 - X7 - X9 - X13 - X16 - X18$	6	$Rp.223.000, 00$

In this research, we will focus on modifying the parameter to fit the edge weight of the multigraph. As shown before, even though we use the same graph but eliminate the double edge by the minimum value of its parameter, we will find a different path that is not optimal since it only appears for one parameter. We have to modify both the parameters used and the algorithm so that the multi-edges do not have to be eliminated in the beginning. We will use an intuitionistic fuzzy approach to convert those three parameters to find the optimal path in the multigraph. The interval-valued sets provide additional degrees of freedom and offer the capability to directly handle uncertainties.

3 Intuitionistic Fuzzy Multigraph

Definition 3.1. *[22] An intuitionistic fuzzy set is a fuzzy set in space so that*

$$A = \Big\{ \big(x, \mu_A(x), v_A(x)\big) | x \in E \Big\},$$

where $\mu_A : E \to [0,1]$, $v_A : E \to [0,1]$ *in which* μ_A *is the membership function that show the possibility that* x *belongs to set* A *and*

$$0 \leq \mu_A(x) + v_A(x) \leq 1.$$

Here, the pair $\big(\mu_A(x), v_A(x)\big)$ *is called the intuitionistic fuzzy number of element* x *in set* A.

Intuitionistic Fuzzy Multigraph (IFM) means that the graph is not simple and has weight in intuitionistic. In this paper, the multigraph means the graph has some vertices connected by more than one edge without any loop, as shown in Figure 1. Before we change the regular graph into an intuitionistic fuzzy graph, we will first set the data as an interval value. Then we will make three different membership and non-membership functions as mentioned in Definition 3.1.

The objective function of the three parameters is to be minimized so that the membership function must be structured accordingly. The higher the degree of membership, the smaller the parameter value, and vice versa. A trapezoidal approach facilitates fuzzification by dividing the data into three interval values.

Definition 3.2. *[12] Let* $\bar{A}$ *be an intuitionistic trapezoidal fuzzy number, and its membership function is*

$$\mu_{\bar{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}\mu_{\bar{A}}, & a \leq x \leq b; \\ \mu_{\bar{A}}, & b \leq x \leq c; \\ \dfrac{d-x}{d-c}\mu_{\bar{A}}, & c \leq x \leq d; \\ 0, & otherwise. \end{cases}$$

Its non-membership function is

$$v_{\bar{A}}(x) = \begin{cases} \dfrac{b-x+v_{\bar{A}}(x-a_1)}{b-a_1}\mu_{\bar{A}}, & a_1 \leq x \leq b; \\ v_{\bar{A}}, & b \leq x \leq c; \\ \dfrac{x-c+v_{\bar{A}}(d_1-x)}{d_1-c}\mu_{\bar{A}}, & c \leq x \leq d_1; \\ 0, & otherwise, \end{cases}$$

where $0 \leq \mu_{\bar{A}} \leq 1$, $0 \leq v_{\bar{A}} \leq 1$ *and* $\mu_{\bar{A}} + v_{\bar{A}} \leq 1$. *The interval* a, b, c, d *is the same. Then we call* $\langle \mu_{\bar{A}}(x), v_{\bar{A}}(x)$ *rangle the Intuitionistic Fuzzy Number* (IFN).

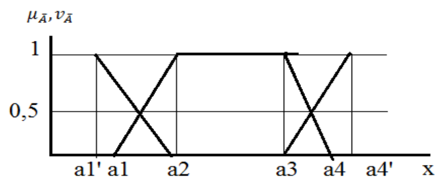


Figure 4: The membership and non-membership function of IFN.

Definition 3.3. *[22] An interval valued intuitionistic fuzzy set A (over a basic set E) is defined by: the membership function* $(M_A : E \to INT([0,1])$*, the non-membership function* $(N_A : E \to INT([0,1]))$*, where* $INT([0,1])$ *is the set of all subsets of the unit interval.*

Definition 3.4. *[9] Let* $G = U, E_1, E_2, ..., E_n)$ *be a graph structure. Then* $IFG\ G = (A, B_1, B_2, ..., B_n)$ *is called an intuitionistic fuzzy graph structure of* G *with underlying vertex set* U *if the following conditions are satisfied:*

i. *A is an intuitioinistic fuzzy set on U with* $\mu_A : U \to [0,1]$ *and* $v_A : U \to [0,1]$*, the degree of membership and the degree of non-membership of* $x \in U$*, respectively, such that*

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \forall x \in U$$

ii. B_i *is an intuitionistic fuzzy set on* E_i *for all* i *and the membership function* $\mu_{B_i} : E_i \to [0,1]$ *and* $v_{B_i} : E_i \to [0,1]$ *are restricted by*

$$\mu_{B_i}(xy) \leq \mu_A(x) \wedge \mu_A(y), v_{b_i}(xy) \leq v_A(x) \vee v_A(y)$$

such that

$$0 \leq \mu_{B_i}(xy) + v_{b_i}(xy) \leq 1 \forall xy \in E_i \subset UXU, i = 1, 2, 3, ..., n.$$

4 Intuitionistic Fuzzy Multigraph with Multi Parameters.

As shown in Figure 1, the multigraph of the transportation network can be transformed into an intuitionistic fuzzy multigraph with all three parameters being the edge weight. First, we have to indicate the membership and non-membership functions to transform each parameter into an interval value intuitionistic fuzzy. The membership and non-membership functions should be related to the data condition. As mentioned before, we need to minimize the road length, travel time, and travel cost for optimal paths from every city. By dividing the data into three interval values but minimizing the parameter then, we have three membership functions and three non-membership functions, such as

$$\mu_1^L(x) = \begin{cases} 0.8, & 25.8 \leq x \leq 61.5 \\ \dfrac{(0.8 \cdot 0.2)(97.2 - x) + (0.2)(35.7)}{35.7}, & 61.5 \leq x \leq 97.2 \\ 0.2, & 97.2 \leq x \leq 132.7 \end{cases}$$

$$\mu_1^U(x) = \begin{cases} 1, & 25.8 \leq x \leq 61.5 \\ \dfrac{(97.2 - x)}{35.7}, & 61.5 \leq x \leq 97.2 \\ 0, & 97.2 \leq x \leq 132.7 \end{cases}$$

$$v_1^L(x) = \begin{cases} 0.2, & 20.8 \leq x \leq 61.5 \\ \dfrac{(0.8 \cdot 0.2)(x - 61.5) + (0.2)(35.7)}{35.7}, & 61.5 \leq x \leq 97.2 \\ 0.8, & 97.2 \leq x \leq 137.7 \end{cases}$$

$$v_1^L(x) = \begin{cases} 0, & 20.8 \leq x \leq 61.5 \\ \dfrac{(x - 61.5)}{35.7}, & 61.5 \leq x \leq 97.2 \\ 1, & 97.2 \leq x \leq 137.7 \end{cases}$$

The road length parameter has 132.7 as the highest data, with a 35.7 range, $\mu_1^L(x)$ and $\mu_1^U(x)$ are the membership function. This membership function has fit the condition of $0 \leq \mu_1^U(x) + v_1^U(x) \leq 1$ also $0 \leq \mu_1^L(x) + v_1^L(x) \leq 1$. As shown in Figure 5, the membership and non-membership function was maximized if the data was low. The lower the $IIFN$, the extended the road length between any two vertices.

This formula fits the other two parameters, such as the membership function of travel time $\left(\mu_2^L(x), \mu_2^U(x)\right)$ and the membership function of travel cost $\left(\mu_3^L(x), \mu_3^U(x)\right)$ With the same function but with different interval values (due to the travel time and travel cost data), we will have the Interval-valued Intuitionistic Fuzzy Numbers $(IIFN)$ of all the data in Table 1, as shown in Table 3. The membership and non-membership degrees in Figure 6 show us the characteristic of $IIFN$ that have been used. All the data was transformed into the same value without losing its meaning.

Table 3: $IIFN$ of the Parameters.

Variable	Road Length	Travel Time	Travel Cost
$X1$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X2$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X3$	$<(0.556;0.594),(0.444;0.406)>$	$<(0.8;1),(0.2;0)>$	$<(0.647;0.745),(0.353;0.255)>$
$X4$	$<(0.8;1),(0.2;0)>$	$<(0.785;0.975),(0.215;0.025)>$	$<(0,8;1),(0.2;0)>$
$X5$	$<(0.2;0),(0.8;1)>$	$<(0.2;0),(0.8;1)>$	$<(0.2,0),(0.8;1)>$
$X6$	$<(0.654;0.756),(0.346;0.244)>$	$<(0.420;0.367),(0.579;0.633)>$	$<(0.5,0.5),(0.5,0.5)>$
$X7$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X8$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.647;0.745),(0.353;0.255)>$
$X9$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X10$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X11$	$<(0.714;0.856),(0.286;0.144)>$	$<(0.8;1),(0.2;0)>$	$<(0.794;0.989),(0.206;0.010)>$
$X12$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.518;0.531),(0.482;0.469)>$
$X13$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X14$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X15$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X16$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X17$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.702;0.837),(0.297;0.163)>$
$X18$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$	$<(0.8;1),(0.2;0)>$
$X19$	$<(0.8;1),(0.2;0)>$	$<(0.318;0.196),(0.682;0.804)>$	$<(0.2;0),(0.8;1)>$

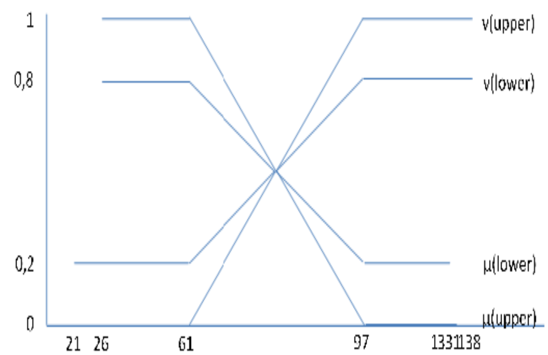


Figure 5: The degree of membership and non-membership of road Length.

5 Optimal Path in Intuitionistic Fuzzy Multigraph with Multi Parameters

The optimal path in intuitionistic fuzzy multigraph with multi-parameters begins with making a weighted function to combine all three parameters. As shown in Table 3, all the parameters have transformed into IIFN. To build a weighted function, we must composite the weight proportional to each parameter [23]. If we have

$$\begin{aligned} M^L(\mu_1^L,\mu_2^L,\mu_3^L)(x) &= a\mu_1^L(x)+b\mu_2^L(x)+c\mu_3^L(x), \\ M^U(\mu_1^U,\mu_2^U,\mu_3^U)(x) &= a\mu_1^U(x)+b\mu_2^U(x)+c\mu_3^U(x), \\ N^L(v_1^L,v_2^L,v_3^L)(x) &= av_1^L(x)+bv_2^L(x)+cv_3^L(x), \\ N^U(v_1^U,v_2^U,v_3^U)(x) &= av_1^U(x)+bv_2^U(x)+cv_3^U(x), \end{aligned}$$

with $M^L(\mu_1^L,\mu_2^L,\mu_3^L)(x)$ and $N^L(v_1^L,v_2^L,v_3^L)(x)$ is a weighted function of IIFN lower part and $M^U(\mu_1^U,\mu_2^U,\mu_3^U)(x)$ and $N^U(v_1^U,v_2^U,v_3^U)(x)$ is weighted function of $IIFN$ upper part, then $0 \leq M^L(\mu_1^L,\mu_2^L,\mu_3^L)(x)+N^L(v_1^L,v_2^L,v_3^L)(x) \leq 1$ and $0 \leq M^U(\mu_1^U,\mu_2^U,\mu_3^U)(x)+N^U(v_1^U,v_2^U,v_3^U)(x) \leq 1$.
For each $0 \leq M^L(\mu_1^L,\mu_2^L,\mu_3^L)(x)+N^L(v_1^L,v_2^L,v_3^L)(x) \leq 1$ because for every $\mu_i^L(x)$ and $v_i^L(x)$ fit $0 \leq \mu_i^L(x)+v_i^L(x) \leq 1$ for $i=1,2,3$, then $0 \leq a+b+c \leq 1$.

For the trial, we will use the combination in Table 4. As shown in the table, the differences in combination do not give much change to the data. As shown in Figure 3(b), the interval data is similar even if we change the priorities of the parameter. The number 33 : 33 : 33 means the priority of parameter road length, travel time, and travel cost is the same so that it is the similar condition for all the parameters. The number 50 : 30 : 20 means the road length has 50% priority, travel time 30%, and travel cost only 20%.

Even though we use only two parameters, for example $(M5, N5)$ use 50 : 50 : 0, which means only road length and travel time that we use with no priority, the difference was not out of the interval value. It fits all other conditions, such as using travel time and travel cost or road length and travel cost. If a weighted function is used in intuitionistic fuzzy, it can still fit definition one if $0 \leq a+b+c \leq 1$ and combined values are still in the same interval range (Figure 6). Now, we solve the optimal path in the multigraph in Figure 1 with weighted edges we use $(M1, N1)$ in Table 4.

Djikstra Algorithm lets us eliminate the minimum weight in every multiple edge. By using the Generalized Improved Score (GIS) function [24], we will get all the weight of the edges, where $GIS = |A| = \dfrac{a+b}{2}$ and $A = \langle (a,b),(c,d) \rangle$ is $IIFN$. After the elimination, the graph becomes a simple graph as can be seen from Figure 3(b). The most negligible weight will be eliminated since our goal is to optimize the $IIFN$. So if $X1 > X2$, then we eliminate $X2$. If $X1 = X2$, we can choose any edge.

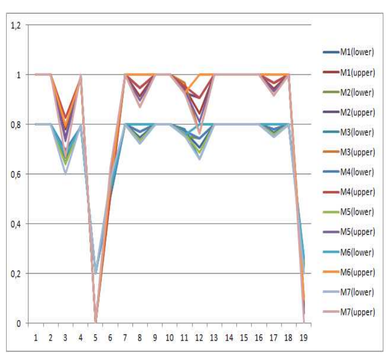


Figure 6: The Effect of Taking Different Weight Value in Weighted Function of $IIFN$.

The optimal weight of the multigraph is 0.999 with $IIFN < (0.999; 1), (0; 0) >$. This weight gave us an optimum path, the same as Figure 3(b). The optimal path by combining three parameters is path $X2 - X7 - X10 - X14 - X16 - X18$ with 262.2 km, 226 $minutes$ or about 4 $hours$ directly and $Rp.223.000, 00$.

Table 4: The Combination of IIFN to be Edges Weight in IFM.

Variable	$(M1, N1)$ 33 : 33 : 33	$(M2, N2)$ 50 : 30 : 20	$(M3, N3)$ 20 : 30 : 50	$(M4, N4)$ 30 : 50 : 20	$(M5, N5)$ 40 : 20 : 40
$X1$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X2$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X3$	$<(0.668; 0.779, (0.332; 0.220)>$	$<(0.647; 0.745), (0.352; 0.254)>$	$<(0.674; 0.791), (0.325; 0.208)>$	$<(0.696; 0.827), (0.303; 0.173)>$	$<(0.641; 0.735), (0.358; 0.264)>$
$X4$	$<(0.795; 0.992), (0.205; 0.008)>$	$<(0.795; 0.992), (0.205; 0.008)>$	$<(0.795; 0.992), (0.205; 0.008)>$	$<(0.792; 0.987), (0.207; 0.013)>$	$<(0.796; 0.994), (0.203; 0.005)>$
$X5$	$<(0.2; 0), (0.8; 1)>$	$<(0.2; 0), (0.8; 1)>$	$<(0.2; 0), (0.8; 1)>$	$<(0.2; 0), (0.8; 1)>$	$<(0.2; 0), (0.8; 1)>$
$X6$	$<(0.525; 0.542), (0.475; 0.458)>$	$<(0.553; 0.588), (0.447; 0.412)>$	$<(0.506; 0.511), (0.493; 0.488)>$	$<(0.506; 0.510), (0.493; 0.489)>$	$<(0.545; 0.575), (0.454; 0.424)>$
$X7$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X8$	$<(0.749; 0.915), (0.251; 0.085)>$	$<(0.769; 0.948), (0.230; 0.051)>$	$<(0.723; 0.872), (0.276; 0.127)>$	$<(0.769; 0.948), (0.230; 0.051)>$	$<(0.738; 0.897), (0.251; 0.085)>$
$X9$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X10$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X11$	$<(0.769; 0.948), (0.230; 0.051)>$	$<(0.755; 0.926), (0.244; 0.073)>$	$<(0.779; 0.966), (0.220; 0.033)>$	$<(0.772; 0.954), (0.227; 0.045)>$	$<(0.763; 0.938), (0.236; 0.061)>$
$X12$	$<(0.706; 0.843), (0.294; 0.156)>$	$<(0.743; 0.906), (0.256; 0.093)>$	$<(0.659; 0.765), (0.341; 0.235)>$	$<(0.744; 0.906), (0.256; 0.093)>$	$<(0.687; 0.812), (0.312; 0.188)>$
$X13$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X14$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X15$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X16$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X17$	$<(0.767; 0, 945), (0.232; 0.054)>$	$<(0.780; 0.967), (0.219; 0.032)>$	$<(0.751; 0.918), (0.248; 0.081)>$	$<(0.780; 0.967), (0.219; 0.032)>$	$<(0.760; 0.934), (0.239; 0.065)>$
$X18$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$	$<(0.8; 1), (0.2; 0)>$
$X19$	$<(0.239; 0.065), (0.761; 0.935)>$	$<(0.235; 0.058), (0.764; 0.941)>$	$<(0.235; 0.058), (0.765; 0.941)>$	$<(0.258; 0.098), (0.741; 0.902)>$	$<(0.223; 0.039), (0.776; 0.960)>$

6 Conclusion

From this discussion, it is possible to do the multi-parameters in the graph to find the shortest path. We can find the optimal path in a multigraph network with the correct transformation. Interval-valued Intuitionistic Fuzzy was chosen because of its ability to minimize uncertainty. We need more discussion about how to build the membership and non-membership functions since they took the primary role in transformation. The weighted function used here is still a linear approach that can be discussed widely in a future research.

References

[1] Haysam Selim, Justin Zhan, "Towards Shortest Path Identification on Large Networks," Journal of Big Data, **3,** no. 10, (2016).

[2] Yaya Sudarya Triana, Indah Syahputri, "Implementation Floyd-Warshall Algorithm for the Shortest Path of Garage," International Journal of Innovative Science and Research Technology, **3,** no. 2, (2018).

[3] Juni Artha Sidabutar, Ujian Sinulingga, "The Implementation of Dijkstra Algorithm in Searching the Shortest Path for Fire Engines in Medan City," Journal of Mathematics Technology and Education, **1,** no. 2, (2022), 160–172.

[4] T. Shafie, "A Multigraph Approach to Social Network Analysis," Journal of Social Structure, **16,** (2015).

[5] Hamza Ben Ticha, Nabil Absi, Dominique Feillet, Alain Quilliot, "Multigraph modeling and adaptive large neighborhood search for the vehicle routing problem with time windows," Computers & Operations Research, **104,** (2019), 113–126.

[6] R. Parvathi , M. G. Karunambigai, Krassimir T Atanassov, "Operations on Intuitionistic Fuzzy Graphs," FUZZ IEEE, (2009), 1396–1401.

[7] Muhammad Akram, Bijan Davvas, "Strong intuitionistic fuzzy graphs," Filomat, **26,** no. 1, (2012), 177–196.

[8] Muhammad Akram, Rabia Akmal, "Operations on Intuitionistic Fuzzy Graph Structures," Fuzzy Inf. Engg., **8,** (2016), 389–410.

[9] Muhammad Akram, Rabia Akmal, "Intuitionistic Fuzzy Graph Structures," Kragujevac Journal of Mathematics, **41,** no. 2, (2017), 219–237.

[10] Zahra Sadri Irani, Hossein Rashmanlou, "Certain Notions of Intuitionistic Fuzzy Graphs," Annals of Pure and Applied Mathematics, **24,** no. 1, (2021), 11–20.

[11] Debaroti Das, P. K. De, "Shortest Path Problem Under Intuitionistic Fuzzy," International Journal of Computer Applications, **105,** no. 1, (2014), 1–4.

[12] P. Jayagowry, G. Geetharamani, "A Critical Path Problem Using Intuitionistic Trapezoidal Fuzzy Number," Applied Mathematical Sciences, **8,** no. 52, (2014), 2555–2562.

[13] P. Jayagowry, G. Geetha Ramani, "Using Trapezoidal Intuitionistic Fuzzy Number to Find Optimized Path in a Network," Advances in Fuzzy Systems, (2014).

[14] Sophia Porchelvi, Sudha G., "A New Approach For Finding Minimum Path In A Network using Triangular Intuitionistic Fuzzy Number," International Journal of Current Research, **6,** no. 8, (2014), 8103–8109.

[15] Naveed Yaqoob, Muhammad Gulistan, Seifedine Kadry, Hafiz Abdul Wahab, "Complex Intuitionistic Fuzzy Graphs with Application in Cellular Network Provider Companies," Mathematics, **7,** no. 35, (2019).

[16] Zehui Shao, Saeed kosari, Hossein Rashmanlou, Muhammad Shoaib, "New Concepts in Intuitionistic Fuzzy Graph with Application in Water Supplier Systems," Mathematics, **8,** (2020).

[17] Gaurav Kumar, Rakesh Kumar Bajaj, Neeraj Gandotra, "Algorithm for Shortest Path Problem in a Network with Interval-valued Intuitionistic Trapezoidal Fuzzy Number," Procedia Computer Science, **70,** (2015), 123–129.

[18] Yang Yang, Hongxu Li, Zhiming Zhang, Xiaowei Liu, "Interval-valued Intuitionistic Fuzzy Analytic Network Process," Information Sciences, **526,** (2020), 102–118.

[19] Siddhartha Sankar Biswas, Bashir Alam, Mohammad Najmud Doja, "Real Time Multigraphs for Communication Networks: An Intuitionistic Fuzzy Mathematical Model," Journal of Computer Science, **9,** no. 7, (2013).

[20] Siddhartha Sankar Biswas, "i-v Fuzzy Shortest Path in a Multigraph," Oriental Journal of Computer Science & Technology, **10,** no. 2, (2017), 364–370.

[21] Sidhartha Sankar Biswas, Bashir Alam, M. N. Doja, "Intuitionistic Fuzzy Shortest Path in a Multigraph," Proceeding of the 4th International Conference on Recent Developments in Science, Engineering and Technology, REDSET 2017, (2018), 533–540.

[22] K. T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Application, Springer-Verlag, Berlin, 1999.

[23] Radko Mesiar, Jana Spirkova, "Weighted Means and Weighting Functions," Kybernetika, **42,** no. 2, (2006), 151–160.

[24] H. Garg, "Generalized Intuitionistic Fuzzy Multiplicative Interactive Geometric Operators and Their Application to Multiple Criteria decision Making," Int. J. Mach. Learn. Cybern., (2016), 1–8.