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Properties of (Λ, p)-submaximal spaces

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Abstract

In this paper, we investigate some properties of (Λ, p)-submaximal spaces by utilizing a $\mathcal{B}(\Lambda, p)$-set.

1 Introduction

The notions of maximality and submaximality of general topological spaces were introduced by Hewitt [5] who discovered a general way of constructing maximal topologies. The existence of a maximal space that is Tychonoff is nontrivial and is due to van Douwen [3]. The first systematic study of submaximal spaces was undertaken in the paper of Arhangel'skiĭ and Collins [1] who gave various necessary and sufficient conditions for a space to be submaximal and showed that every submaximal space is left-separated. This led to the question on whether every submaximal space is σ-discrete [1]. Mashhour et al. [6] introduced and investigated the concept of preopen sets and preclosed sets. Ganster et al. [4] introduced the concepts of a pre-Λ-set and a pre-V-set in topological spaces and investigated their fundamental

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properties. Boonpok and Viriyapong [2] introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets which are defined by utilizing the notions of Λ_p-sets and preclosed sets. The concept of (Λ, p)-submaximal spaces was introduced by Srisarakham and Boonpok [7]. In this paper, we investigate some properties of (Λ, p)-submaximal spaces.

2 Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [6] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [4] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p*-set* [2] (*pre-*Λ*-set* [4]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)*-closed* [2] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)*-open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)*-cluster point* [2] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)*-closure* [2] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets contained in A is called the (Λ, p)*-interior* [2] of A and is denoted by $A_{(\Lambda,p)}$. A subset A of a topological space (X, τ) is said to be $s(\Lambda, p)$*-open* [2] if $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$.

Lemma 2.1. [2] *For subsets A, B of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq A^{(\Lambda,p)}$ and $[A^{(\Lambda,p)}]^{(\Lambda,p)} = A^{(\Lambda,p)}$.

(2) If $A \subseteq B$, then $A^{(\Lambda,p)} \subseteq B^{(\Lambda,p)}$.

(3) $A^{(\Lambda,p)} = \cap\{F | A \subseteq F$ and F is (Λ, p)-closed$\}$.

(4) $A^{(\Lambda,p)}$ is (Λ, p)-closed.

(5) A is (Λ, p)-closed if and only if $A = A^{(\Lambda,p)}$.

Lemma 2.2. [2] *Let A and B be subsets of a topological space (X, τ). For the (Λ, p)-interior, the following properties hold:*

(1) $A_{(\Lambda,p)} \subseteq A$ *and* $[A_{(\Lambda,p)}]_{(\Lambda,p)} = A_{(\Lambda,p)}$.

(2) *If* $A \subseteq B$, *then* $A_{(\Lambda,p)} \subseteq B_{(\Lambda,p)}$.

(3) $A_{(\Lambda,p)}$ *is* (Λ, p)*-open.*

(4) A *is* (Λ, p)*-open if and only if* $A_{(\Lambda,p)} = A$.

(5) $[X - A]^{(\Lambda,p)} = X - A_{(\Lambda,p)}$.

3 Properties of (Λ, p)-submaximal spaces

In this section, we investigate some properties of (Λ, p)-submaximal spaces.

Definition 3.1. [7] *A subset* A *of a topological space* (X, τ) *is said to be:*

(i) (Λ, p)*-dense if* $A^{(\Lambda,p)} = X$;

(ii) (Λ, p)*-codense if its complement is* (Λ, p)*-dense.*

Definition 3.2. [7] *A topological space* (X, τ) *is said to be* (Λ, p)*-submaximal if, for each* (Λ, p)*-dense subset of* X *is* (Λ, p)*-open.*

Lemma 3.3. [7] *For a subset* A *of a topological space* (X, τ)*, the following properties are equivalent:*

(1) A *is locally* (Λ, p)*-closed;*

(2) $A = U \cap A^{(\Lambda,p)}$ *for some* $U \in \Lambda_p O(X, \tau)$;

(3) $A^{(\Lambda,p)} - A$ *is* (Λ, p)*-closed;*

(4) $A \cup [X - A_{(\Lambda,p)}]$ *is* (Λ, p)*-open;*

(5) $A \subseteq [A \cup [X - A^{(\Lambda,p)}]]_{(\Lambda,p)}$.

Theorem 3.4. *A topological space* (X, τ) *is* (Λ, p)*-submaximal if and only if for each* (Λ, p)*-codense subset of* X *is* (Λ, p)*-closed.*

Proof. Let A be a (Λ, p)-codense subset of X. Then $X - A$ is (Λ, p)-dense. Since (X, τ) is (Λ, p)-submaximal, we have $X - A$ is (Λ, p)-open and hence A is (Λ, p)-closed.

Conversely, let A be a (Λ, p)-dense subset of X. Then $X - A$ is (Λ, p)-codense and hence $X - A$ is (Λ, p)-closed. Thus, A is (Λ, p)-open. This shows that (X, τ) is (Λ, p)-submaximal. □

Definition 3.5. *A subset A of a topological space (X, τ) is said to be:*

(i) *a $t(\Lambda, p)$-set if $A_{(\Lambda,p)} = [A^{(\Lambda,p)}]_{(\Lambda,p)}$;*

(ii) *a $s(\Lambda, p)$-regular set if A is a $t(\Lambda, p)$-set and $s(\Lambda, p)$-open;*

(iii) *a $\mathcal{B}(\Lambda, p)$-set if $A = U \cap V$, where $U \in \Lambda_p O(X, \tau)$ and V is a $t(\Lambda, p)$-set;*

(iv) *an $\mathcal{AB}(\Lambda, p)$-set if $A = U \cap V$, where $U \in \Lambda_p O(X, \tau)$ and V is a $s(\Lambda, p)$-regular set.*

Theorem 3.6. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is (Λ, p)-submaximal;*

(2) *$A^{(\Lambda,p)} - A$ is (Λ, p)-closed for every subset A of X;*

(3) *every subset of X is locally (Λ, p)-closed;*

(4) *every subset of X is a $\mathcal{B}(\Lambda, p)$-set;*

(5) *every (Λ, p)-dense set of X is a $\mathcal{B}(\Lambda, p)$-set.*

Proof. (1) $\Rightarrow$ (2): Let A be any subset of X. Then, $[X - [A^{(\Lambda,p)} - A]]^{(\Lambda,p)} = [A \cup [X - A^{(\Lambda,p)}]]^{(\Lambda,p)} = X$ and hence $X - [A^{(\Lambda,p)} - A]$ is (Λ, p)-dense. By the hypothesis, $X - [A^{(\Lambda,p)} - A]$ is (Λ, p)-open. Thus, $A^{(\Lambda,p)} - A$ is (Λ, p)-closed.

(2) $\Rightarrow$ (3): This is obvious by Lemma 3.3.

(3) $\Rightarrow$ (4): This follows from the fact that every locally (Λ, p)-closed set is a $\mathcal{B}(\Lambda, p)$-set.

(4) $\Rightarrow$ (5): The proof is obvious.

(5) $\Rightarrow$ (1): Let A be a (Λ, p)-dense subset of X. By (5), A is a $\mathcal{B}(\Lambda, p)$-set and hence $A = U \cap F$, where U is (Λ, p)-open and $F_{(\Lambda,p)} = [F^{(\Lambda,p)}]_{(\Lambda,p)}$. Since $A \subseteq F$, $A^{(\Lambda,p)} \subseteq F^{(\Lambda,p)}$ and $X = F^{(\Lambda,p)}$. Thus, $X = [F^{(\Lambda,p)}]_{(\Lambda,p)} = F_{(\Lambda,p)}$ and hence $F = X$. Therefore, $A = U \cap F = U \cap X = U$. This shows that A is (Λ, p)-open. Thus, (X, τ) is (Λ, p)-submaximal. □

Theorem 3.7. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is (Λ, p)-submaximal;*

(2) *every subset of X is a $\mathcal{B}(\Lambda, p)$-set;*

(3) every $\beta(\Lambda, p)$-open set is a $\mathcal{B}(\Lambda, p)$-set;

(4) every (Λ, p)-dense set is a $\mathcal{B}(\Lambda, p)$-set.

Proof. (1) $\Rightarrow$ (2): It follows from Theorem 3.6.

(2) $\Rightarrow$ (3): This is obvious.

(3) $\Rightarrow$ (4): It follows from the fact that every (Λ, p)-dense set is a $\beta(\Lambda, p)$-open set.

(4) $\Rightarrow$ (1): It follows from Theorem 3.6. □

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