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Characterizations of (Λ, p)-extremally disconnected spaces

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Abstract

Our main purpose is to investigate some characterizations of (Λ, p)-extremally disconnected spaces.

1 Introduction

In 1960, Gillman and Jerison [5] introduced the concept of extremally disconnected topological spaces. Thompson [9] introduced the notion of S-closed spaces. Cameron [3] proved that every maximally S-closed space is extremally disconnected. Niori [8] introduced the notion of locally S-closed spaces which is strictly weaker than that of S-closed spaces. In 1988, Noiri [7] investigated some characterizations of extremally disconnected spaces by utilizing preopen sets and semi-preopen sets. Mashhour et al. [6] introduced and investigated the concept of preopen sets and preclosed sets. In 2002, Ganster et al. [4] introduced the notions of a pre-Λ-set and a pre-V-set in topological spaces and investigated their fundamental properties. Quite recently, Boonpok and Viriyapong [1] introduced the notions of (Λ, p)-open sets

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and (Λ, p)-closed sets which are defined by utilizing the notions of Λ_p-sets and preclosed sets. The concept of (Λ, p)-extremally disconnected spaces was introduced in [10]. In this paper, we investigate some characterizations of (Λ, p)-extremally disconnected spaces.

2 Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [6] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [4] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [1] (*pre*-Λ-*set* [4]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [1] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_pO(X, \tau)$ (resp. $\Lambda_pC(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [1] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets contained in A is called the (Λ, p)-*interior* [1] of A and is denoted by $A_{(\Lambda,p)}$. A subset A of a topological space (X, τ) is said to be $s(\Lambda, p)$-*open* (resp. $p(\Lambda, p)$-*open*, $\beta(\Lambda, p)$-*open*, $r(\Lambda, p)$-*open*) [1] if $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$ (resp. $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A \subseteq [[A^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}$, $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$). A subset A of a topological space (X, τ) is called $\alpha(\Lambda, p)$-*open* [10] (resp. $b(\Lambda, p)$-*open*) if $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$ (resp. $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)} \cup [A_{(\Lambda,p)}]^{(\Lambda,p)}$). The complement of a $s(\Lambda, p)$-open (resp. $p(\Lambda, p)$-open, $r(\Lambda, p)$-open, $\beta(\Lambda, p)$-open, $\alpha(\Lambda, p)$-open, $b(\Lambda, p)$-open) set is called $s(\Lambda, p)$-*closed* (resp. $p(\Lambda, p)$-*closed*, $r(\Lambda, p)$-*closed*, $\beta(\Lambda, p)$-*closed*, $\alpha(\Lambda, p)$-*closed*, $b(\Lambda, p)$-*closed*). The family of all $s(\Lambda, p)$-open (resp. $p(\Lambda, p)$-open, $r(\Lambda, p)$-open, $\beta(\Lambda, p)$-open, $\alpha(\Lambda, p)$-open, $b(\Lambda, p)$-open) sets in a topological space (X, τ) is denoted by $s\Lambda_pO(X, \tau)$ (resp. $p\Lambda_pO(X, \tau)$, $r\Lambda_pO(X, \tau)$, $\beta\Lambda_pO(X, \tau)$, $\alpha\Lambda_pO(X, \tau)$, $b\Lambda_pO(X, \tau)$). The family of all $s(\Lambda, p)$-closed (resp. $p(\Lambda, p)$-closed, $r(\Lambda, p)$-closed, $\beta(\Lambda, p)$-closed, $\alpha(\Lambda, p)$-closed, $b(\Lambda, p)$-closed) sets in a topological space (X, τ) is denoted by $s\Lambda_pC(X, \tau)$ (resp. $p\Lambda_pC(X, \tau)$, $r\Lambda_pC(X, \tau)$, $\beta\Lambda_pC(X, \tau)$, $\alpha\Lambda_pC(X, \tau)$, $b\Lambda_pC(X, \tau)$).

3 Characterizations of (Λ, p)-extremally disconnected spaces

In this section, we investigate some characterizations of (Λ, p)-extremally disconnected spaces.

Recall that a topological space (X, τ) is said to be (Λ, p)-*extremally disconnected* [10] if $V^{(\Lambda,p)}$ is (Λ, p)-open in X for every (Λ, p)-open set V of X.

Theorem 3.1. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, p)-extremally disconnected.

(2) For every $r(\Lambda, p)$-open set of X is (Λ, p)-closed.

(3) For every $r(\Lambda, p)$-closed set of X is (Λ, p)-open.

Proof. (1) ⇒ (2): Let U be a $r(\Lambda, p)$-open set. Then, $U = [U^{(\Lambda,p)}]_{(\Lambda,p)}$. Since U is (Λ, p)-open, we have $U^{(\Lambda,p)}$ is (Λ, p)-open. Thus, $U = [U^{(\Lambda,p)}]_{(\Lambda,p)} = U^{(\Lambda,p)}$ and hence U is (Λ, p)-closed.

(2) ⇒ (1): Let U be a (Λ, p)-open set. Since $[U^{(\Lambda,p)}]_{(\Lambda,p)}$ is $r(\Lambda, p)$-open, we have $[U^{(\Lambda,p)}]_{(\Lambda,p)}$ is (Λ, p)-closed and hence $U^{(\Lambda,p)} \subseteq [[U^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)} = [U^{(\Lambda,p)}]_{(\Lambda,p)}$. Thus, $U^{(\Lambda,p)}$ is (Λ, p)-open. This shows that (X, τ) is (Λ, p)-extremally disconnected.

(2) ⇔ (3): This is obvious. □

Lemma 3.2. *For a subset A of a topological space (X, τ), the following properties are equivalent:*

(1) $A \in \beta\Lambda_p O(X, \tau)$.

(2) $A^{(\Lambda,p)} \in r\Lambda_p C(X, \tau)$.

(3) $A^{(\Lambda,p)} \in \beta\Lambda_p O(X, \tau)$.

(4) $A^{(\Lambda,p)} \in s\Lambda_p O(X, \tau)$.

(5) $A^{(\Lambda,p)} \in b\Lambda_p O(X, \tau)$.

Theorem 3.3. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X,τ) *is* (Λ,p)*-extremally disconnected.*

(2) For each $V\in\beta\Lambda_pO(X,\tau)$, $V^{(\Lambda,p)}\in r\Lambda_pO(X,\tau)$.

(3) For each $V\in b\Lambda_pO(X,\tau)$, $V^{(\Lambda,p)}\in r\Lambda_pO(X,\tau)$.

(4) For each $V\in s\Lambda_pO(X,\tau)$, $V^{(\Lambda,p)}\in r\Lambda_pO(X,\tau)$.

(5) For each $V\in\alpha\Lambda_pO(X,\tau)$, $V^{(\Lambda,p)}\in r\Lambda_pO(X,\tau)$.

(6) For each $V\in\Lambda_pO(X,\tau)$, $V^{(\Lambda,p)}\in r\Lambda_pO(X,\tau)$.

(7) For each $V\in r\Lambda_pO(X,\tau)$, $V^{(\Lambda,p)}\in r\Lambda_pO(X,\tau)$.

(8) For each $V\in p\Lambda_pO(X,\tau)$, $V^{(\Lambda,p)}\in r\Lambda_pO(X,\tau)$.

Proof. The proof follows from Theorem 2 of [2]. □

Theorem 3.4. *For a topological space* (X,τ)*, the following properties are equivalent:*

(1) (X,τ) *is* (Λ,p)*-extremally disconnected.*

(2) $r\Lambda_pC(X,\tau)\subseteq\Lambda_pO(X,\tau)$.

(3) $r\Lambda_pC(X,\tau)\subseteq\alpha\Lambda_pO(X,\tau)$.

(4) $r\Lambda_pC(X,\tau)\subseteq p\Lambda_pO(X,\tau)$.

(5) $s\Lambda_pO(X,\tau)\subseteq\alpha\Lambda_pO(X,\tau)$.

(6) $s\Lambda_pC(X,\tau)\subseteq\alpha\Lambda_pC(X,\tau)$.

(7) $s\Lambda_pC(X,\tau)\subseteq p\Lambda_pC(X,\tau)$.

(8) $s\Lambda_pO(X,\tau)\subseteq p\Lambda_pO(X,\tau)$.

(9) $\beta\Lambda_pO(X,\tau)\subseteq p\Lambda_pO(X,\tau)$.

(10) $\beta\Lambda_pC(X,\tau)\subseteq p\Lambda_pC(X,\tau)$.

(11) $b\Lambda_pC(X,\tau)\subseteq p\Lambda_pC(X,\tau)$.

(12) $b\Lambda_pO(X,\tau)\subseteq p\Lambda_pO(X,\tau)$.

(13) $r\Lambda_pO(X,\tau)\subseteq p\Lambda_pC(X,\tau)$.

(14) $r\Lambda_p O(X, \tau) \subseteq \Lambda_p C(X, \tau)$.

(15) $r\Lambda_p O(X, \tau) \subseteq \alpha\Lambda_p C(X, \tau)$.

Proof. The proof follows from Theorem 3 of [2]. □

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