International Journal of Mathematics and
Computer Science, **18**(2023), no. 3, 563–570

Engineering Mathematics for radiative heat transfer in the magnetohydrodynamic

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(Received February 1, 2023, Accepted March 19, 2023,
Published March 31, 2023)

Abstract

We study the distribution of temperatures for the radiative MHD Ekman layer on a porous flat plate by including Joule's dissipation function for the following cases:
(i) A steady temperature is preserved on the plate.
(ii) The plate is insulated.
We obtain exact solutions in the two instances of sucking and blowing (such a solution does not exist in the event that there is no radiative heat emission present and blowing is taking place).

1 Introduction

In recent years, several studies have been carried out specifically concerned with elucidating the influence of various mechanisms of transmission of heat through the Ekman layer of a plate with pores that are located in the vicinity of a magnetic field and involving a fluid that is capable of conducting electricity. These investigations have applications in many astrophysical and astronautical problems.

Key words and phrases: Steady temperature, Joule's dissipation, Ekman Layer, Flat Plate, Temperature Distribution, Insulation, exact solutions.
AMS (MOS) Subject Classifications: 65K10, 49M37, 90C06.
ISSN 1814-0432, 2023, http://ijmcs.future-in-tech.net

Recently Gupta [1], Erdoğan [2], Murthy and Ram [3], Das et al. [4] have considered the boundary layer due to rotations that are not coaxial of a porous disc and a fluid at infinity, yet they did not consider the consequences of the radiative heat transfer. Nagara et al. [5], Aziz [6] have discussed the effect of radiation in the MHD Ekman layer but did not consider Joule's dissipation function in the energy equation that arises in the MHD heat transfer analysis [7]. The purpose of this paper is to investigate the same problem, including Joule's dissipation function representing the conversion of magnetic energy into heat.

2 Mathematical Formulation

Consider a coordinate system based on the Cartesian transform that rotates continuously at angular velocity Ω that is constant concerning the z-axis, with a positive sign indicating movement vertically upward, with the plate having the same level of porosity throughout and coinciding with the plane $z = 0$.

A magnetic field B_0 that is constant and uniform is applied in the direction of the z-axis. The basic equations of motion for such a configuration are well established and are given by:

$$-2v\Omega - \ w_0\frac{du}{dz} = -\frac{\sigma B_0^2}{\rho}(u-v) + \frac{d^2u}{dz^2}\} \tag{2.1}$$

$$-2(u-v)\Omega - \ w_0\frac{dv}{dz} = -\frac{\sigma B_0^2}{\rho}v + \frac{d^2v}{dz^2}\} \tag{2.2}$$

where $u, v, w = w_0$ are the components of the fluid velocity and ρ is the fluid density.

Assuming horizontal uniformity everywhere, the equation for continuity yields $w = w_0$ which is a constant.

$w_0 > 0$ is representative of suction and $w_0 < 0$ is indicative of blowing. Since the magnetic Reynolds number is extremely low, the induced magnetic field is considered to be insignificant in relation to the magnetic field that is being applied.

The energy equation describing the transport of thermal energy in the presence of the magnetic field can be written as:

$$pc_pw_0\frac{dT}{dz} = K\frac{d^2T}{dz^2} - \frac{dq_R}{dz} + [\rho\mu(\frac{du}{dz}^2 + (\frac{dv}{dz})^2] + \sigma\mu_e^2B_0^2[(u-v)^2 + v^2]\} \tag{2.3}$$

where c_p is the specific heat at a pressure that is held constant, K is the coefficient of thermal conductivity, μ is the viscosity, μ_e the magnetic permeability, σ the electrical conductivity, and q_R the radiative flux. The fluid is assumed to move with constant velocity $\cup$ in the x-direction outside the frictional region.

The boundary conditions are:

For velocity

$$\left.\begin{array}{ll} u = v = 0 & as\ z = 0 \\ u \to \cup,\ v \to 0 & as\ z = \infty \end{array}\right\} \tag{2.4}$$

For case (i)

$$\left.\begin{array}{ll} T = T_0 & at\ z = 0 \\ T = T_\infty & as\quad z = \infty \end{array}\right\} \tag{2.5}$$

For case (ii)

$$\left.\begin{array}{ll} \frac{dT}{dz} = 0 & at\ z = 0 \\ T = T_\infty & as\ z = \infty \end{array}\right\} \tag{2.6}$$

We olve the equations of motion 2.1 and 2.2, with the boundary conditions taken into account 2.4. Accordingly, the velocity field is given by:

$$u = \cup[1 - exp(-a\eta)]cos\beta\eta$$

$$v = \cup[exp(-a\eta)]sin\beta\eta$$

where

$$a = \pm\frac{s}{2} \pm \left(\frac{\left[(4N + s^2)^2 + 16E^2\right] + (4N + s^2)}{8}\right)^{\frac{1}{2}}$$

$$\beta = \left(\frac{\left[(4N + s^2)^2 + 16E^2\right]^{\frac{1}{2}} - (4N + s^2)}{8}\right)$$

and

$$N = \frac{\mu_e^2 B_0^2}{\rho\Omega}, E = \frac{2\Omega v}{\cup^2}, s = \frac{w_0}{(2\Omega v)^{\frac{1}{2}}}$$

Denote the parameter related to magnetism, the Taylor number, as the suction parameter in their respective forms, $\eta = (\frac{\Omega}{2v)}^{\frac{1}{2}} z$ being a non-dimensional coordinate.

3 Solution of the problem

Under approximation that is extremely thin and optically clear, Radiation with a mean free path is significantly greater than the typical duration of the flow field along the length. Sometimes, the situation can be summarized as being similar to radiation, in which there is almost no self-absorption, to the point when the Planck coefficient of the mean absorption $K_p = \frac{1}{L_R}$ is not particularly large, where L_R is the typical length of the radiation's mean free path. There is no radiative interaction between the different components of the fluid because all the components of the fluid exchange radiation directly with the boundary surface.

Vogler et al. [8], Travgott [9], Aeronautics and Astronautics [10] have demonstrated that the equation for radiative heat transfer for a non-grey gas that is approaching equilibrium and under the optically thin limit has the simple form:

$$\frac{dq_r}{dz} = 4\rho k_p \widetilde{\sigma} T^4\} \tag{3.7}$$

where $\widetilde{\sigma}$ is the Stefan Boltzmann constant.

In the situation of optically exceedingly thin $k_p \to 0$ as $L_R \to \infty$ accordingly the radiative heat flux will be constant as has been shown by Liu et al. [11]. Considering the temperature T_0 at the porous disc is the greatest and the temperature of liquids at infinity, denoted by T_∞, is the lowest. It is reasonable to suppose that the liquid will reach an isothermal state of equilibrium when it reaches infinity, whereas the overall flow field's variation in temperature is minimal. In certain circumstances, the equation 3.7 can be linearized as discussed by Aziz [6]. In the present circumstance, the most appropriate illustration of a linear approximation of the equation 3.7 can be found in the form using curve fitting and the principle of least squares of:

$$\frac{dq_R}{dz} = c_1 T - c_2 \tag{3.8}$$

where

$$c_1 = \frac{16}{5}\rho k_p \sigma T_0^3 (1 - \frac{T_\infty}{T_0})^3, c_2 = \frac{4}{5}\rho k_p \sigma T_0^4 (1 - \frac{T_\infty}{T_0})^4$$

The liquid is in equilibrium and can be described as isothermal. It is possible to rewrite the boundary conditions 2.5 and 2.6 that need to be accomplished via the temperature profile as:

$$T \to \frac{1}{5}\, T_0 \ as \ z \to \infty\} \tag{3.9}$$

For both cases (i) and (ii) $T = T_0$ at $z = 0$ and $\frac{dT}{dz} = 0$ at $z = 0$ for case (ii). This means that the porous plate is kept at a fixed temperature T_0 for case (i) and insulated for case (ii). Assuming the temperature of the fluid at infinity has a constant value of $\frac{1}{5}$ T_0 , these are some dimensionless parameters that we present:

$$\theta = \frac{T - T_0}{T_\infty - T_0}, P_r = \frac{2\rho vc}{k}p, E_c = \frac{\cup^2}{2(T_0 - T)c_p}$$

where P_r referred to as a Prandtl number and the number E_c is referred to as an Eckert number.

Transforming equation 2.3 into dimensionless from, we get:

$$\frac{d\theta^2}{d\eta^2} + p_r s\frac{d\theta}{d\eta}F\theta = -F + \frac{5}{4}p_r E_c(a^2 + \beta^2).exp(-2a\eta) + 2Np_r E_c exp(-2\ a\eta) \tag{3.10}$$

where

$$F = \frac{2(\frac{4}{5})^4 \widetilde{\sigma} k_p p_r T_0^3}{c_p \Omega} > 0$$

The boundary condition 3.10 gives:

$$\theta\,(0) = 0, \theta\,(\infty) = 1, for\ case(i)\} \tag{3.11}$$

$$\frac{d\theta}{d\eta}|_{\eta=0}^{=0}, \theta(\infty) = 1, for\ case(ii)\ \} \tag{3.12}$$

Case (i)

The solution of equation 3.10 satisfying 3.11 is given by:

$$\theta(\eta) = 1 - e^{-\lambda\eta} + \frac{5p_r E_c\left(\alpha^2 + \beta^2 + \frac{8}{5}N\right)}{4\left(4a^2 - 2ap_r S - F\right)}\ (e^{-2a\eta} - e^{-\lambda\eta}) \tag{3.13}$$

For

$$4a^2 - 2ap_r S - F\ \neq 0$$

$$\theta(\eta) = 1 - e^{-\lambda\eta} - \frac{5p_r E_c(a^2 + \beta^2 + \frac{8}{5}N)}{4(4a{-}p_r S)}e^{-2a\eta} \tag{3.14}$$

For

$$4a^2 - 2ap_r S - F = 0$$

where

$$\lambda = \frac{1}{2}p_r s + \frac{1}{2}\sqrt{p_r^2 s^2 + \ 4F > 0}$$

Case (ii)

The solution of equation 3.10 satisfying 3.12 is given by:

$$\theta(\eta) = 1 + \frac{5p_r E_c(a^2 + \beta^2 + \frac{8}{5}N)}{4(4a^2 - 2ap_r S - F)} e^{-2a\eta} - \frac{2a}{\lambda} e^{-\lambda\eta} \tag{3.15}$$

For

$$4a^2 - 2ap_r s - F = 0$$

$$\theta(\eta) = 1 + \frac{5p_r E_c(\sigma^2 + \beta^2 + \frac{8}{5}N)}{4(4\sigma - \ p_r S)} e^{-2\sigma\eta} \tag{3.16}$$

For

$$4a^2 - 2ap_r s - F = 0$$

Since F and λ are greater than zero in both of these instances of suction S being more than zero and blowing S being less than zero respectively, the asymptotic solutions 3.13, 3.14, 3.15, and 3.16 are legitimate throughout the entirety of the flow field, whereas there would be no asymptotic solution in the absence of radiative heat transfer exists on the possibility of exceeding the limit case when it comes to blowing, when F=0, this solution is reduced to that obtained by Steven and Ellingson [3] for N=0 the solution is reduced to that obtained by Das et al. [4].

4 Conclusion

In this study, we analyzed the distribution of temperatures in radiative MHD Ekman layer on a porous flat. By considering Joules's dissipation function, we obtained exact solutions in the distinct cases:

(i) when a steady temperature was maintained on the plate, and

(ii) when the plate was insulated.

The analysis included both sucking and blowing which are common in many practical applications. The obtained solutions provided valuable information into the behavior of temperature distribution in these scenarios which can help in the design and optimization of thermal management systems. However, it is worth nothing that a solution couldn't be found when radiative heat emission is absent and blowing is taking place which highlights the complexity of the problems and the need for further investigation. Over all, this study contributed to our understanding of radiative MHD Ekman layer on a porous flat plate and demonstrated the importance of considering Joule's dissipation function when analyzing such a system.

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