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Symmetric Generalized Bi-Derivations with Prime Ideals

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Abstract

The primary objective of this paper is to demonstrate that a quotient ring $\mathbb{S}/\Gamma$ is commutative by studying the behavior of symmetric generalized bi-derivations of $\mathbb{S} \times \mathbb{S}$ that fulfill some algebraic identities concerning a prime ideal. The result of Vukman is extended by proposing identities dealing with different symmetric generalized bi-additive mappings: for all elements ℓ_1 of a ring $\mathbb{S}$, the statement $\lambda(\ell_1, \ell_1)\ell_1 + \ell_1\zeta(\ell_1, \ell_1) - \eta(\ell_1, \ell_1)$ belongs to a prime ideal Γ.

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1 Introduction

Algebra is considered one of the sciences with different disciplines that have an impact on most areas of life, either directly or indirectly. Abstract algebra, applied algebra, and others are known as disciplines that are closely related to many sciences, with the most important one being computer science, and in particular, image processing [1, 2, 3]. In modern algebra, the generalizations of the derivation concept hold significant importance in various fields such as engineering calculations, quantum physics, business, and the computation of eigenvalues of matrices.

Throughout this article, the center of an associative ring $\mathbb{S}$ will be indicated by $\mathfrak{C}(\mathbb{S})$. An ideal Γ of $\mathbb{S}$ is prime if Γ is not equal to $\mathbb{S}$ and whenever the product $\eta_1\mathbb{S}\eta_2$ is contained in Γ for every pair of elements, η_1, η_2 in $\mathbb{S}$, then at least one η_1 or η_2 is in Γ. Furthermore, if the ideal (0) of a ring $\mathbb{S}$ is prime, then $\mathbb{S}$ is referred to as a prime ring. For elements χ_1 and χ_2 in $\mathbb{S}$, the symbol $[\chi_1, \chi_2]$ refers to the commutator $\chi_1\chi_2 - \chi_2\chi_1$. In 1957, Posner defined the term derivation (D, for short) which sparked the interest of numerous researchers who explored the derivation theory on a variety of algebraic structures based on Posner's definition of derivation on a ring [4].

Several different types of D mappings can be found in the literature, as documented in reference [5, 6, 7]. The concept of D mapping has been generalized in diverse ways over the past few decades by many authors, resulting in remarkable outcomes and expanded applicability in various domains. Brešar introduced the term "generalized derivation" (GD, for short) as one of these methods of expansion, as described in reference [8]. Hvala initiated the algebraic investigation of GD mappings and expanded certain results concerning D mappings to GD mappings, as detailed in reference [9]. Numerous researchers have examined the invariance of specific ideals under derivations. Nadeem et al. proposed some differential identities using GD mappings within a prime ideal of a ring without requiring the ring to be prime, as stated in reference [10]. Additional generalizations of D mappings are available in reference [11, 12]. In 1987, Maksa [13] introduced the concept of a symmetric bi-derivation (SBD) as a bi-additive mapping that satisfies specific identities. In [14], the author presented two results concerning SBD mappings on prime rings, depending on the product of traces of these maps. Additionally, the existence of a nonzero SBD mapping on a prime ring of characteristic different from two and three was studied. Following [15], the author became interested in a type of mapping that is closely related to the idea of bi-derivations, which is termed a commuting mapping. Regarding commuting mappings, Posner's theorem, as discussed in [4], is a crucial finding in the investigation of these mappings. It states that a prime ring possessing a nonzero commuting D mapping must be commutative. This theorem marks an important initial outcome in the study of commuting mappings. Vukman in [16], studied the commuting traces of SBD mappings on a noncommutative prime ring of characteristic different two. Moreover, when specific conditions are met regarding prime rings, the characterization of SBD mappings can be determined based on their traces [17]. Following [18], Argaç introduced the thought of generalized bi-derivation (GBD, for short). Further, a GBD mapping that fulfills symmetric property is said to be symmetric generalized bi-derivation (SGBD, for short) [19]. According to [20], Ali et al. extended Vukman's results [16] to SGBD mappings in a suitable subset of a prime ring. This paper aims to study the demeanor SGBD mappings by presenting many algebraic identities that comprise different SGBD mappings with prime ideals to generalize Vukman's result. The arrangement of the article is as follows: Section 2 is devoted to giving a background for some needed concepts while Section 3 is dedicated to thoroughly exploring the proposed identities of SGBD mappings with prime ideals. A main theorem is proved in addition to several corollaries.

2 Background

This section provides the fundamental concepts that are required.

Definition 2.1. Derivation (D) [4]

A mapping $\xi : \mathbb{S} \to \mathbb{S}$ that maintains the addition operation and satisfies $\xi(\chi_1\chi_2) = \xi(\chi_1)\chi_2 + \chi_1\xi(\chi_2)$ for all $\chi_1, \chi_2 \in \mathbb{S}$ is said to be a D mapping.

Definition 2.2. Generalized Derivation (GD) [8]

An additive mapping $\vartheta : \mathbb{S} \to \mathbb{S}$ which satisfies $\vartheta(\chi_1\chi_2) = \vartheta(\chi_1)\chi_2 + \chi_1\xi(\chi_2)$ for all $\chi_1, \chi_2 \in \mathbb{S}$ is called GD mapping, where $\xi : \mathbb{S} \to \mathbb{S}$ is a D mapping.

It is evident that the concept of GD mapping encompasses the concept of D mapping.

Definition 2.3. Symmetric (SB) [21]

A bi-additive mapping $\Omega : \mathbb{S} \times \mathbb{S} \to \mathbb{S}$ is called SB mapping if $\Omega(\rho_1, \rho_2) = \Omega(\rho_2, \rho_1)$ is true for all elements ρ_1, ρ_2 of $\mathbb{S}$. Furtherer, a mapping $\alpha : \mathbb{S} \to \mathbb{S}$ defined by $\alpha(\chi) = \Omega(\chi, \chi)$ is called the trace of a SB mapping $\Omega : \mathbb{S} \times \mathbb{S} \to \mathbb{S}$.

Definition 2.4. Symmetric Bi-Derivation (SBD) [13]

A bi-additive mapping $\xi : \mathbb{S} \times \mathbb{S} \to \mathbb{S}$ is said to be SBD mapping if satisfies the condition $\xi(\chi_1\chi_2, \chi_3) = \chi_1\xi(\chi_2, \chi_3) + \xi(\chi_1, \chi_3)\chi_2$ for all elements χ_1, χ_2, χ_3 of the ring $\mathbb{S}$.

Definition 2.5. Commuting [15]

A mapping $\phi : \mathbb{S} \times \mathbb{S} \to \mathbb{S}$ is commuting on $\mathbb{S}$ if the condition $[\phi(\kappa), \kappa] = 0$ holds for all $\kappa \in \mathbb{S}$.

Definition 2.6. Generalized Bi-Derivation (GBD) [18]

Let $\pi : \mathbb{S} \times \mathbb{S} \to \mathbb{S}$ be a BD mapping. Then the biadditive mapping $\lambda : \mathbb{S} \times \mathbb{S} \to \mathbb{S}$ is GBD mapping (it referred to as (λ, π)) whenever the map $\beta \longrightarrow (\alpha, \beta)$ is a GD mapping of $\mathbb{S}$ associated with π for all element for all element α in addition to that, the map $\alpha \longrightarrow (\alpha, \beta)$ is a GD mapping associated with π for all element β, in the other words, $\lambda(\alpha\beta, \upsilon) = \lambda(\alpha, \upsilon)\beta + \alpha\pi(\beta, \upsilon)$ and $\lambda(\alpha, \beta\upsilon) = \lambda(\alpha, \beta)\upsilon + \beta\pi(\alpha, \upsilon)$ for all $\alpha, \beta, \upsilon \in \mathbb{S}$.

Definition 2.7. Symmetric Generalized Bi-Derivation (SGBD) [19]

A SB bi-additive mapping $\zeta : \mathbb{S} \times \mathbb{S} \to \mathbb{S}$ is said to be SGBD if there exists a SBD $\pi : \mathbb{S} \times \mathbb{S} \to \mathbb{S}$ such that $\zeta(\chi_1\chi_2, \chi_3) = \zeta(\chi_1, \chi_3)\chi_2 + \chi_1\pi(\chi_1, \chi_3)$ for all $\chi_1, \chi_2, \chi_3 \in \mathbb{S}$.

3 Main Theorm

The main focus of this section is to prove the fundamental theorem of this study by utilizing two categories of mappings, namely SGBD with prime ideals. An algebraic identity comprising SGBD mappings and bi-additive mappings with a prime ideal is incorporated in the following theorem, which is an extension of Vukman's result.

Lemma 3.1. *Let Γ be a prime ideal of a ring $\mathbb{S}$ such that $Char(\mathbb{S}/\Gamma) \neq 2$. Suppose that the SGBD mappings (λ, π) and (ζ, π) of $\mathbb{S} \times \mathbb{S}$ and the bi-additive mapping η of $\mathbb{S} \times \mathbb{S}$ are satisfying the statement $\lambda(\ell_1, \ell_1)\ell_1 + \ell_1\zeta(\ell_1, \ell_1) - \eta(\ell_1, \ell_1) \in \Gamma$ for all $\ell_1 \in \mathbb{S}$. Then $2\lambda(\ell_1, \ell_2)\ell_2 + \lambda(\ell_2, \ell_2)\ell_1 + 2\ell_2\zeta(\ell_1, \ell_2) + \ell_1\zeta(\ell_2, \ell_2) \in \Gamma$ for all $\ell_1, \ell_2 \in \mathbb{S}$.*

Proof. Assume that,

$$\lambda(\ell_1, \ell_1)\ell_1 + \ell_1\zeta(\ell_1, \ell_1) - \eta(\ell_1, \ell_1) \in \Gamma \text{ for all } \ell_1 \in \mathbb{S}. \tag{3.1}$$

Linearize Equation (3.1) (i.e, substitute $\ell_1 + \ell_2$ instead of ℓ_1), then for all $\ell_1, \ell_2 \in \mathbb{S}$

$$\begin{aligned}&\lambda(\ell_1,\ell_1)\ell_1 + \lambda(\ell_1,\ell_1)\ell_2 + 2\lambda(\ell_1,\ell_2)\ell_1 + 2\lambda(\ell_1,\ell_2)\ell_2 + \lambda(\ell_2,\ell_2)\ell_1\\ &+ \lambda(\ell_2,\ell_2)\ell_2 + \ell_1\zeta(\ell_1,\ell_1) + \ell_2\zeta(\ell_1,\ell_1) + 2\ell_1\zeta(\ell_1,\ell_2) + 2\ell_2\zeta(\ell_1,\ell_2)\\ &+ \ell_1\zeta(\ell_2,\ell_2) + \ell_2\zeta(\ell_2,\ell_2) - \eta(\ell_1,\ell_1) - 2\eta(\ell_1,\ell_2) - \eta(\ell_2,\ell_2) \in \Gamma.\end{aligned} \tag{3.2}$$

Substituting Equation (3.1) in Equation (3.2) gives the following for all $\ell_1, \ell_2 \in \mathbb{S}$

$$\begin{aligned}&\lambda(\ell_1,\ell_1)\ell_2 + 2\lambda(\ell_1,\ell_2)\ell_1 + 2\lambda(\ell_1,\ell_2)\ell_2 + \lambda(\ell_2,\ell_2)\ell_1 + \ell_2\zeta(\ell_1,\ell_1)\\ &+ 2\ell_1\zeta(\ell_1,\ell_2) + \ell_1\zeta(\ell_2,\ell_2) + 2\ell_2\zeta(\ell_1,\ell_2) - 2\eta(\ell_1,\ell_2) \in \Gamma.\end{aligned} \tag{3.3}$$

Replace ℓ_2 by $-\ell_2$ in Equation (3.3) and again add the result to Equation (3.3) to get the following for all $\ell_1, \ell_2 \in \mathbb{S}$

$$4\lambda(\ell_1,\ell_2)\ell_2 + 2\lambda(\ell_2,\ell_2)\ell_1 + 4\ell_2\zeta(\ell_1,\ell_2) + 2\ell_1\zeta(\ell_2,\ell_2) \in \Gamma. \tag{3.4}$$

In view of the primeness of Γ and $Char(\mathbb{S}/\Gamma)$ not 2 yields

$$2\lambda(\ell_1,\ell_2)\ell_2 + \lambda(\ell_2,\ell_2)\ell_1 + 2\ell_2\zeta(\ell_1,\ell_2) + \ell_1\zeta(\ell_2,\ell_2) \in \Gamma \text{ for all } \ell_1, \ell_2 \in \mathbb{S}. \square$$

Lemma 3.2. *Let Γ be a prime ideal of a ring $\mathbb{S}$ such that $Char(\mathbb{S}/\Gamma) \neq 2$. Suppose that the SGBD mapping (λ, π) of $\mathbb{S} \times \mathbb{S}$ is satisfying the condition $[\wp, \ell]\mathbb{S}\pi(\ell, \ell) \subseteq \Gamma$ for all $\wp, \ell \in \mathbb{S}$. Then either $\mathbb{S}/\Gamma$ is commutative or $\pi(\mathbb{S}, \mathbb{S}) \subseteq \Gamma$.*

Proof. By hypothesis,

$$[\wp, \ell]\mathbb{S}\pi(\ell, \ell) \subseteq \Gamma \text{ for all } \wp, \ell \in \mathbb{S}. \tag{3.5}$$

Using the primeness property of Γ to conclude that either $[\wp, \ell] \in \mathbb{S}$ or $\pi(\ell, \ell) \in \Gamma$ for all $\wp, \ell \in \mathbb{S}$. Let $\Lambda_1 = \{\ell \in \mathbb{S} : [\wp, \ell] \in \Gamma \text{ for all } \wp \in \mathbb{S}\}$ and $\Lambda_2 = \{\ell \in \mathbb{S} : \pi(\ell, \ell) \in \Gamma\}$. Clearly, $\mathbb{S} = \Lambda_1 \cup \Lambda_2$ and Λ_1, Λ_2 are two additive subgroups of $\mathbb{S}$. According to Brauer's trict this is not true. Hence, either $\mathbb{S} = \Lambda_1$ or $\mathbb{S} = \Lambda_2$. The first case implies that $[\wp, \ell] \in \Gamma$ for all $\wp \in \mathbb{S}$ and this means that $\ell + \Gamma \in \mathfrak{C}(\mathbb{S}/\Gamma)$ for all $\ell \in \mathbb{S}$. So, $\mathbb{S}/\Gamma$ is commutative. While the second assumption gives $\pi(\ell, \ell) \in \Gamma$ for all $\ell \in \mathbb{S}$. Linearize the last relation on ℓ and in view of $Char(\mathbb{S}/\Gamma) \neq 2$ follows that $\pi(\omega, \eta) \in \Gamma$ for all $\omega, \eta \in \mathbb{S}$. That is, $\pi(\mathbb{S}, \mathbb{S}) \subseteq \Gamma$. $\square$

The following theorem generalizes Vukman's result by utilizing an algebraic equation that incorporates two mappings for SGBD.

Theorem 3.3. *Let Γ be a prime ideal of a ring $\mathbb{S}$ such that $Char(\mathbb{S}/\Gamma) \neq 2$. Suppose that the SGBD mappings (λ, π) and (ζ, π) of $\mathbb{S}\times\mathbb{S}$ and the bi-additive mapping η of $\mathbb{S} \times \mathbb{S}$ are satisfying the statement $\lambda(\ell_1, \ell_1)\ell_1 + \ell_1\zeta(\ell_1, \ell_1) - \eta(\ell_1, \ell_1) \in \Gamma$ for all $\ell_1 \in \mathbb{S}$. Then either $\mathbb{S}/\Gamma$ is commutative or $\pi(\mathbb{S}, \mathbb{S}) \subseteq \Gamma$.*

Proof. Suppose that, $\lambda(\ell_1, \ell_1)\ell_1 + \ell_1\zeta(\ell_1, \ell_1) - \eta(\ell_1, \ell_1) \in \Gamma$ for all $\ell_1 \in \mathbb{S}$. Then Lemma 3.1 gives

$$2\lambda(\ell_1, \ell_2)\ell_2 + \lambda(\ell_2, \ell_2)\ell_1 + 2\ell_2\zeta(\ell_1, \ell_2) + \ell_1\zeta(\ell_2, \ell_2) \in \Gamma \text{ for all } \ell_1, \ell_2 \in \mathbb{S}. \quad (3.6)$$

Substituting $\ell_1\ell_3$ for ℓ_1 in Equation (3.6) implies that

$$\begin{aligned} &2\lambda(\ell_1, \ell_2)\ell_3\ell_2 + 2\ell_1\pi(\ell_3, \ell_2)\ell_2 + \lambda(\ell_2, \ell_2)\ell_1\ell_3 + 2\ell_2\zeta(\ell_1, \ell_2)\ell_3 \\ &+ 2\ell_2\ell_1\pi(\ell_3, \ell_2) + \ell_1\ell_3\zeta(\ell_2, \ell_2) \in \Gamma \text{ for all } \ell_1, \ell_2 \in \mathbb{S}. \end{aligned} \quad (3.7)$$

Multiply Equation (3.6) by ℓ_3 from the right and subtract the result from Equation (3.7) leads to the following for all $\ell_1, \ell_2, \ell_3 \in \mathbb{S}$

$$2\lambda(\ell_1, \ell_2)[\ell_2, \ell_3] + \ell_1[\zeta(\ell_2, \ell_2), \ell_3] - 2\ell_1\pi(\ell_3, \ell_2)\ell_2 - 2\ell_2\ell_1\pi(\ell_3, \ell_2) \in \Gamma. \quad (3.8)$$

Putting $\ell_4\ell_1$ for ℓ_1 in Equation (3.8) gives

$$\begin{aligned} &2\lambda(\ell_4, \ell_2)\ell_1[\ell_2, \ell_3] + 2\ell_4\pi(\ell_1, \ell_2)[\ell_2, \ell_3] + \ell_4\ell_1[\zeta(\ell_2, \ell_2), \ell_3] \\ &- 2\ell_4\ell_1\pi(\ell_3, \ell_2)\ell_2 - 2\ell_2\ell_4\ell_1\pi(\ell_3, \ell_2) \in \Gamma \text{ for all } \ell_1, \ell_2, \ell_3, \ell_4 \in \mathbb{S}. \end{aligned} \quad (3.9)$$

Now, multiply Equation (3.8) from left side by ℓ_4 and comparing it with Equation (3.9) to have

$$\begin{aligned} &2\ell_4\lambda(\ell_1, \ell_2)[\ell_2, \ell_3] - 2\ell_4\ell_2\ell_1\pi(\ell_3, \ell_2) - 2\lambda(\ell_4, \ell_2)\ell_1[\ell_2, \ell_3] \\ &- 2\ell_4\pi(\ell_1, \ell_2)[\ell_2, \ell_3] + 2\ell_2\ell_4\ell_1\pi(\ell_3, \ell_2) \in \Gamma \text{ for all } \ell_1, \ell_2, \ell_3, \ell_4 \in \mathbb{S}. \end{aligned} \quad (3.10)$$

Putting $\ell_2 = \ell_3$ in Equation (3.10) and using the property of Γ to conclude that $[\ell_2, \ell_4]\ell_1\pi(\ell_2, \ell_2) \in \Gamma$ for all $\ell_1, \ell_2, \ell_4 \in \mathbb{S}$. That is, $[\ell_2, \ell_4]\mathbb{S}\pi(\ell_2, \ell_2) \subseteq \Gamma$ for all $\ell_2, \ell_4 \in \mathbb{S}$. According to Lemma 3.2, it is feasible to obtain the needed result. □

Especially, in Theorem 3.3 if λ and ζ are considered as GSBD mappings such that $\lambda = -\zeta$, it follows from Theorem 3.3 that the subsequent corollary holds.

Corollary 3.4. *Let Γ be a prime ideal of a ring $\mathbb{S}$ such that $Char(\mathbb{S}/\Gamma) \neq 2$. Suppose that the SGBD mapping (λ, π) and the bi-additive mapping η of $\mathbb{S}\times\mathbb{S}$ are satisfying the statement $[\lambda(\ell_1, \ell_1), \ell_1] - \eta(\ell_1, \ell_1) \in \Gamma$ for all $\ell_1 \in \mathbb{S}$. Then the next statments are fulfilled:*

(i) $\eta(\mathbb{S},\mathbb{S}) \subseteq \Gamma$.

(ii) Either $\mathbb{S}/\Gamma$ *is commutative or* $\pi(\mathbb{S},\mathbb{S}) \subseteq \Gamma$ *is true.*

Proof. Assume that,

$$[\lambda(\ell_1,\ell_1),\ell_1] - \eta(\ell_1,\ell_1) \in \Gamma \text{ for all } \ell_1 \in \mathbb{S}. \tag{3.11}$$

(i) In Theorem 3.3 substitute $\lambda = -\zeta$ and $\ell_2 = \ell_3$ in Equation (3.8) to have

$$\ell_1[\lambda(\ell_2,\ell_2),\ell_2] + 2\ell_1\pi(\ell_2,\ell_2)\ell_2 + 2\ell_2\ell_1\pi(\ell_2,\ell_2) \in \Gamma \text{ for all } \ell_1,\ell_2 \in \mathbb{S}. \tag{3.12}$$

Left Multiplication of Equation (3.11) by ℓ_1 and comparing the result with Equation (3.12) yields

$$2\ell_1\pi(\ell_2,\ell_2)\ell_2 + 2\ell_2\ell_1\pi(\ell_2,\ell_2) + \ell_1\eta(\ell_2,\ell_2) \in \Gamma \text{ for all } \ell_1,\ell_2 \in \mathbb{S}. \tag{3.13}$$

Replacing ℓ_2 by $-\ell_2$ in Equation (3.13) leads to

$$-2\ell_1\pi(\ell_2,\ell_2)\ell_2 - 2\ell_2\ell_1\pi(\ell_2,\ell_2) + \ell_1\eta(\ell_2,\ell_2) \in \Gamma \text{ for all } \ell_1,\ell_2 \in \mathbb{S}. \tag{3.14}$$

Adding Equation (3.13) and Equation (3.14) with applying the hypothesis of Γ and $Char(\mathbb{S}/\Gamma)$, then $\eta(\ell_2,\ell_2) \in \Gamma$ for all $\ell_2 \in \mathbb{S}$. Making the last equation linear and using $Char(\mathbb{S}/\Gamma) \neq 2$, we get $\eta(\mathbb{S},\mathbb{S}) \subseteq \Gamma$.
(ii) This follows immediately from Theorem 3.3 by considering $\lambda = -\zeta$. □

In particular, in Corollary 3.4 if $\Gamma = (0)$ is a prime ideal and the bi-addtive mapping η is zero, then one can get the next corollary.

Corollary 3.5. *Let* $\mathbb{S}$ *be a noncommutative prime ring of* $Char(\mathbb{S}) \neq 2$. *Suppose that the SGBD mappings* (λ,π) *of* $\mathbb{S}\times\mathbb{S}$ *that satisfies the condition* $[\lambda(\ell,\ell),\ell] = 0$ *for all* $\ell \in \mathbb{S}$. *Then* π *equals zero.*

If $\lambda = \pi$ is considered as SBD mapping in Corollary 3.5, it can be inferred that the subsequent corollary follows directly from Corollary 3.5.

Corollary 3.6. *[16, Theorem 1] Let* $\mathbb{S}$ *be a noncommutative prime ring of* $Char \neq 2$. *Let* π *be a SBD mapping of* $\mathbb{S}\times\mathbb{S}$ *and* $[\pi(\ell,\ell),\ell] = 0$ *for all* $\ell \in \mathbb{S}$. *Then* $\pi = 0$.

Proposition 3.7. *Consider a prime ideal* Γ *of the ring* $\mathbb{S}$ *and* $\pi : \mathbb{S}\times\mathbb{S} \longrightarrow \mathbb{S}$ *be a SBD mapping such that* $\pi(\eta,\eta) \in \Gamma$ *for all element* η *of a nonzero ideal* $\mathcal{A}$ *of* $\mathbb{S}$ *doesn't containes* Γ. *Then either* $Char(\mathbb{S}/\Gamma) \neq 2$ *or* $\pi(\mathbb{S},\mathbb{S}) \subseteq \Gamma$.

Proof. Assume that $Char(\mathbb{S}/\Gamma) \neq 2$ and

$$\pi(\eta, \eta) \in \Gamma \text{ for all } \eta \in \mathcal{A}. \tag{3.15}$$

Making Equation (3.15)linear and using the assumption, we get

$$\pi(\eta, \gamma) \in \Gamma \text{ for all } \eta, \gamma \in \mathcal{A}. \tag{3.16}$$

In Equation (3.16), substitute $\upsilon\eta$ instead of η where $\upsilon \in \mathbb{S}$ and apply [22, Fact 1] to obtain $\pi(\upsilon, \gamma) \in \Gamma$ for all $\eta \in \Gamma$. Similarly, replace η by $\omega\eta$ for $\omega \in \mathbb{S}$ in the last relation to conclude that $\pi(\mathbb{S}, \mathbb{S}) \subseteq \Gamma$. $\square$

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