International Journal of Mathematics and
Computer Science, **18**(2023), no. 4, 637–645

# $m$-Polar Fuzzy BRK-ideals and BRK-algebras

**Areej Almuhaimeed**

Department of Mathematics
College of Science
Taibah University
Madinah, Saudi Arabia

email: aamuhaimeed@taibahu.edu.sa

(Received March 6, 2023, Accepted April 19, 2023,
Published May 31, 2023)

**Abstract**

In this paper, we introduce the concept of an $m$-polar BRK-algebra and investigate the idea of $m$-polar fuzzy BRK-subalgebras and its properties. Moreover, we present $m$-polar fuzzy BRK-ideals with some of their characteristics. Furthermore, we provide some conditions that connect $m$-polar fuzzy BRK-subalgebras and $m$-polar fuzzy BRK-ideals.

## 1 Introduction

Fuzzy sets, defined by Zadeh [1], are generalizations of classical sets. This idea applied on several classes of algebras and developed some fundamental characteristics of these algebras.

In 1965, two new classes of algebras were intoduced; namely, BCK-algebra and BCI-algebra [2]. Many other algebraic classes were then introduced and investigated [8], [9], [11] and [10].

Bandaru [3] introduced a generalization of BCK-algebra and BCI-algebra which is the class of BRK-algebra. Elgendy [4] applied fuzzy sets on BRK-algebras and BRK-ideals and applied the concept of cubic on BRK-algebras

**Key words and phrases:** Fuzzy BRK-subalgebras, fuzzy BRK-ideals-
$m$-polar fuzzy sets.
**AMS (MOS) Subject Classifications**: 08A72, 03B52.
**ISSN** 1814-0432, 2023, http://ijmcs.future-in-tech.net

and BRK-ideals [5]. In addition, anti fuzzy BRK-ideals was investigated [7]. In [6], the concept of bipolar fuzzy was investigated on BRK-ideals and BRK-algebras.

In this paper, we introduce the concept of an $m$-polar BRK-algebra. First, we investigate $m$-polar fuzzy BRK-subalgebras and its properties. In addition, $m$-polar fuzzy BRK-ideals are presented with its characteristics. Finally, we provide some conditions that connect $m$-polar fuzzy BRK-subalgebras and $m$-polar fuzzy BRK-ideals.

## 2 Preliminaries

In this part, we present the basic notions regarding BRK-algebra which will be used in later sections. The results in this section are taken from [3] and [4].

We first present the class of BRK-algebras.

**Definition 2.1.** *Let $A \neq \phi$. Define a binary operation on the set $A$, $*$ and consider a constant $0 \in A$. If $(A, 0, *)$ satisfies the following criteria, it is referred to as a BRK-algebra:*

*(1) $f * 0 = f$, for all $f \in A$.*

*(2) $(f * g) * f = 0 * g$, for all $f, g \in A$.*

*We also define a partial ordered relation on $(A, 0, *)$ by*

$$f \leq g \Leftrightarrow f * g = 0$$

Recall that the following characteristics are true if $(A, 0, *)$ is a BRK-algebra:

(1) $f * f = 0$, for all $f \in A$.

(2) $0 * (f * g) = (0 * f) * (0 * g)$, for every $f, g \in A$.

**Definition 2.2.** *Suppose that $\phi \neq B \subseteq A$, where $A$ be a BRK-algebra. Then $B$ is referred to as a BRK-subalgebra of $A$ if*

$$f * g \in B, \qquad \forall f, g \in B.$$

**Definition 2.3.** *Let $A$ be a BRK-algebra. Suppose that $\phi \neq C \subseteq A$. Then $C$ is called a BRK-ideal of $A$ if:*

*(1)* $0 \in C$.

*(2)* *If* $0 * (f * g) \in C$ *and* $0 * g \in C$, *then* $0 * f \in C$, *for all* $f, g \in A$.

Now, we are ready to apply fuzzy settings on BRK-algebra.

**Definition 2.4.** *Assume that $A$ is a BRK-algebra and $\zeta$ a fuzzy subset of $A$. Then $\zeta$ is referred to as a fuzzy BRK-subalgebra of $A$ if*

$$\zeta(f * g) \geq \min\{\zeta(f), \zeta(g)\}, \quad \forall f, g \in A$$

**Definition 2.5.** *Let $A$ be a BRK-algebra and $\zeta$ a fuzzy subset of $A$. Then $\zeta$ is is referred to as a fuzzy BRK-ideal of $A$ if*

*(1)* $\zeta(0) \geq \zeta(f)$, *for all* $f \in A$.

*(2)* $\zeta(0 * f) \geq \min\{\zeta(0 * (f * g)), \zeta(0 * g)\}$, *for all* $f, g \in A$

In [12], the notation $m$-polar fuzzy set is explained. Assume that $A \neq \phi$. Then the function

$$\zeta : A \longrightarrow [0, 1]^m$$

is referred to as an $m$-polar fuzzy set of $A$,

$$\zeta(f) := \{(\xi_1 \circ \zeta)(f), (\xi_2 \circ \zeta)(f), \cdots, (\xi_m \circ \zeta)(f)\},$$

where

$$\xi_k : [0, 1]^m \longrightarrow [0, 1]$$

is the $k^{th}$ projection for all $k \in \{1, \cdots, m\}$.

# 3 $m$-polar fuzzy BRK-subalgebras

**Definition 3.1.** *Assume that $A$ is a BRK-algebra. If an $m$-polar fuzzy subset zeta of $A$ fulfills the criteria:*

$$\zeta(f * g) \geq \min\{\zeta(f), \zeta(g)\}, \quad \forall f, g \in A,$$

*then $\zeta$ is an $m$-polar fuzzy BRK-algebra:*

**Example 1.** *Let $A = \{0, f, g, h\}$ be a BRK-algebra presented by the following table:*

| $*$ | $0$ | $f$ | $g$ | $h$ |
|---|---|---|---|---|
| $0$ | $0$ | $f$ | $0$ | $f$ |
| $f$ | $f$ | $0$ | $f$ | $0$ |
| $g$ | $g$ | $f$ | $0$ | $f$ |
| $h$ | $h$ | $g$ | $h$ | $0$ |

*BRK-algebra in example 1*

*Consider the 2-polar fuzzy set $\zeta : A \longrightarrow [0,1]^2$ described as:*

$$\zeta(0) = (0.5, 0.8)$$
$$\zeta(f) = (0.4, 0.6)$$
$$\zeta(g) = (0.3, 0.7)$$
$$\zeta(h) = (0.4, 0.5)$$

Then $\zeta$ is a 2-polar fuzzy BRK-subalgebra of $A$.

Now, we introduce some propositions regarding properties of $m$-polar fuzzy BRK-subalgebras.

**Proposition 3.2.** *If $A$ is a BRK-algebra, then any $m$-polar fuzzy subalgebra of $A$ satisfies;*

$$\zeta(0) \geq \zeta(f) \quad \forall f \in A.$$

*Proof.* Since $A$ is a BRK-algebra, then $f * f = 0$. Thus, for all $f \in A$, we have

$$\begin{aligned} \zeta(0) &= \zeta(f * f) \\ &\geq \min\{\zeta(f) * \zeta(f)\} \\ &= \zeta(f) \end{aligned}$$

□

**Proposition 3.3.** *Let $A$ be a BRK-algebra. Suppose that for any $m$-polar fuzzy subalgebra $\zeta$ of $A$,*

$$\zeta(f * g) \geq \zeta(g) \quad \forall f, g \in A.$$

*Then $\zeta(f) = \zeta(0)$.*

*Proof.* Since $A$ is a BRK-algebra, then $a * 0 = a$. Thus, for all $a \in A$, we obtain

$$\begin{aligned} \zeta(f) &= \zeta(f * 0) \\ &\geq \zeta(0) \qquad \text{( By assumption)} \end{aligned}$$

By proposition 3.2, $\zeta(0) \geq \zeta(f)$. Hence $\zeta(f) = \zeta(0)$. □

# 4 $m$-polar fuzzy BRK-ideals

The $m$-polar fuzzy BRK-ideals and its properties are presented in this section. In addition, we provide some conditions that connect $m$-polar fuzzy BRK-subalgebras and $m$-polar fuzzy BRK-ideals

**Definition 4.1.** *Assume that $A$ is a BRK-algebra and $\zeta$ an $m$-polar fuzzy subset of $A$. Then $\zeta$ is said to be an $m$-polar fuzzy BRK-ideal of $A$ if*

*(1)* $\zeta(0) \geq \zeta(f)$, *for all* $f \in A$.

*(2)* $\zeta(0 * f) \geq \min\{\zeta(0 * (f * g)), \zeta(0 * g)\}$, *for all* $f, g \in A$.

Now, we prove some propositions regarding several properties of $m$-polar fuzzy BRK-ideals.

**Proposition 4.2.** *Assume that $A$ is a BRK-algebra and $\zeta$ is an $m$-polar fuzzy BRK-ideal. Now,*

$$0 * f \leq 0 * g \text{ implies that } \zeta(0 * f) \geq \zeta(0 * g), \quad f, g \in A.$$

*Proof.* Suppose that $0 * f \leq 0 * g$. Then

$$(0 * f) * (0 * g) = 0.$$

From the properties of BRK-algebras, we obtain

$$0 * (f * g) = 0.$$

That $*$ is an $m$-polar fuzzy BRK-ideal, implies

$$\begin{aligned} \zeta(0 * f) &\geq \min\{\zeta(0 * (f * g)), \zeta(0 * g)\} \\ &= \min\{\zeta(0), \zeta(0 * g)\} \\ &= \zeta(0 * g) \end{aligned}$$

Therefore, $\zeta(0 * f) \geq \zeta(0 * g)$ as required. □

**Proposition 4.3.** *Assume that $A$ is a BRK-algebra and $\zeta$ an $m$-polar fuzzy BRK-ideal. Then*

$$f * g \leq h \text{ implies that } \zeta(0 * (f * g)) \geq \zeta(0 * h), \quad f, g, h \in A.$$

*Proof.* Assume that $af * g \leq h$. Then

$$(f * g) * h = 0.$$

By assumption, we obtain

$$\begin{aligned}\zeta(0 * (f * g)) &\geq \min\{\zeta(0 * ((f * g) * h), \zeta(0 * h)\}\\ &= \min\{\zeta(0 * 0), \zeta(0 * h)\}\\ &= \min\{\zeta(0), \zeta(0 * h)\}\\ &= \zeta(0 * h)\end{aligned}$$

Therefore,

$$\zeta(0 * (f * g)) \geq \zeta(0 * h)$$

as required. □

**Theorem 4.4.** *Assume that $A$ is a BRK-algebra and $\zeta$ is an $m$-polar fuzzy BRK-ideal of $A$. If*

$$0 * f = f, \quad \forall f \in A,$$

*$\zeta$ is an $m$-polar fuzzy BRK-subalgebra of $A$*

*Proof.* Suppose that

$$0 * f = f, \quad \forall f \in A.$$

Then we obtain

$$\begin{aligned}\zeta(f * g) &= \zeta(0 * (f * g))\\ &\geq \min\{\zeta(0 * ((f * g) * h)), \zeta(0 * h)\} && (\zeta \text{ is an } m\text{-polar fuzzy BRK-ideal})\\ &= \min\{\zeta(((f * g) * h), \zeta(h)\} && (\text{ By assumption})\\ &= \min\{\zeta(((f * g) * f), \zeta(f)\} && (\text{ Set } h = f)\\ &\geq \min\{\zeta(0 * g), \zeta(f)\}\\ &= \min\{\zeta(g), \zeta(f)\} && (\text{ By assumption})\\ &= \min\{\zeta(f), \zeta(g)\}\end{aligned}$$

Therefore, $\zeta$ is an $m$-polar fuzzy BRK-subalgebra of $A$ □

**Theorem 4.5.** *Assume that $A$ is a BRK-algebra and $\zeta$ is an $m$-polar fuzzy BRK-subalgebra of $A$. If*

$$(f * g) * f = f, \quad \forall f, g \in A,$$

*$\zeta$ is an $m$-polar fuzzy BRK-ideal of $A$.*

*Proof.* Suppose that

$$(f * g) * f = f, \quad \forall f, g \in A,$$

Then

(1)

$$\begin{aligned}\zeta(0) &= \zeta(f * f)\\ &\geq \min\{\zeta(f), \zeta(f)\}\\ &= \zeta(f)\end{aligned}$$

(2) That $A$ is a BRK-algebra, implies

$$(f * g) * f = 0 * g.$$

By assumption,

$$(f * g) * f = f.$$

Thus $f = 0 * g$. Now, we have

$$\begin{aligned}\zeta(0 * f) &\geq \min\{\zeta(0), \zeta(f)\}\\ &\geq \min\{\zeta(0 * (f * g)), \zeta(f)\}\\ &= \min\{\zeta(0 * (f * g)), \zeta(0 * g)\}\end{aligned}$$

Therefore, $\zeta$ is an $m$-polar fuzzy BRK-ideal of $A$. □

**Theorem 4.6.** *Assume that $A$ is a BRK-algebra. If*

$$0 * f = 0, \quad \forall f \in A,$$

*then any $m$-polar fuzzy subset of $A$ is an $m$-polar fuzzy BRK-ideal of $A$.*

*Proof.* Suppose that

$$0 * f = 0, \quad \forall f \in A,$$

Then

(1) We have

$$\begin{aligned}\zeta(0) &= \zeta(f * f)\\ &\geq \min\{\zeta(f), \zeta(f)\}\\ &= \zeta(f).\end{aligned}$$

(2) Note that

$$\begin{aligned}\zeta(0*f) &= \zeta(0)\\ &= \min\{\zeta(0),\zeta(0)\}\\ &= \min\{\zeta(0*(f*g)),\zeta(0*g)\} \qquad \text{( By assumption)}\end{aligned}$$

Therefore, $\zeta$ is an $m$-polar fuzzy BRK-ideal of $A$. □

**Theorem 4.7.** *Assume that $A$ is a BRK-algebra, $\zeta$ is an $m$-polar fuzzy BRK-ideal of $A$ and*

$$D = \{f \in A : \zeta(f) = \zeta(0)\}.$$

*Then $D$ is an $m$-polar BRK-ideal.*

*Proof.* Assume that

$$0*(f*g) \in D \text{ and } 0*g \in D.$$

By the definition of $D$, we obtain

$$\zeta(0*(f*g)) = \zeta(0),$$

$$\zeta(0*g) = \zeta(0).$$

Now, we have

$$\begin{aligned}\zeta(0*f) &\geq \min\{\zeta(0*(f*g)),\zeta(0*g)\}\\ &= \min\{\zeta(0),\zeta(0)\}\\ &= \zeta(0)\end{aligned}$$

But $\zeta(0) \geq \zeta(0*f)$. Thus

$$\zeta(0*f) = \zeta(0).$$

Hence $f \in D$, and therefore, $D$ is an $m$-polar BRK-ideal. □

# 5 Conclusion

In this paper, we introduced the aspect of an $m$-polar BRK-algebra. First, we investigated $m$-polar fuzzy BRK-subalgebras and its properties. Secondly, we presented $m$-polar fuzzy BRK-ideals with their characteristics. Finally, we provided some conditions that connect $m$-polar fuzzy BRK-subalgebras and $m$-polar fuzzy BRK-ideals

# References


[1] Lotfi A. Zadeh, Fuzzy sets, Information and control, **8**, no. 3, (1965), 338–353.

[2] Yasuyuki Imai, Kiyoshi Iséki, On axiom systems of propositional calculi. I, Proceedings of the Japan Academy, **41**, no. 6, (1965), 436–439.

[3] Ravi Kumar Bandaru, On BRK-algebras, International Journal of Mathematics and Mathematical Sciences, 2012.

[4] Osama Rashad Elgendy, Fuzzy BRK-ideal of BRK-algebra, JP Journal Algebra and Number Theory and Applications , **36**, no. 3, (2015), 231–240.

[5] Osama Rashad Elgendy, Cubic BRK-ideal of BRK-algebra, Ann. Fuzzy Math. Inf., **11**, (2016), 1–9.

[6] Khizar Hayat, Xiao-Chu Liu, Bing-Yuan Cao, Bipolar fuzzy BRK-ideals in BRK-algebras, Fuzzy Information and Engineering and Decision, (2018), 3–15.

[7] Osama Rashad Elgendy, Anti fuzzy BRK-ideal of BRK-algebra, British J. math. Comp. Sci., **10**, no. 6, (2015), 1–9.

[8] Chang Bum Kim, Hee Sik Kim On BG-algebras, Demonstratio Mathematica, **41**, no. 3, (2008), 497–506.

[9] Joseph Neggers, Kim Sik, On B-algebras, Matematički vesnik, **54**, nos. 1-2, (2002), 21–29.

[10] Joseph Neggers, Sun Shin Ahn, Hee Sik Kim, On Q-algebras,International Journal of Mathematics and Mathematical Sciences, **27**, no. 12, (2001), 749–757.

[11] Andrzej Walendziak, On BF-algebras, Mathematica Slovaca, **57**, (2007), 119–128.

[12] Juanjuan Chen, Shenggang Li, Shengquan Ma, Xueping Wang, Polar fuzzy sets: an extension of bipolar fuzzy sets, The scientific world journal, (2014).